Causal Data Science:

Estimating Identifiable Causal Effects

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References

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   Unified Covariate Adjustment for Causal Inference

5. **Jung, Y.**, Park, W., and Lee, S. (NeurIPS-2024)
   Complete Graphical Criterion for Sequential Covariate Adjustment in Causal Inference

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6. **Jung, Y.**, Tian, J. and Bareinboim, E.. (AAAI-2020)
   Estimating Causal Effects using Weighting-based Estimators.
7. **Jung, Y.**, Tian, J. and Bareinboim, E. (NeurIPS-2020)
   Learning Causal Effects via Weighted Empirical Risk Minimization.
8. **Jung, Y.**, Tian, J. and Bareinboim, E. (ICML-2021)
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9. **Jung, Y.**, Tian, J. and Bareinboim, E. (NeurIPS-2021)
   Double Machine Learning Density Estimation for Local Treatment Effects with Instruments.
10. **Jung, Y.**, Kasiviswanathan, S., Tian, J., Janzing D., Blöbaum, P., Bareinboim, E. (ICML-2022)
    On Measuring Causal Contributions via do-Interventions.
11. **Jung, Y*.** and Bellot, A*. (NeurIPS-2024)
    Efficient Policy Evaluation Across Multiple Different Experimental Datasets

“ *Remdesivir use is associated with lower mortality in patients with COVID*

Clinical Infectious Diseases, 2019

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CNN, 2020

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CNN, 2020

“ *WHO recommends against use of Remdesivir for COVID patients*

CNN, 2020

What’s going on?

Story Behind the Data

Observational Study (FDA)

	Mortality Rate
Remdesivir	11%
Non Remdesivir	20%

Positive Correlation with Lower Mortality

vs.

Randomized Trial (WHO)

	Mortality Rate
Remdesivir	15%
Non Remdesivir	15%

No Causal Effect to Lower Mortality

Story Behind the Data

Since Remdesivir costs over $2000, wealthier patients are more likely to receive it.

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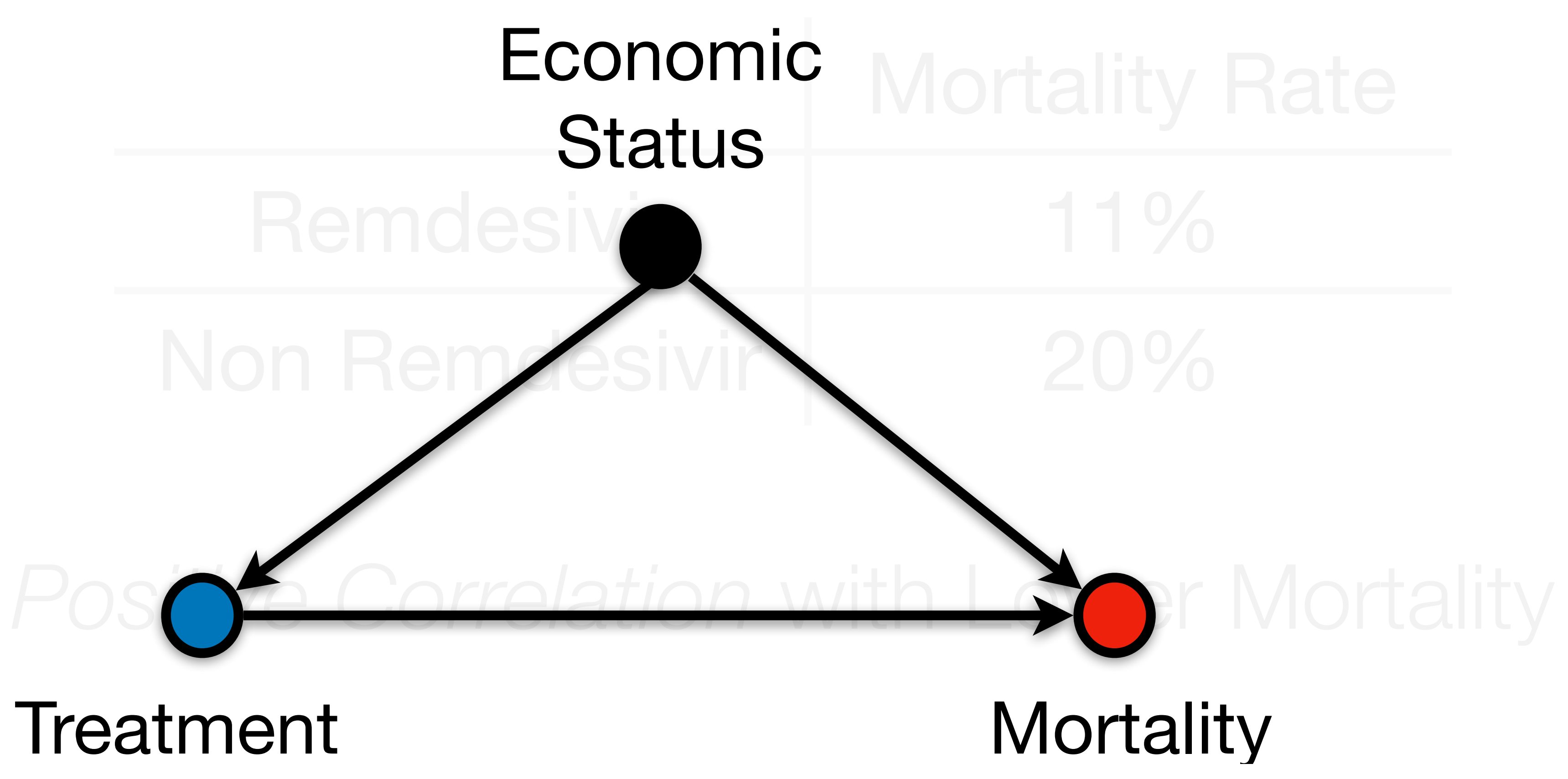

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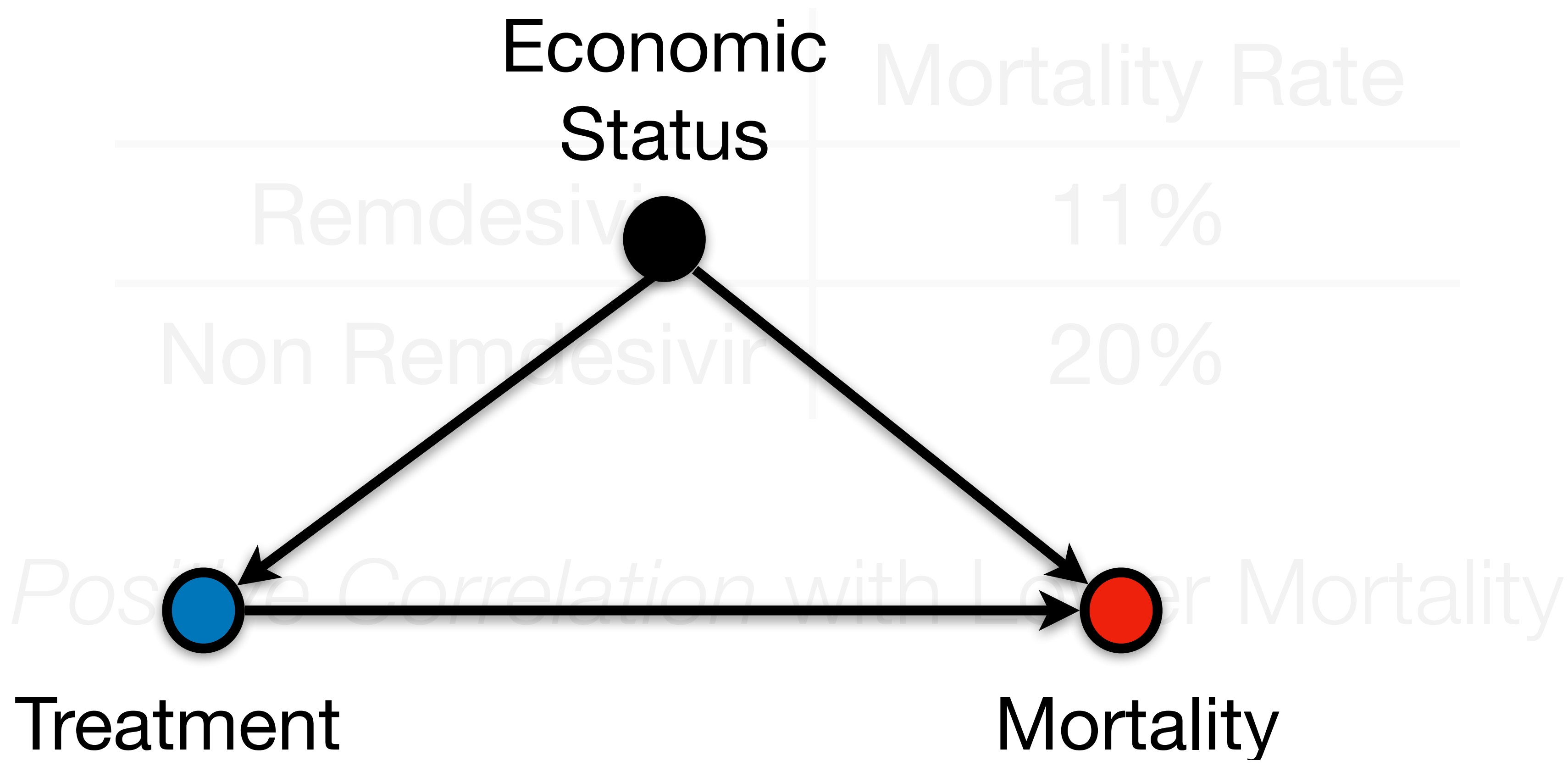

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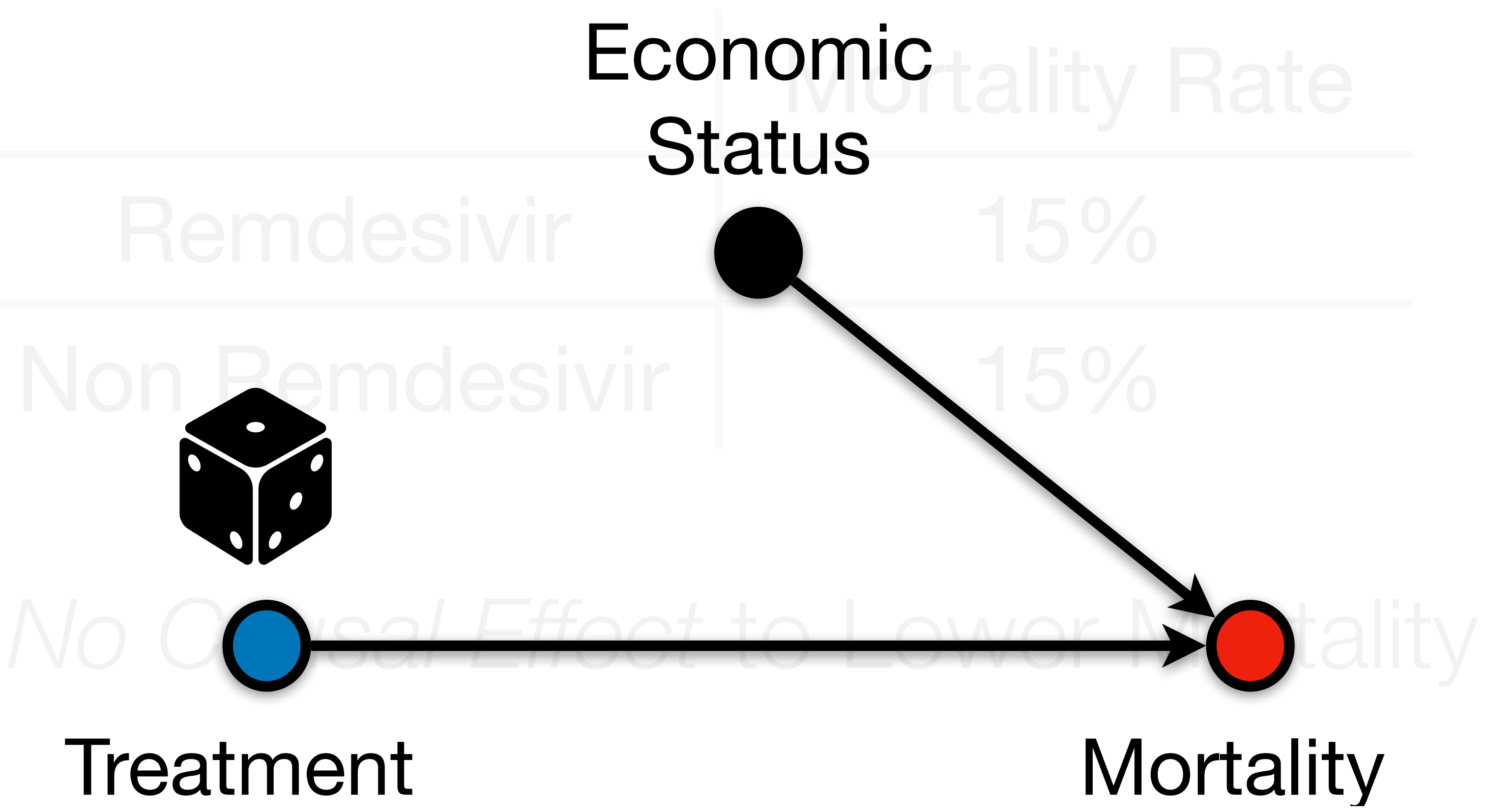

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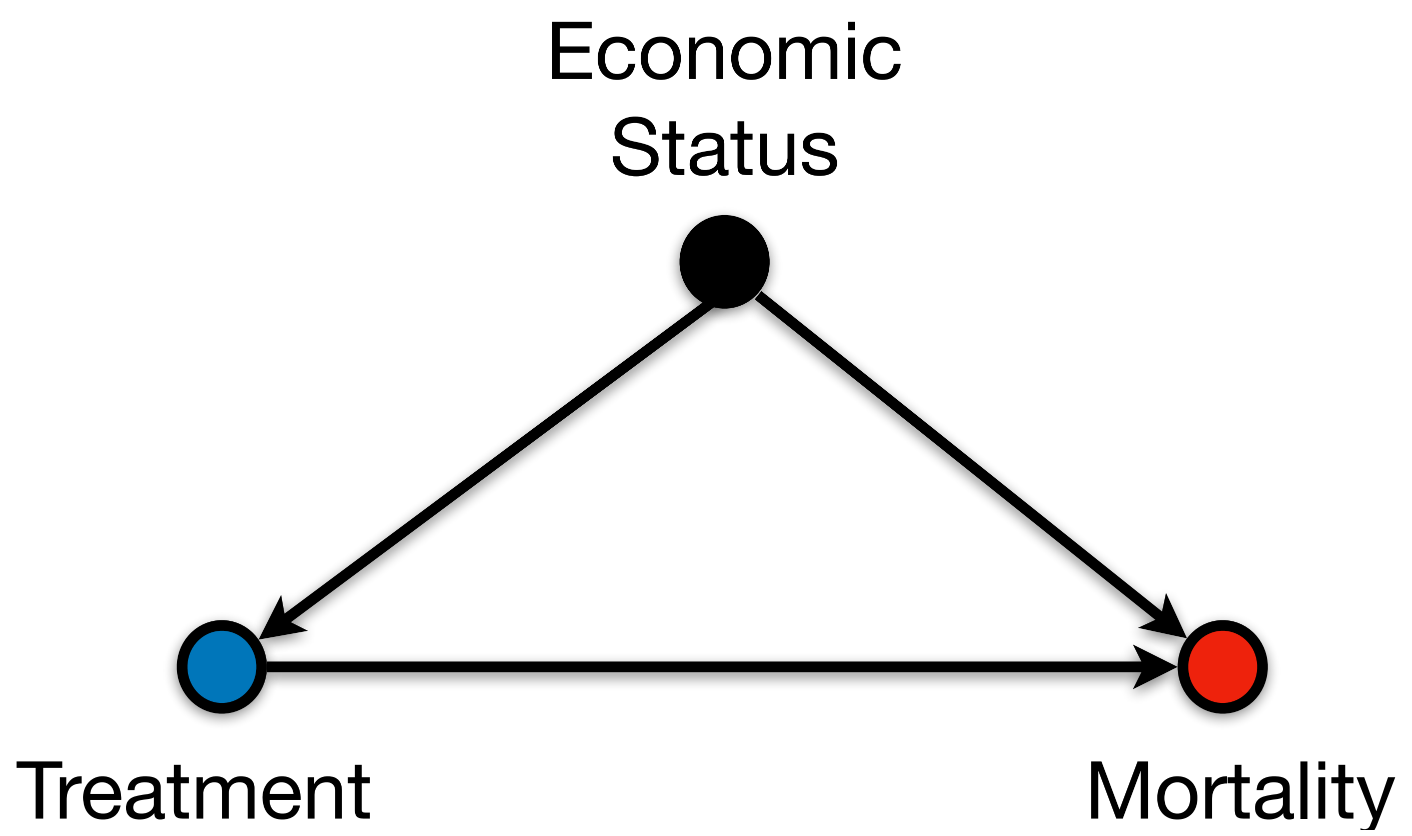

"Causal Inference Engine"

Causal Effect

	Mortality Rate
Remdesivir	15%
Non Remdesivir	15%

Structural Causal Models & Causal Graph

Structural Causal Models & Causal Graph

[Def 1] Structural Causal Model

(Pearl 95)

A structural causal model (SCM) is a 4-tuple $\langle \mathbf{V}, \mathbf{U}, \mathbf{F}, P(\mathbf{U}) \rangle$ where

- $\mathbf{V} = \{V_1, \ldots, V_n\}$ are endogenous variables;
- $\mathbf{U} = \{U_1, \ldots, U_m\}$ are exogenous variables;
- $\mathbf{F} = \{f_1, \ldots, f_n\}$are functions determining $\mathbf{V}$ ($V_i \leftarrow f_i(\mathbf{PA}_i, \mathbf{U}_i)$ for $\mathbf{PA}_i \subseteq \mathbf{V}$, $\mathbf{U}_i \subseteq \mathbf{U}$)
- $P(\mathbf{U})$ is a distribution over $\mathbf{U}$.

Structural Causal Models & Causal Graph

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$$\mathbf{U} = \{U_1, U_2, U_3\}, \mathbf{V} = \{X, Y, Z\},$$

$$\mathbf{F} = \begin{cases} Z & \leftarrow U_1 \\ X & \leftarrow U_1 \oplus U_2 \oplus Z \\ Y & \leftarrow X \oplus Z \oplus U_3 \end{cases}$$

and $\mathbf{U} = \{U_1, U_2, U_3\}$ are independent.

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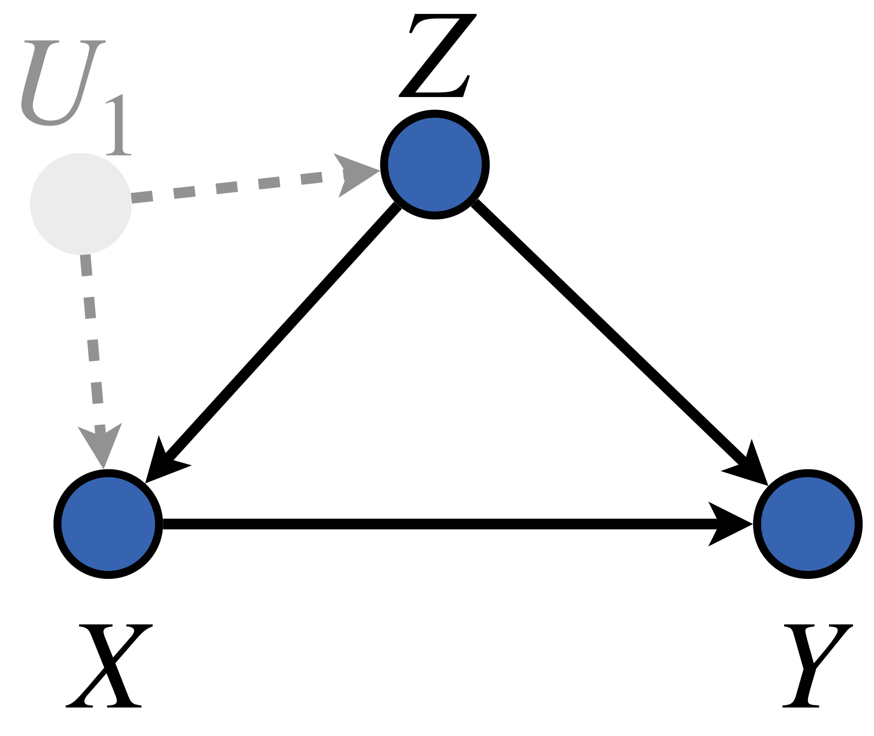

Causal Diagram $\mathcal{G}$

Do-Interventions & Counterfactual

Do-Interventions & Counterfactual

[Def 4] Intervention on $\mathbf{X} = \mathbf{x}$

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For an SCM $\mathscr{M} \triangleq \langle \mathbf{V}, \mathbf{U}, \mathbf{F}, P(\mathbf{U}) \rangle$, an *intervention* is to replace $\mathbf{F}$ to
$\mathbf{F}_{\mathbf{x}} \triangleq \{f_i : V_i \notin \mathbf{X}\} \cup \{\mathbf{X} \leftarrow \mathbf{x}\}$ (“do($\mathbf{x}$)”),

which induces an *interventional SCM*
$\mathscr{M}_{\mathbf{x}} \triangleq \langle \mathbf{V}, \mathbf{U}, \mathbf{F}_{\mathbf{x}}, P(\mathbf{U}) \rangle$

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Potential Response (Pearl 2000)

The potential response of $\mathbf{Y} \subseteq \mathbf{V}$ to an intervention $\text{do}(\mathbf{x})$ is $\mathbf{Y}_{\mathbf{x}} \triangleq \mathbf{Y}_{\mathscr{M}_{\mathbf{x}}}$, induced by the interventional SCM

Do-Interventions & Counterfactual

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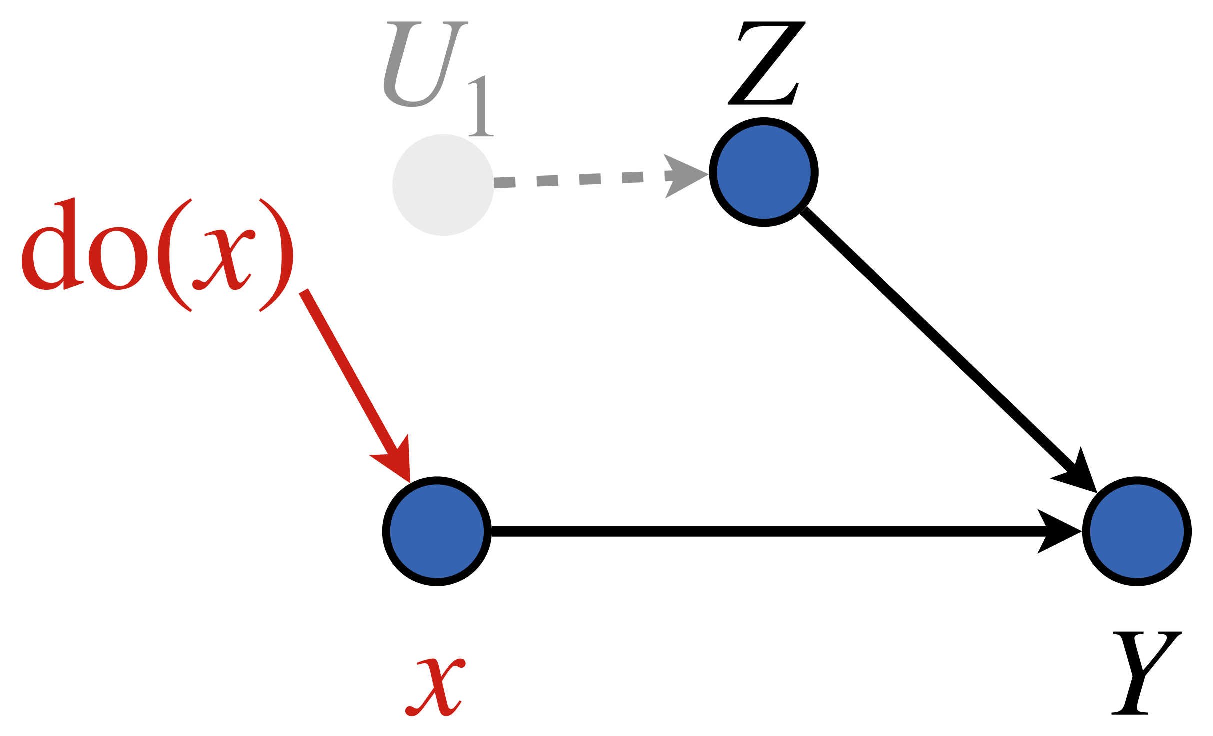

Causal Diagram $\mathcal{G}_{\overline{X}}$

Standard Causal Inference Engine

Standard Causal Inference Engine

Input

Effect (Q)

$\mathbb{E}[Y \mid \mathrm{do}(x)]$

Graph

Encode a story (or assumptions) behind the dataset

Samples

D from a distribution P

Standard Causal Inference Engine

Input	Identification	Estimation
	"When is the causal effect computable from available data?"	"How do we compute the effect from data?"

Effect (Q)

$\mathbb{E}[Y \mid \mathrm{do}(x)]$

Graph

Samples

D from a distribution P

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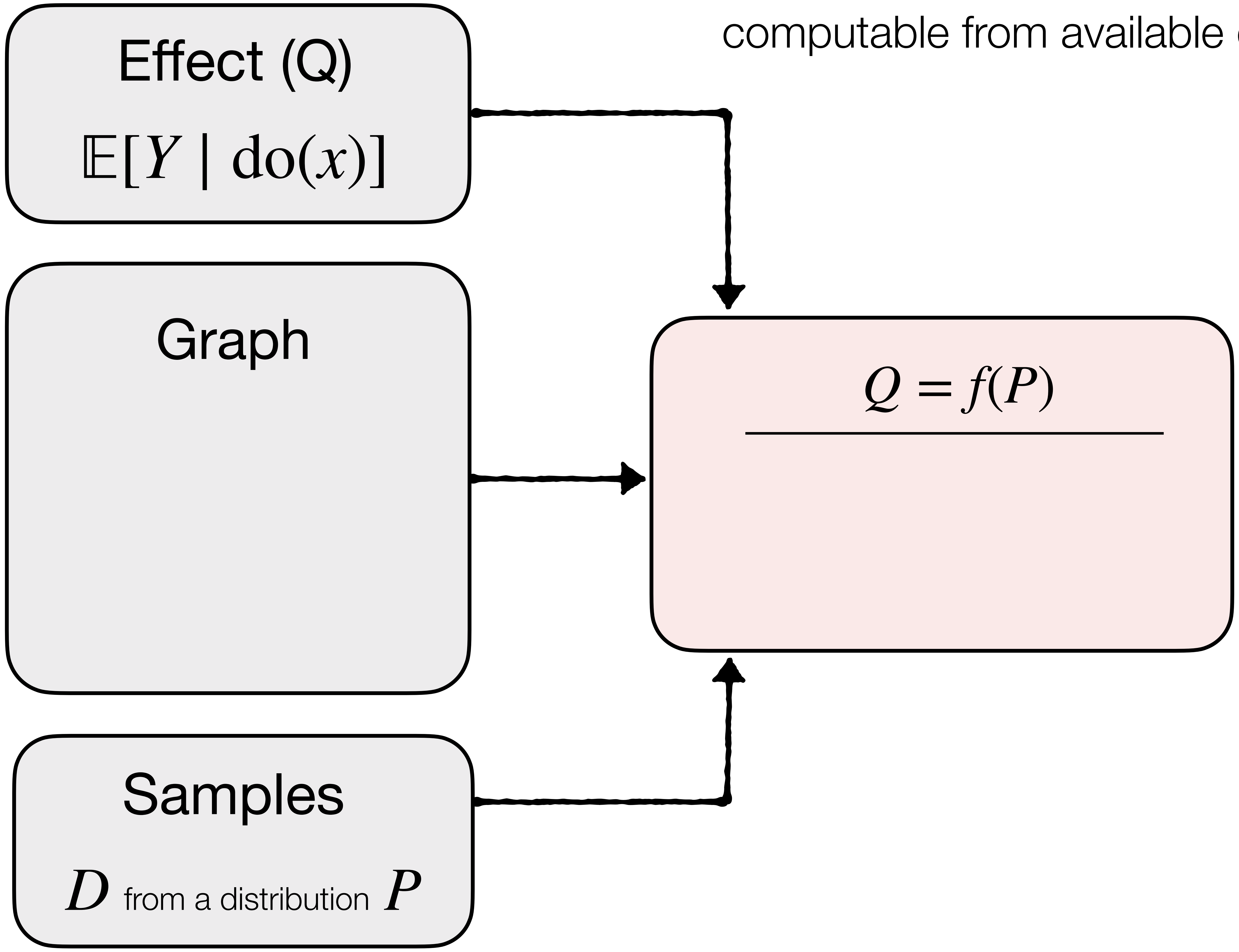

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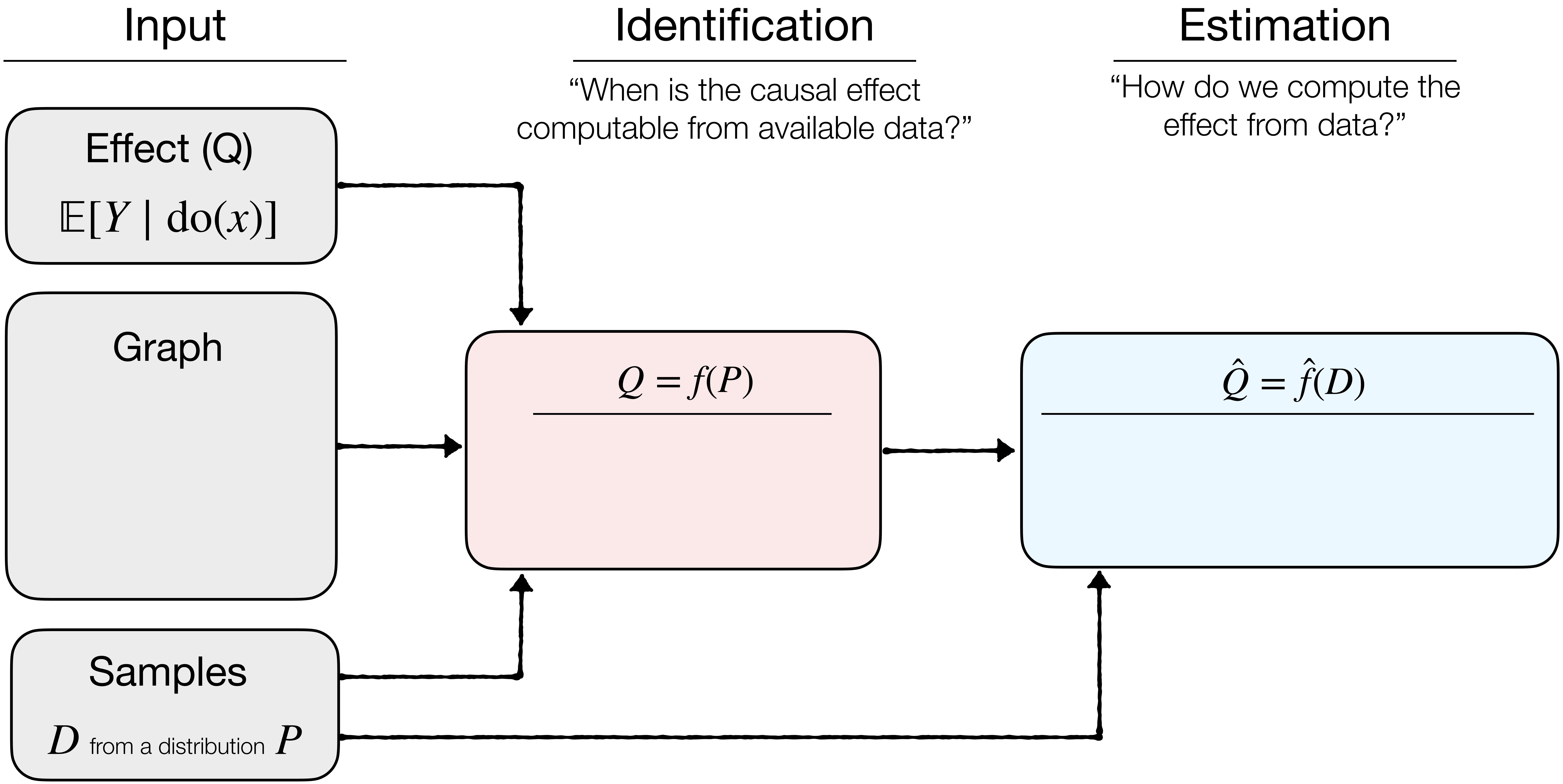


Standard Causal Inference Engine

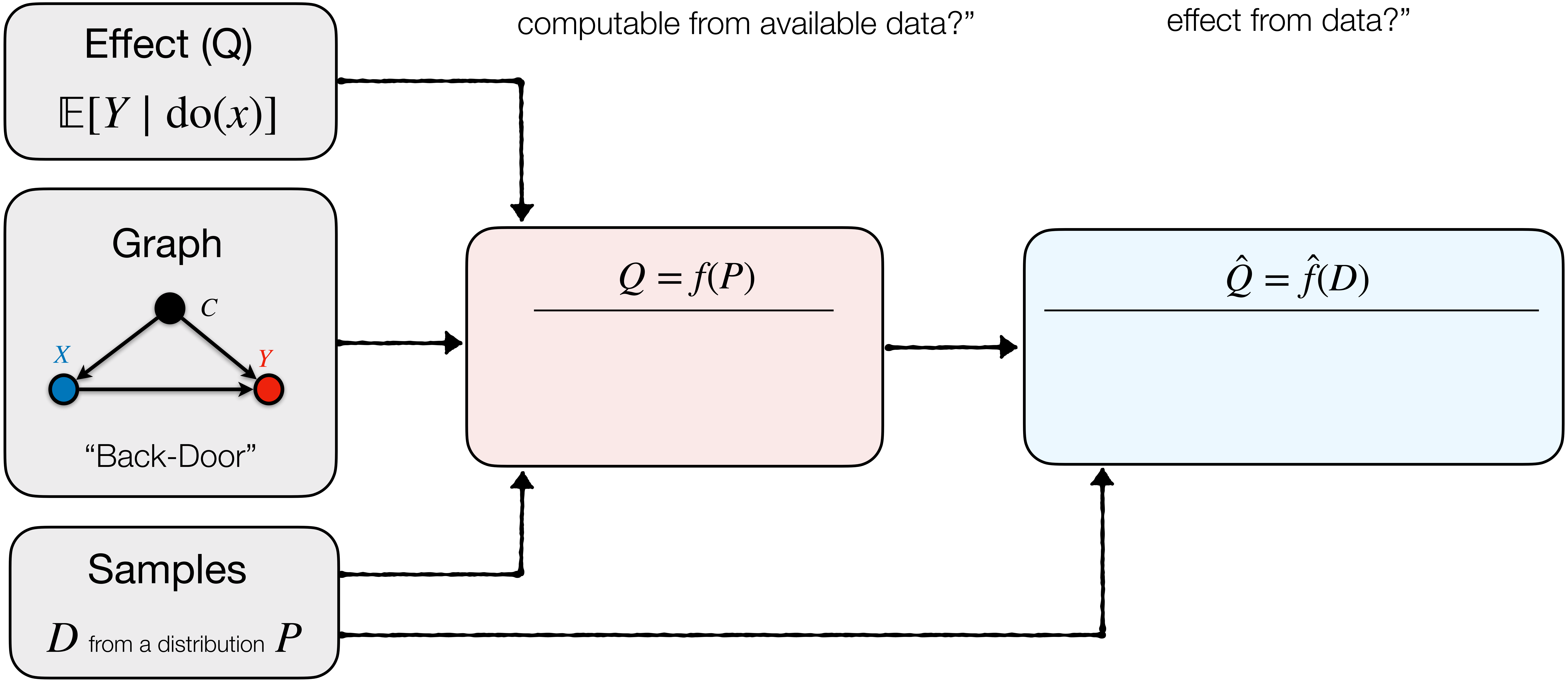
Input

Identification

"When is the causal effect computable from available data?"

Estimation

"How do we compute the effect from data?"



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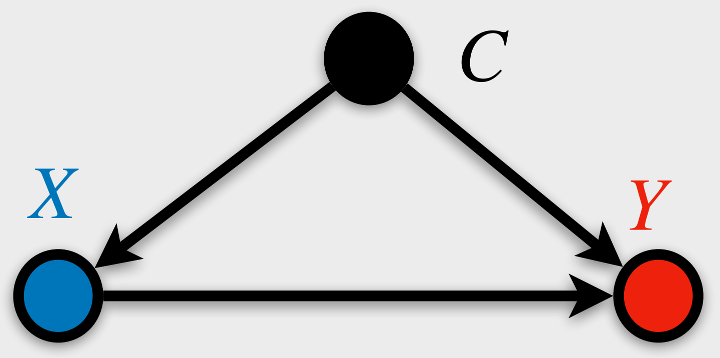

"Back-Door"

Samples

D from a distribution P

$$Q = f(P)$$

$$Q = \sum_c \mathbb{E}[Y \mid x, c] P(c)$$

"Back-Door Adjustment"

Rubin, 74; Robins, 86; Pearl, 95

$$\hat{Q} = \hat{f}(D)$$

Standard Causal Inference Engine

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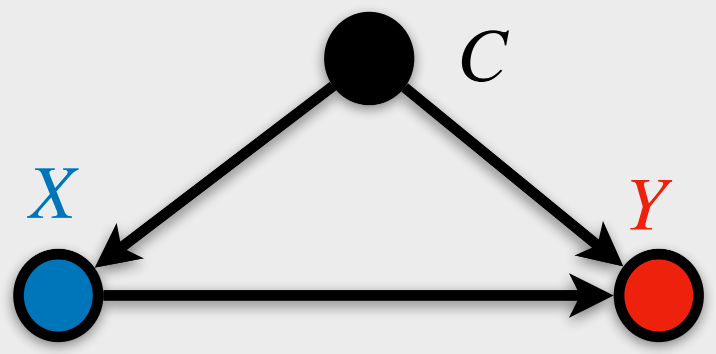
Effect (Q)

$\mathbb{E}[Y \mid \mathrm{do}(x)]$

Graph



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Samples

D from a distribution P

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"When is the causal effect computable from available data?"

$Q = f(P)$

$$Q = \sum_c \mathbb{E}[Y \mid x, c] P(c)$$

"Back-Door Adjustment"

Rubin, 74; Robins, 86; Pearl, 95

Estimation

"How do we compute the effect from data?"

$\hat{Q} = \hat{f}(D)$

One-Step Estimator, TMLE, DML

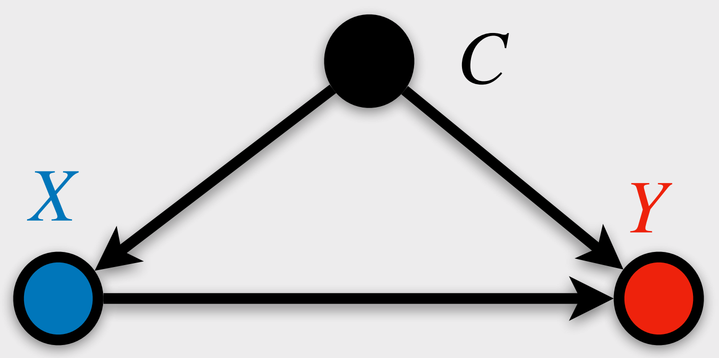
Bickel et al., 93; van der Laan and Rubin 06, van der Laan and Gruber12; Chernozhukov et al, 2018

Challenges in Standard Setting

Effect (Q)

$\mathbb{E}[Y \mid \mathrm{do}(x)]$

Graph



"Back-Door"

Samples

D from P

Challenges in Standard Setting

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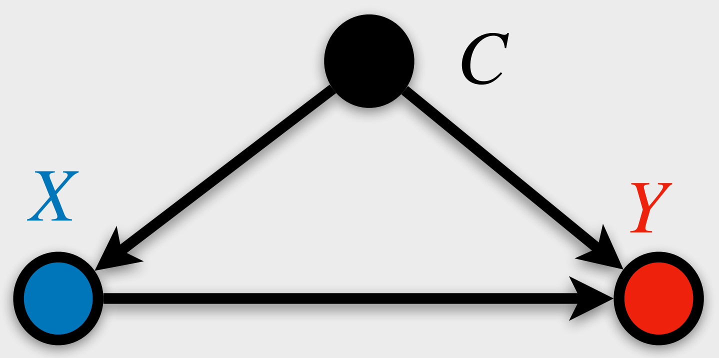

"Back-Door"

Samples

D from P

1 Complex dependences

Challenges in Standard Setting

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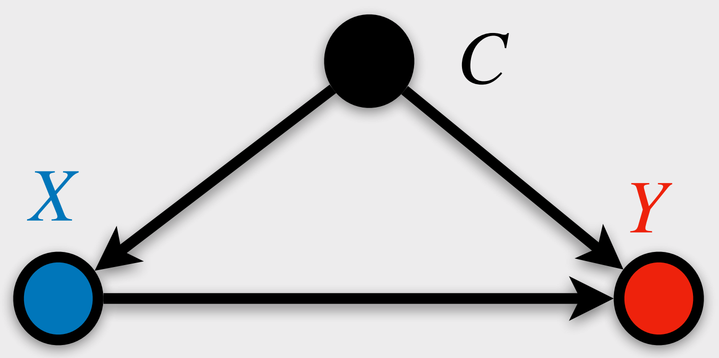

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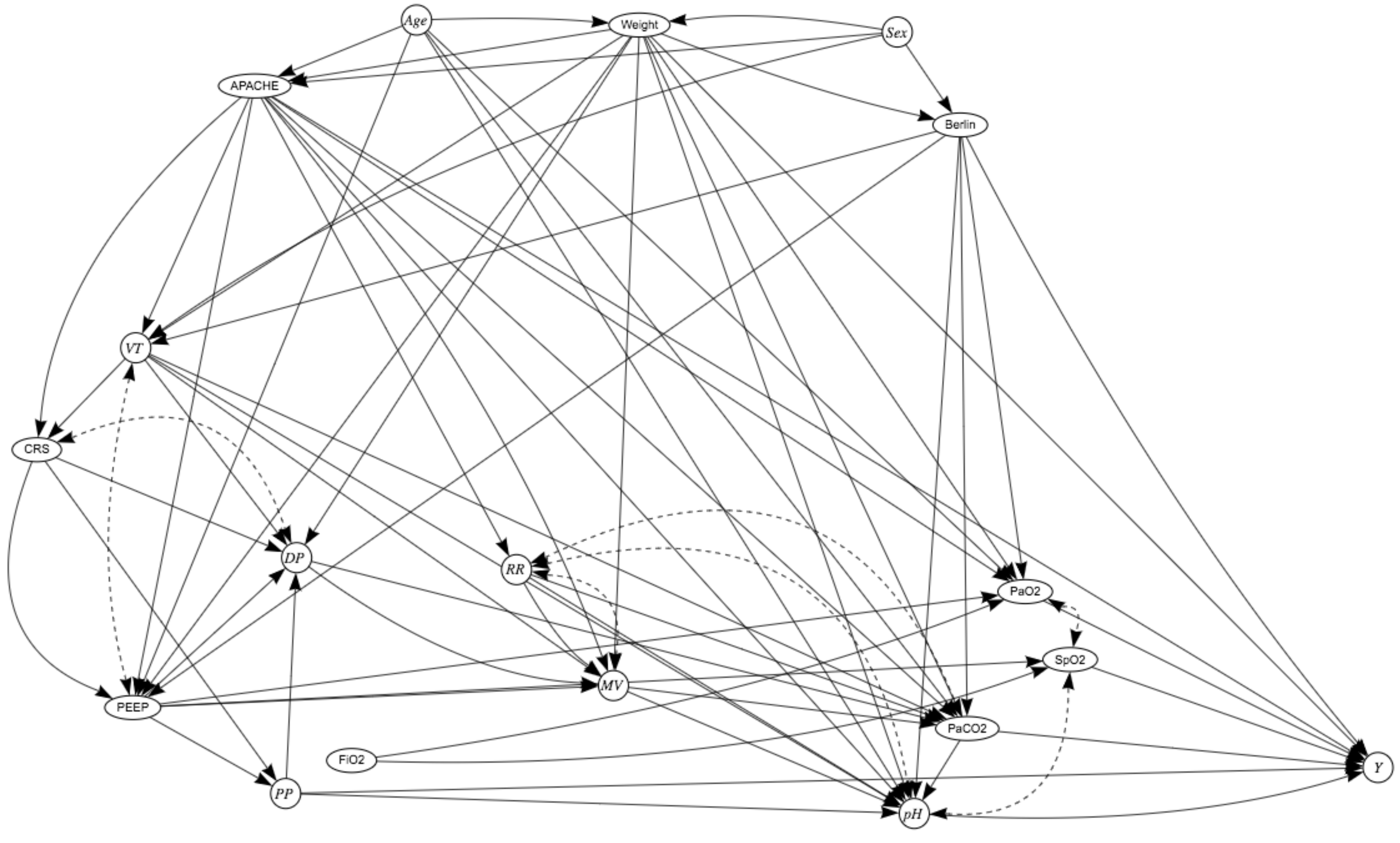


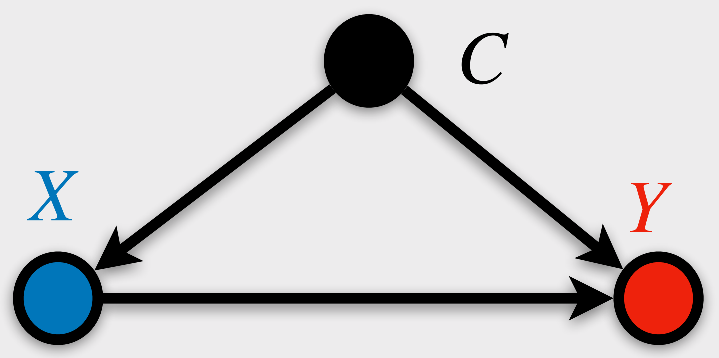
Causal graph on acute respiratory distress syndrome (ARDS)

Challenges in Standard Setting

Effect (Q)

$\mathbb{E}[Y \mid do(x)]$

Graph



"Back-Door"

Samples

D from P

1. Complex dependences
2. Data fusion (observations & experiments)

Challenges in Standard Setting

Effect (Q)

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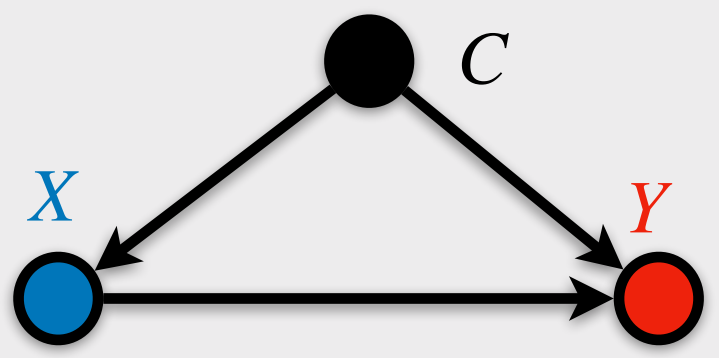
Graph



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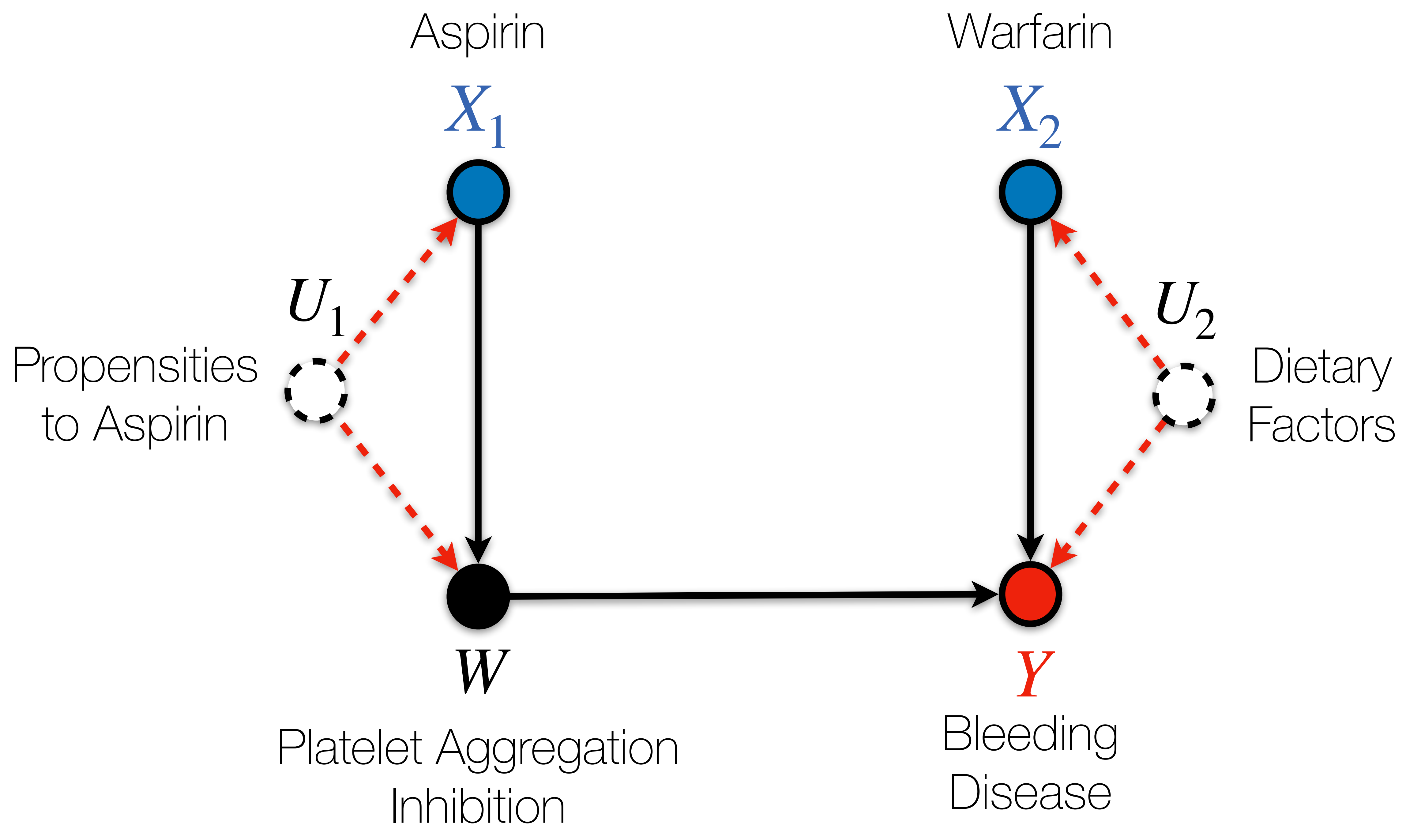

- Goal: Estimate $\mathbb{E}[Y \mid do(x_1, x_2)]$ from single interventions $do(x_1)$ and $do(x_2)$.

Challenges in Standard Setting

Effect (Q)

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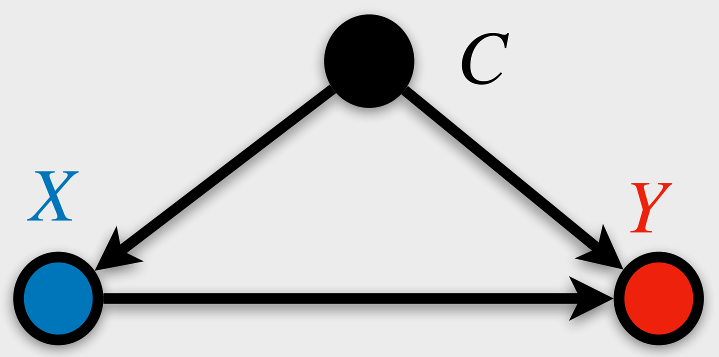

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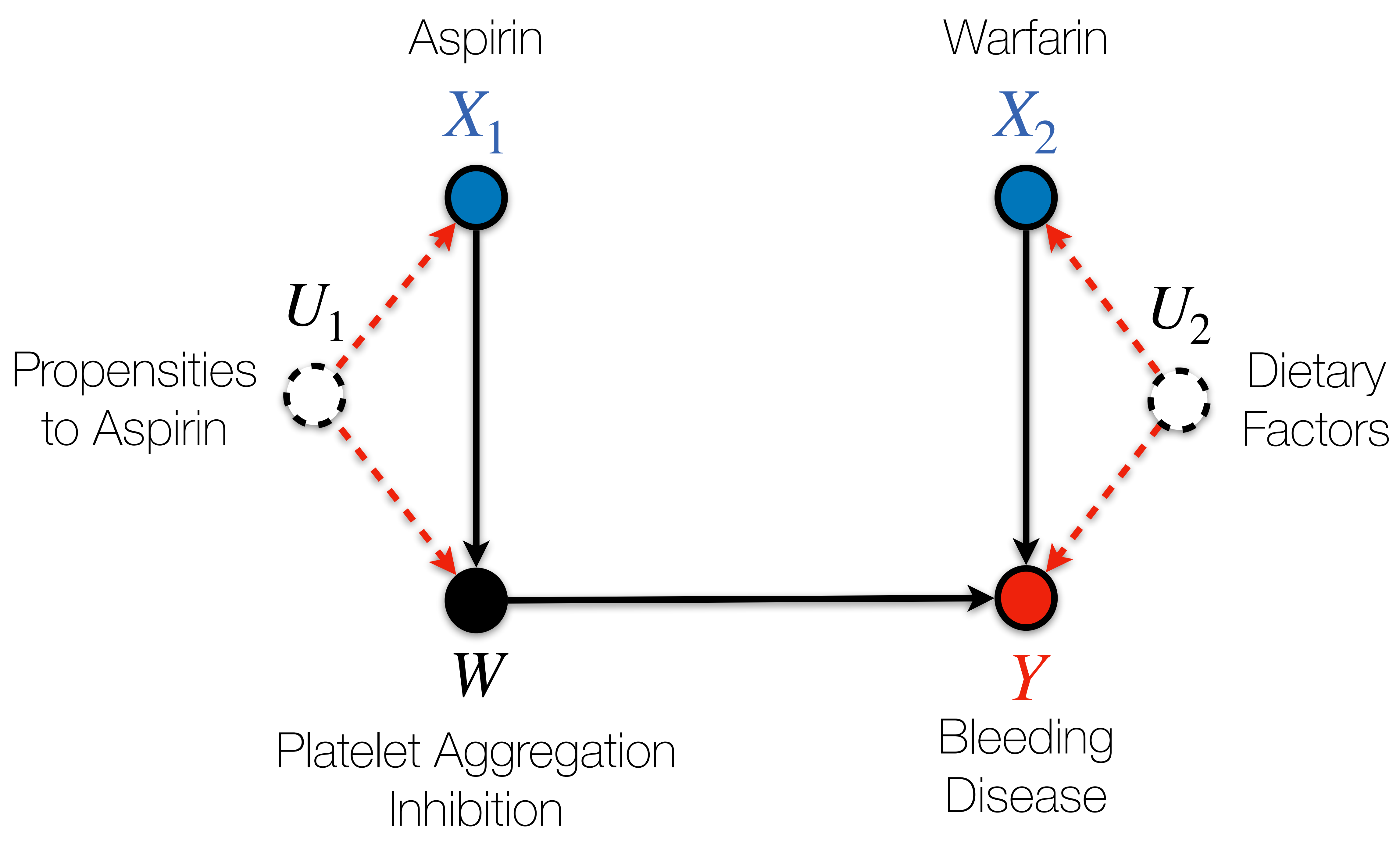

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Challenges in Standard Setting

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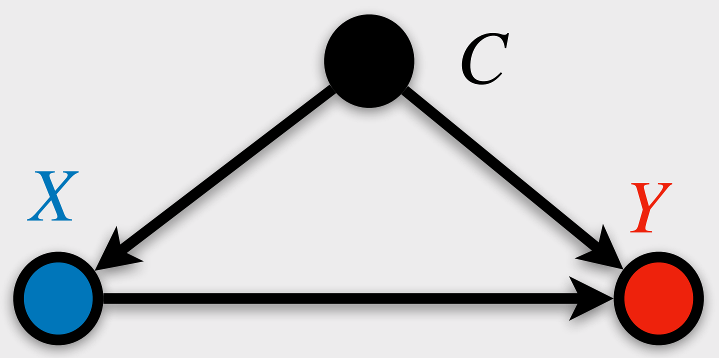

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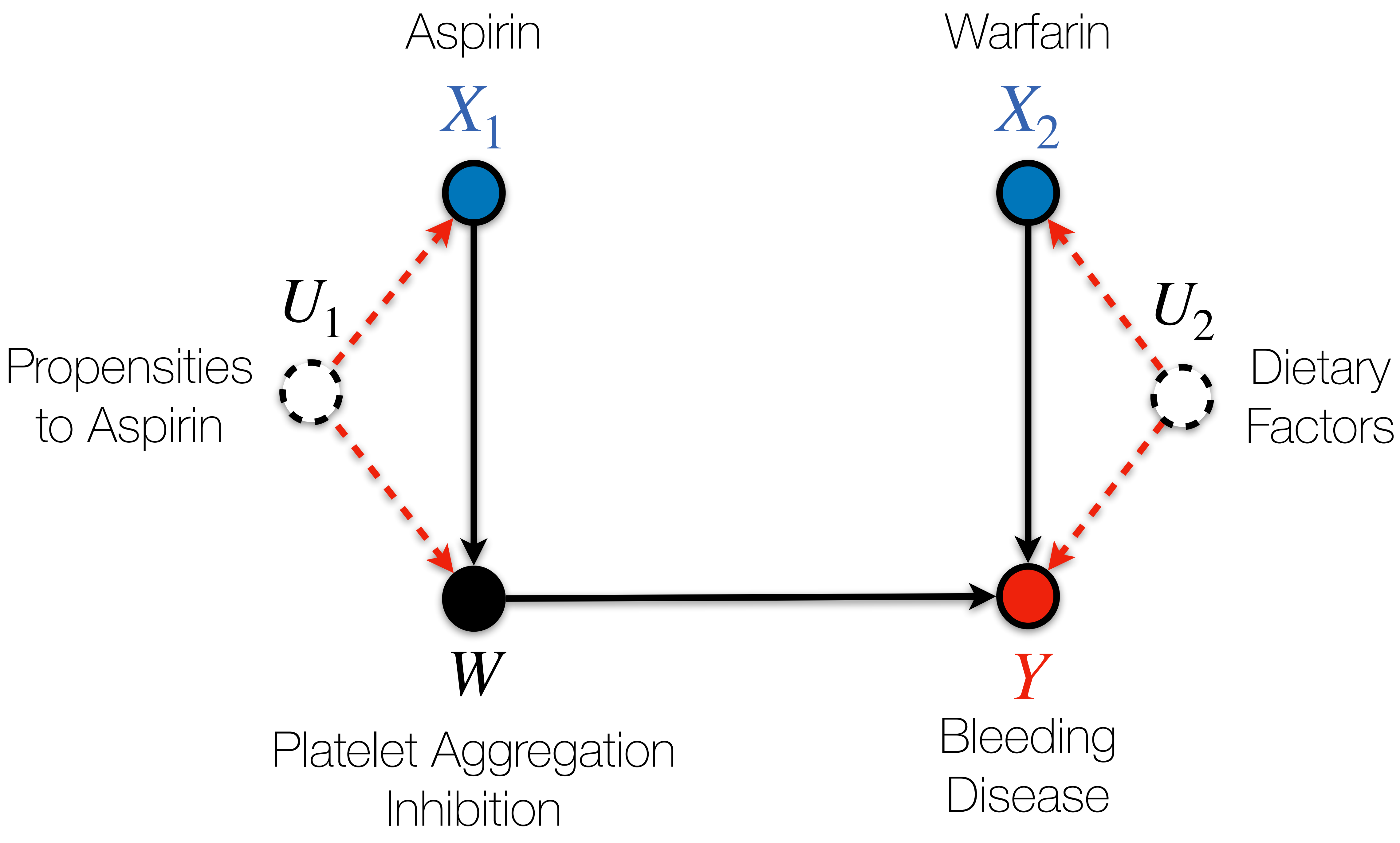

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- Drug interactions between X_1 and X_2
- Not identifiable from observations

Challenges in Standard Setting

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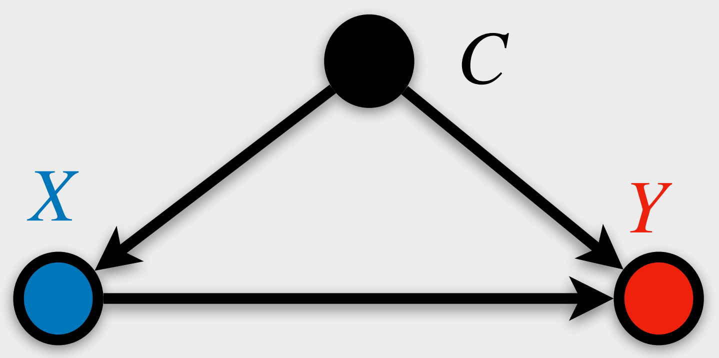
Graph



"Back-Door"

Samples

D from P

1 Complex dependences

2 Data fusion (observations & experiments)

3 More general scenarios

Challenges in Standard Setting

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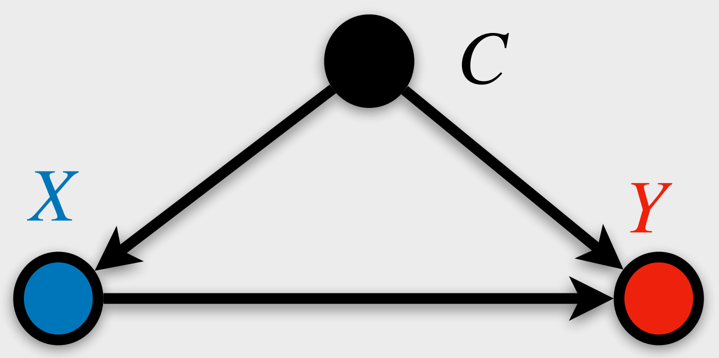


"Back-Door"

Samples

D from P

1. Complex dependences
2. Data fusion (observations & experiments)
3. More general scenarios

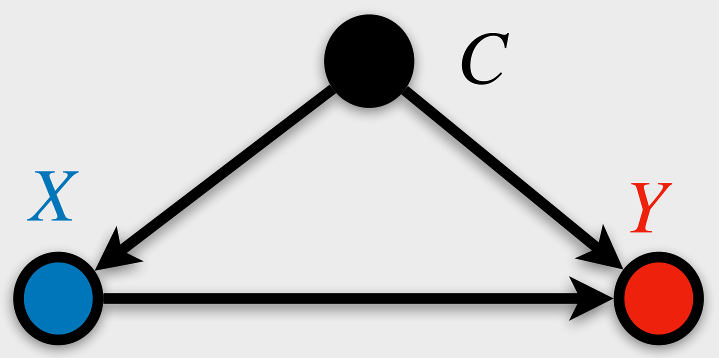
(Fairness) Salary (Y) a man ($X = x$) would earn if he is given the opportunities (M) that other genders ($X \neq x$) had received

Challenges in Standard Setting

Effect (Q)

$\mathbb{E}[Y \mid \mathrm{do}(x)]$

Graph



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$$\mathbb{E}[\ Y_{x, M_{\neg x}}\ ]$$

Challenges in Standard Setting

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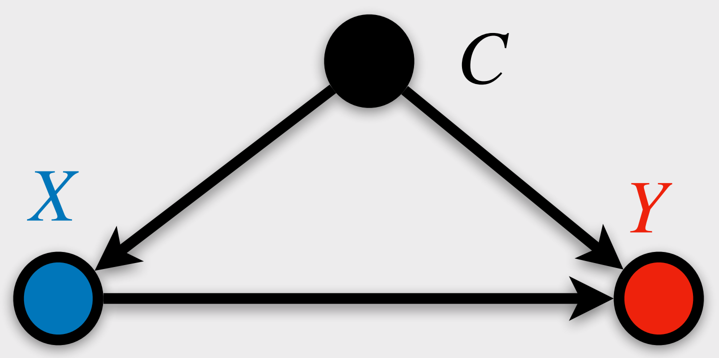

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$$\mathbb{E}[\, Y_{x, M_{\neg x}} \,]$$

expected — salary — a man would earn — given the opportunity — other genders had received

Estimating Identifiable Causal Effects

Tasks	Challenges
	1 Complicated dependences
	2 Data fusion (observations + experiments)
	3 More general scenarios

Estimating Identifiable Causal Effects

Tasks

1 **[Ch. 3]** Estimating causal effects from observations → ✔

Challenges

2 Data fusion (observations + experiments)

3 More general scenarios

Estimating Identifiable Causal Effects

Tasks		Challenges
1 **[Ch. 3]** Estimating causal effects from observations	→	✔
2 **[Ch. 4]** Estimating causal effects from data fusion	→	✔
		3 More general scenarios

Estimating Identifiable Causal Effects

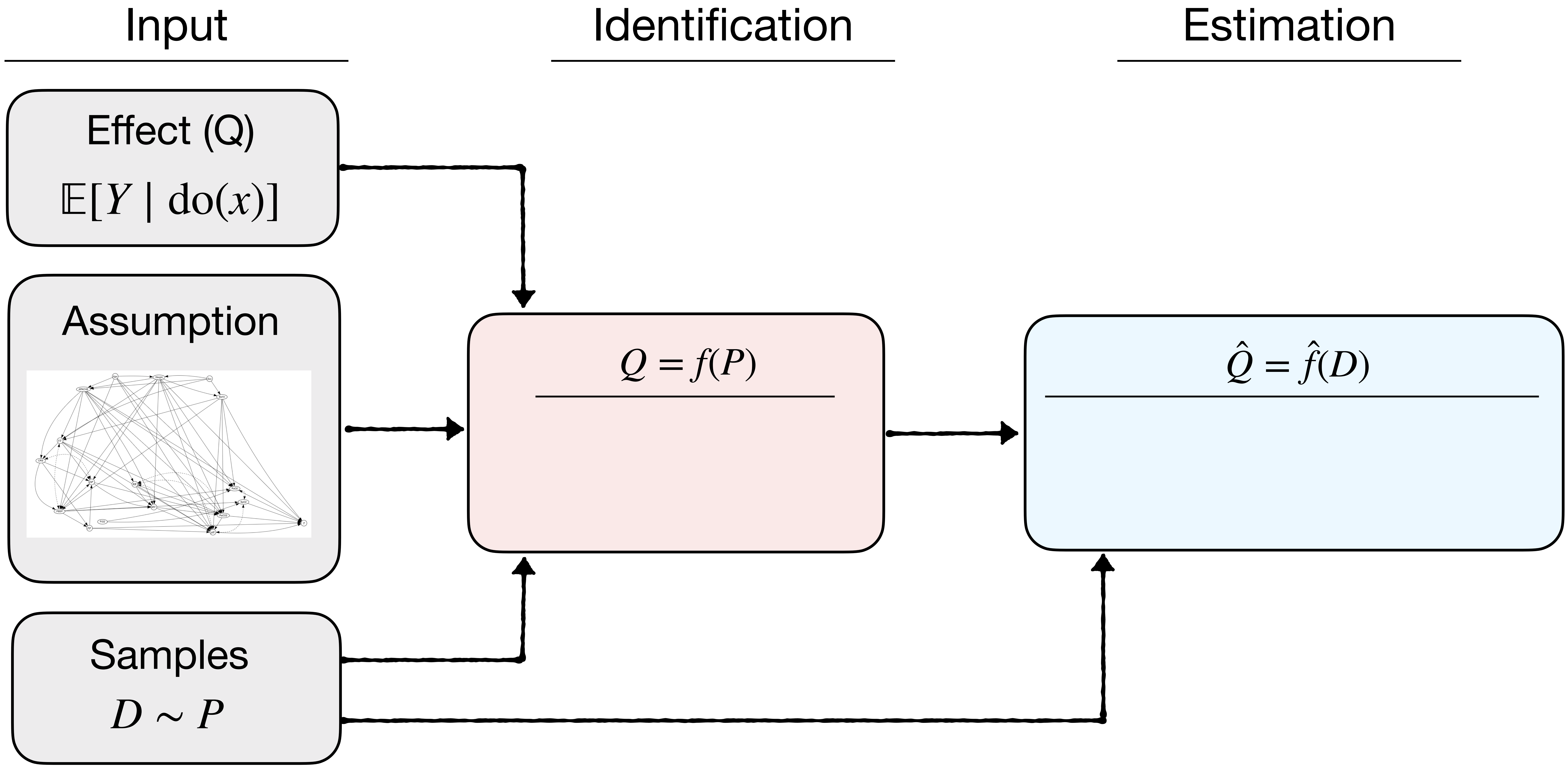
Tasks		Challenges
1 **[Ch. 3]** Estimating causal effects from observations	→	✔
2 **[Ch. 4]** Estimating causal effects from data fusion	→	✔
3 **[Ch. 5]** Unified causal effect estimation method	→	✔

Talk Outline

1. **Ch. 3** Estimating causal effects from observations
2. **Ch. 4** Estimating causal effects from data fusion
3. **Ch. 5** Unified causal effect estimation method
4. Conclusion

Talk Outline

1 **Ch. 3** Estimating causal effects from observations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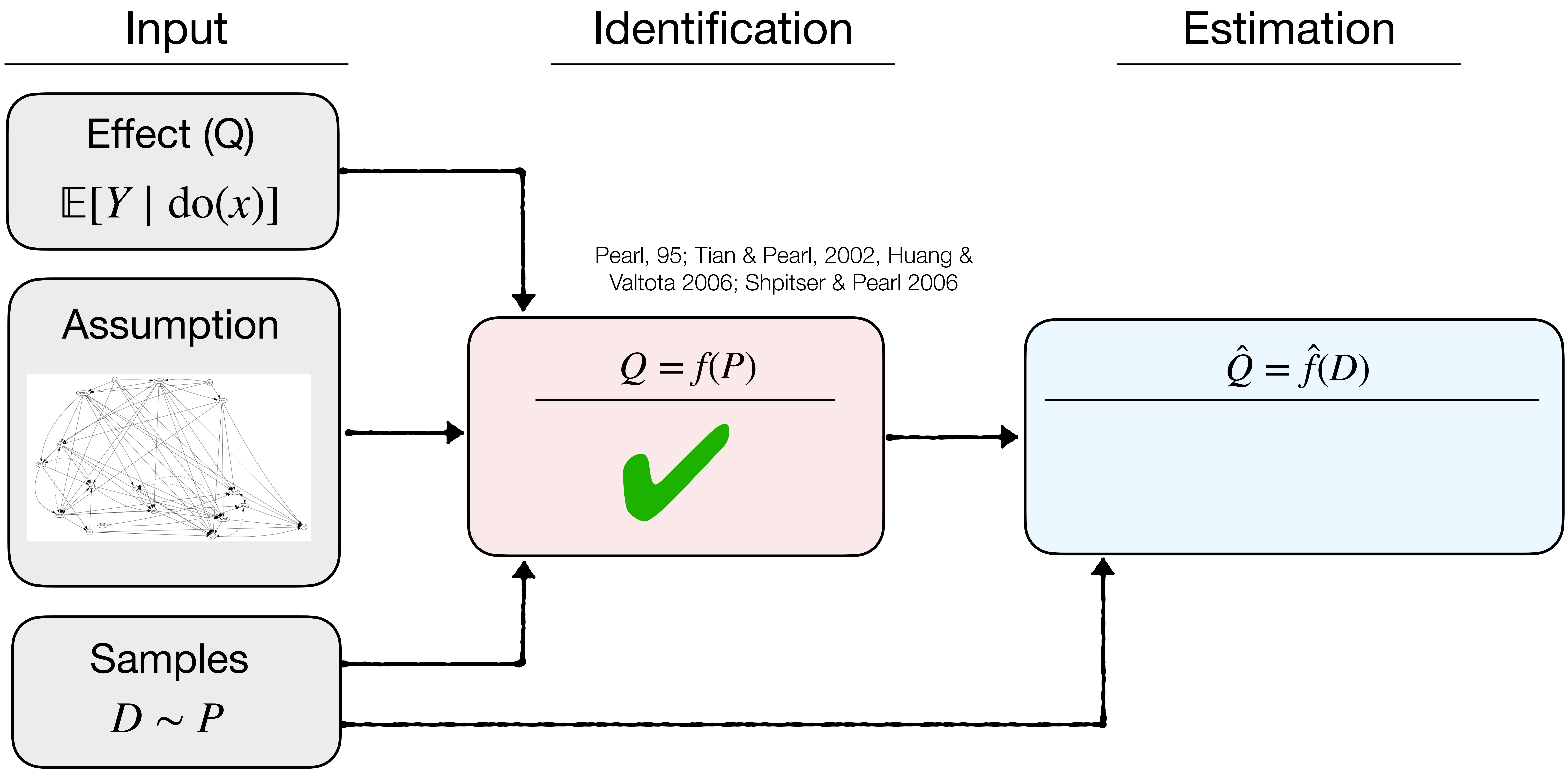
Estimation
Effect (Q)
$\mathbb{E}[Y \mid \mathrm{do}(x)]$
Assumption
Samples
$D \sim P$
$Q = f(P)$
$\hat{Q} = \hat{f}(D)$



Input
Identification
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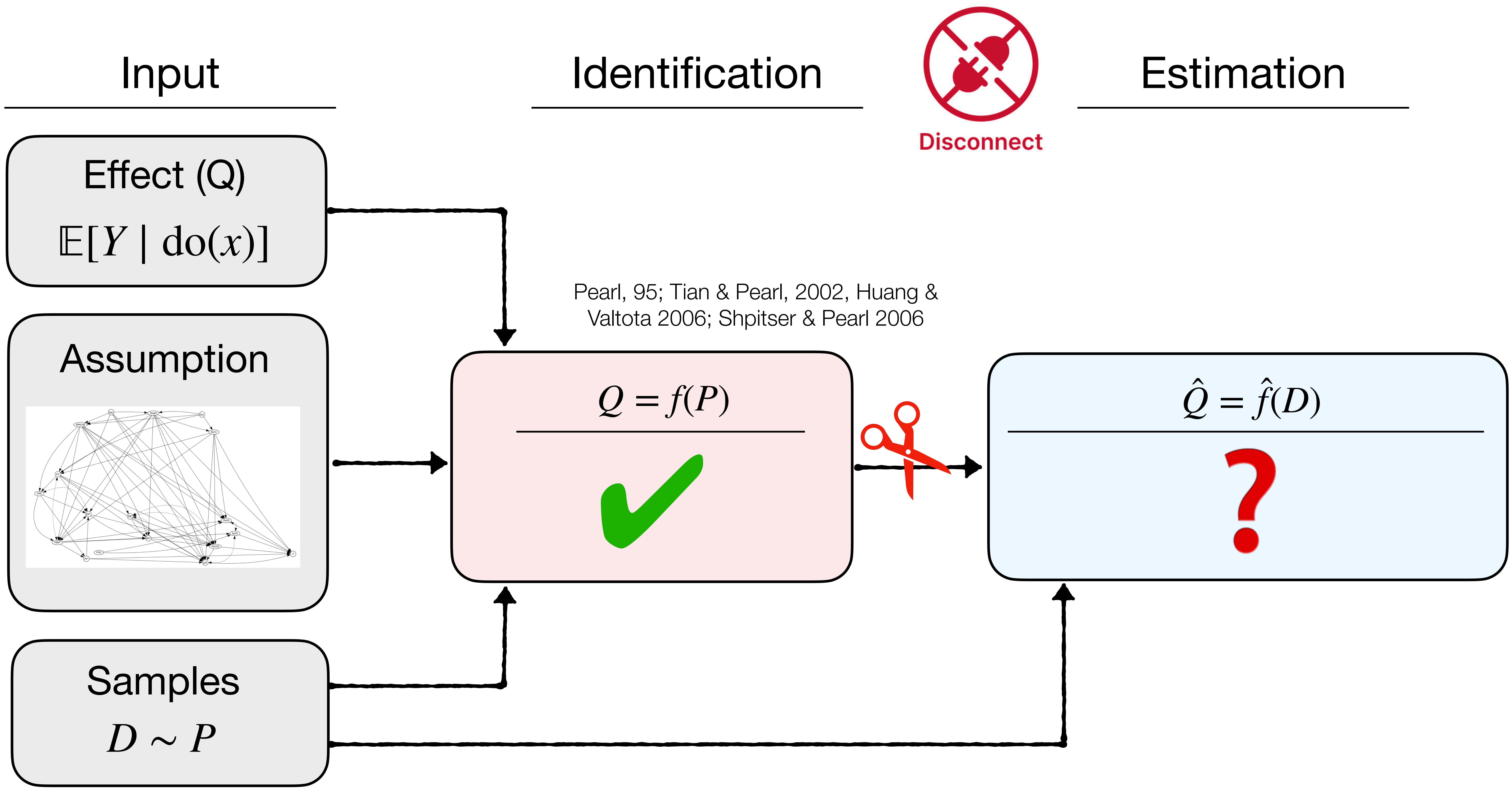
Input
Identification
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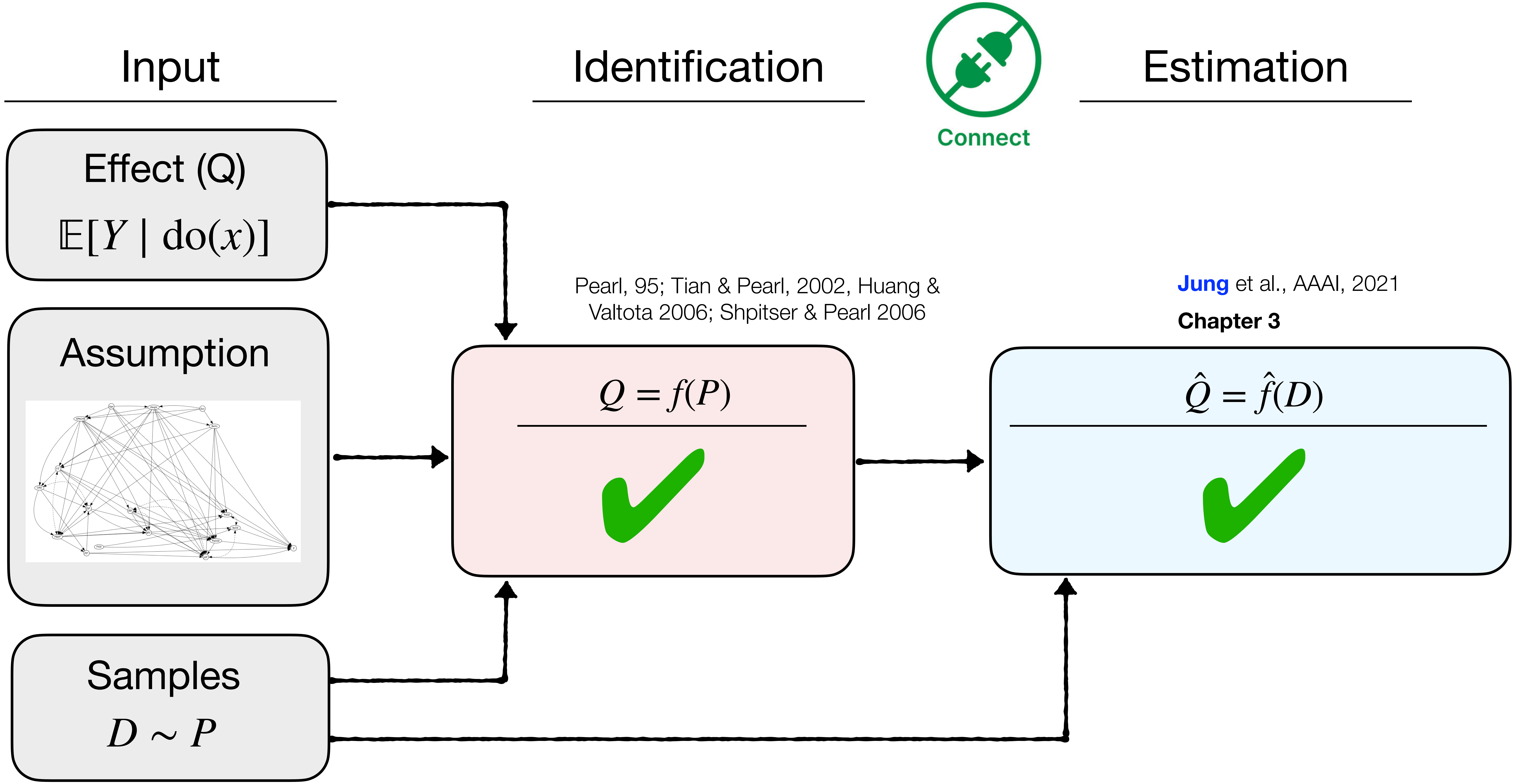
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Jung et al., AAAI, 2021
Chapter 3
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Background: Back-door Adjustment (BD)

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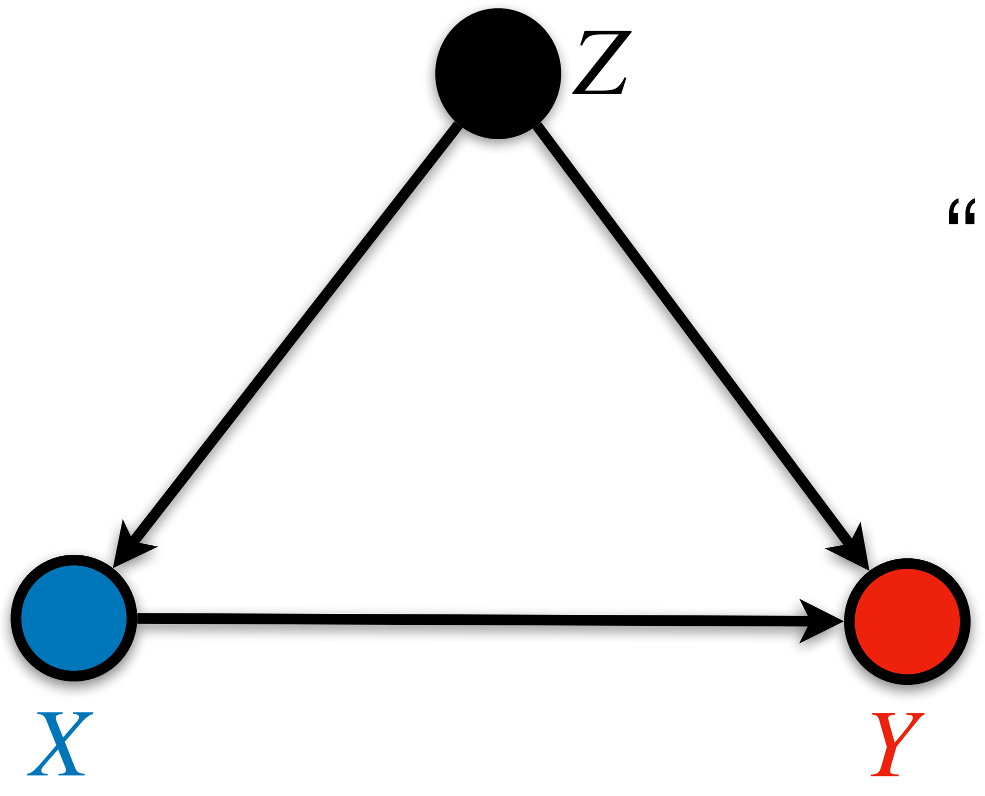

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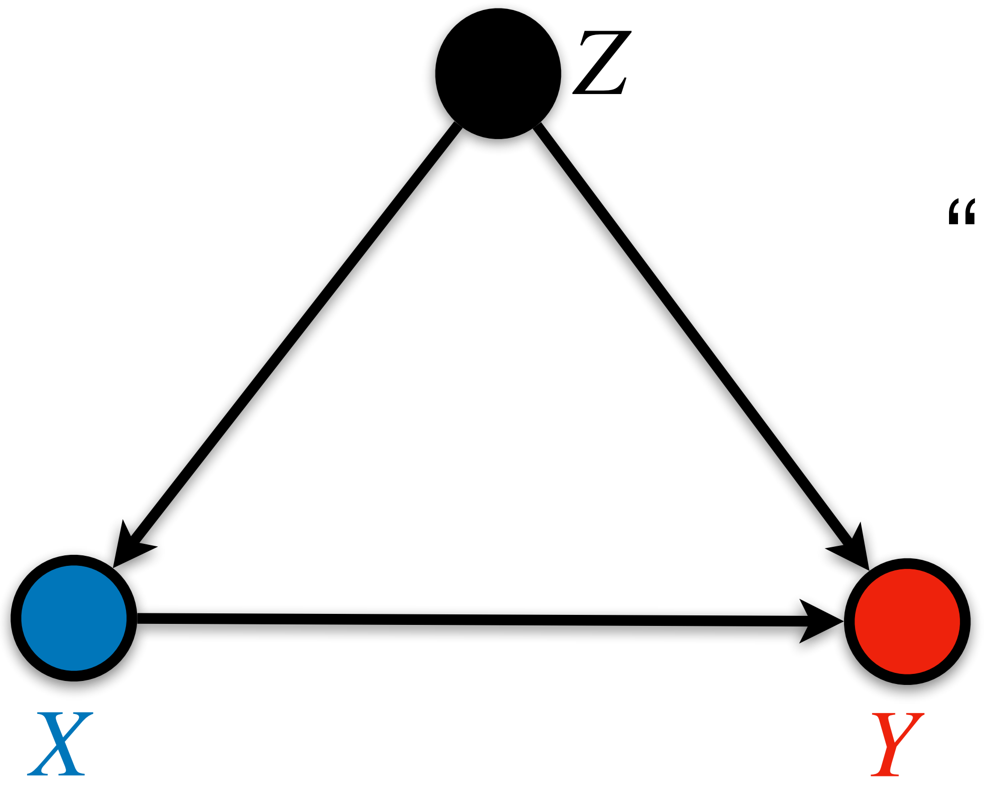

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1. $\mathbf{Z}$ is not a descendent of treatment;
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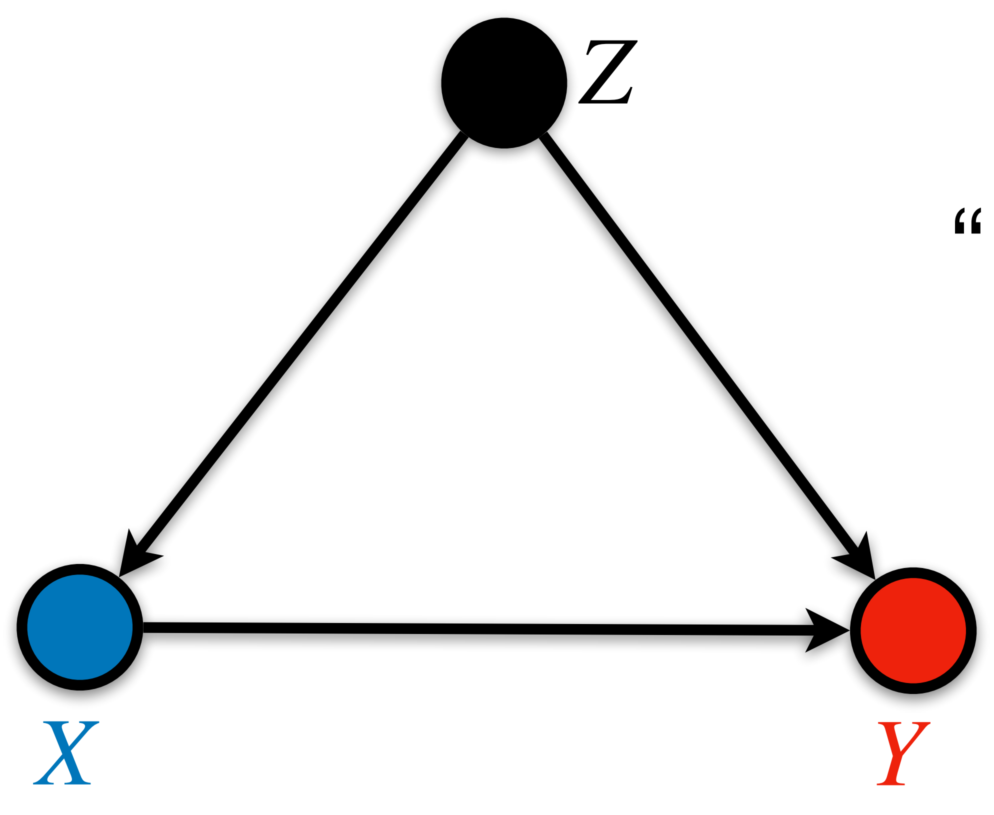

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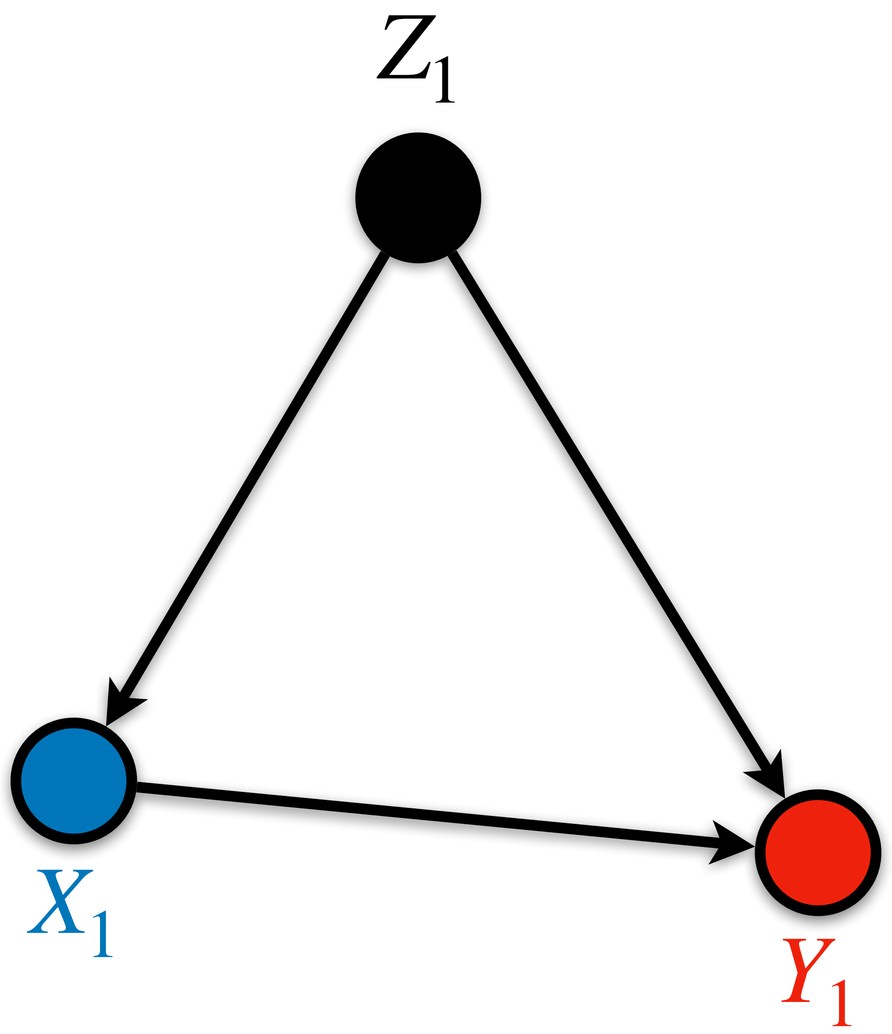

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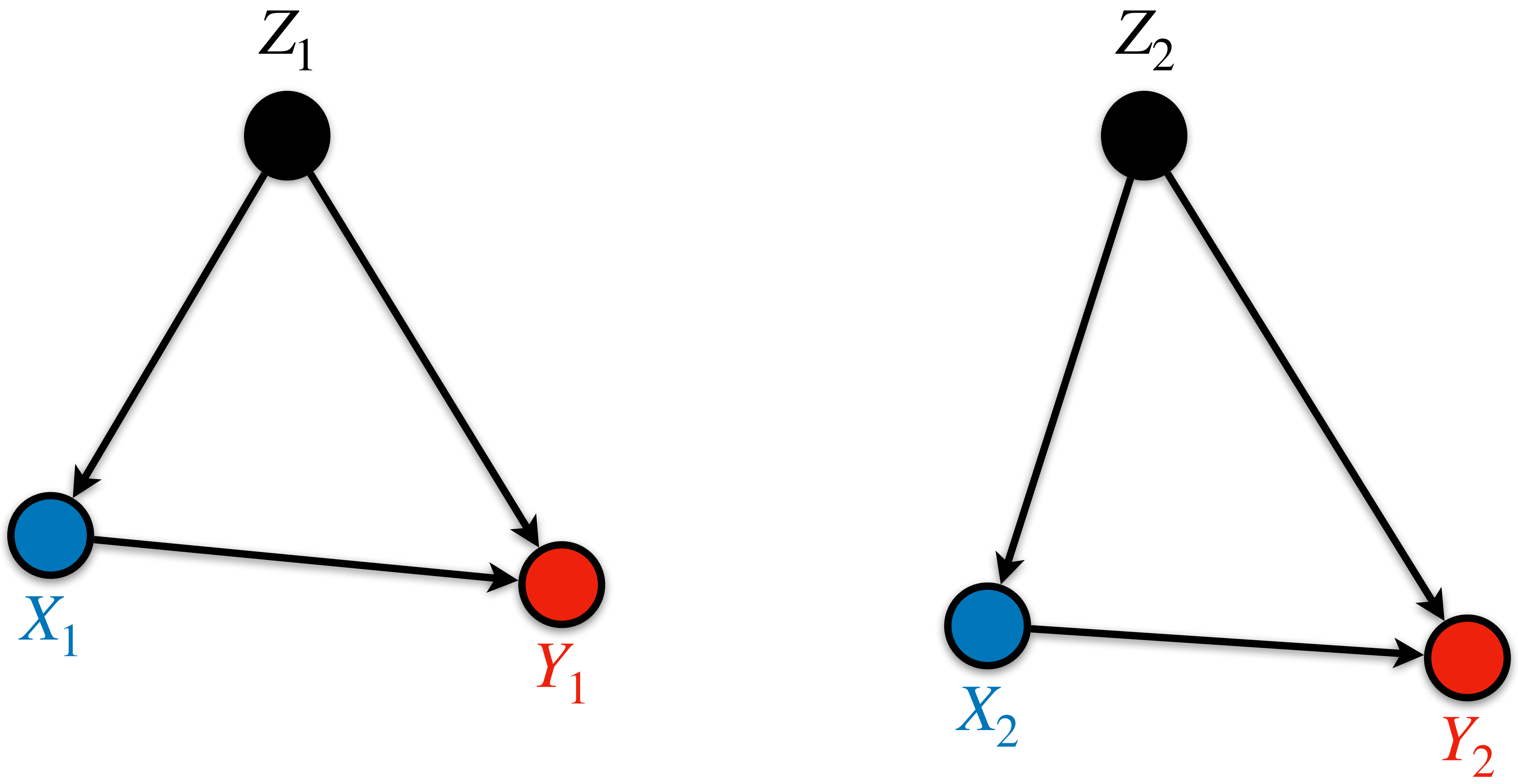

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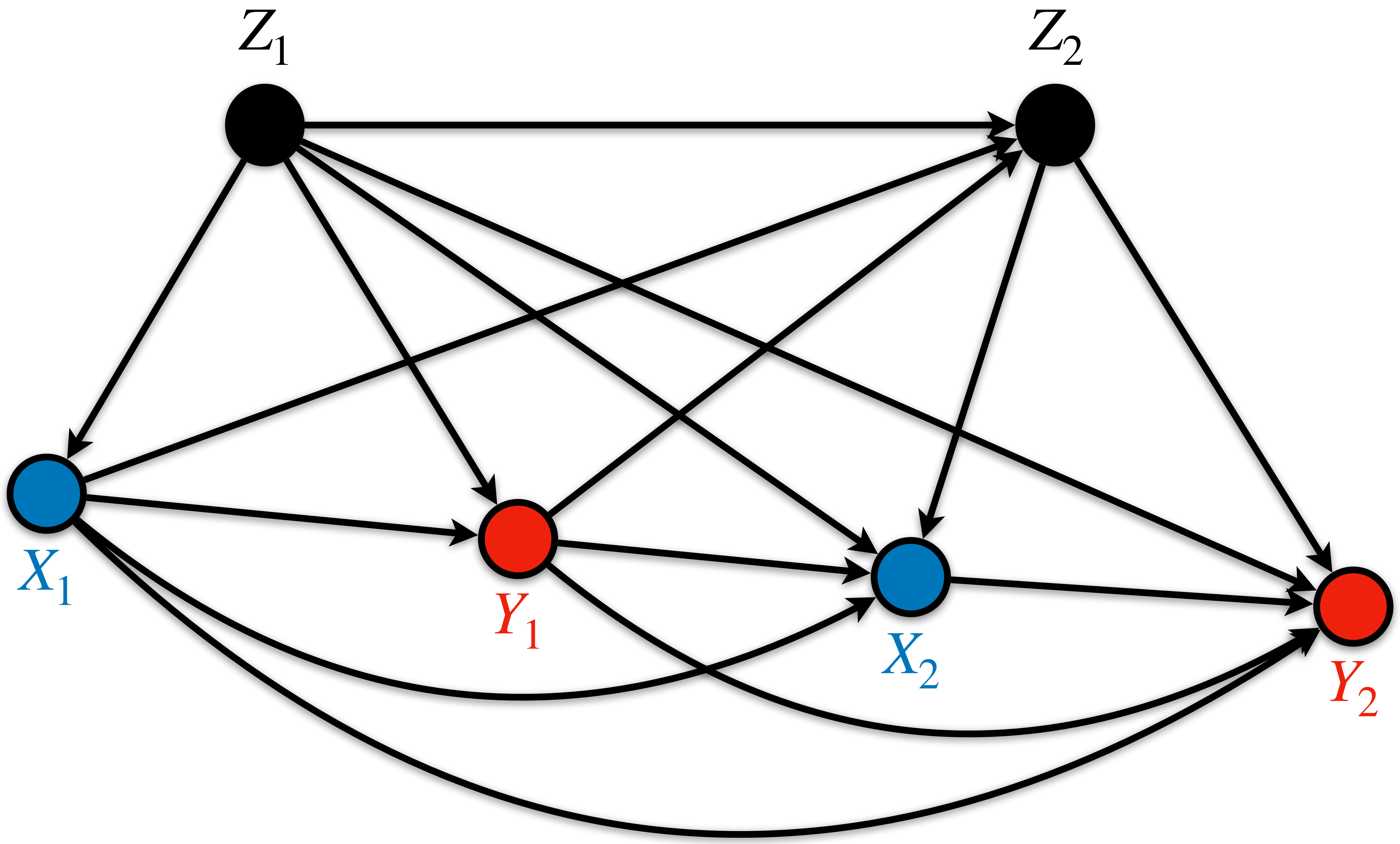

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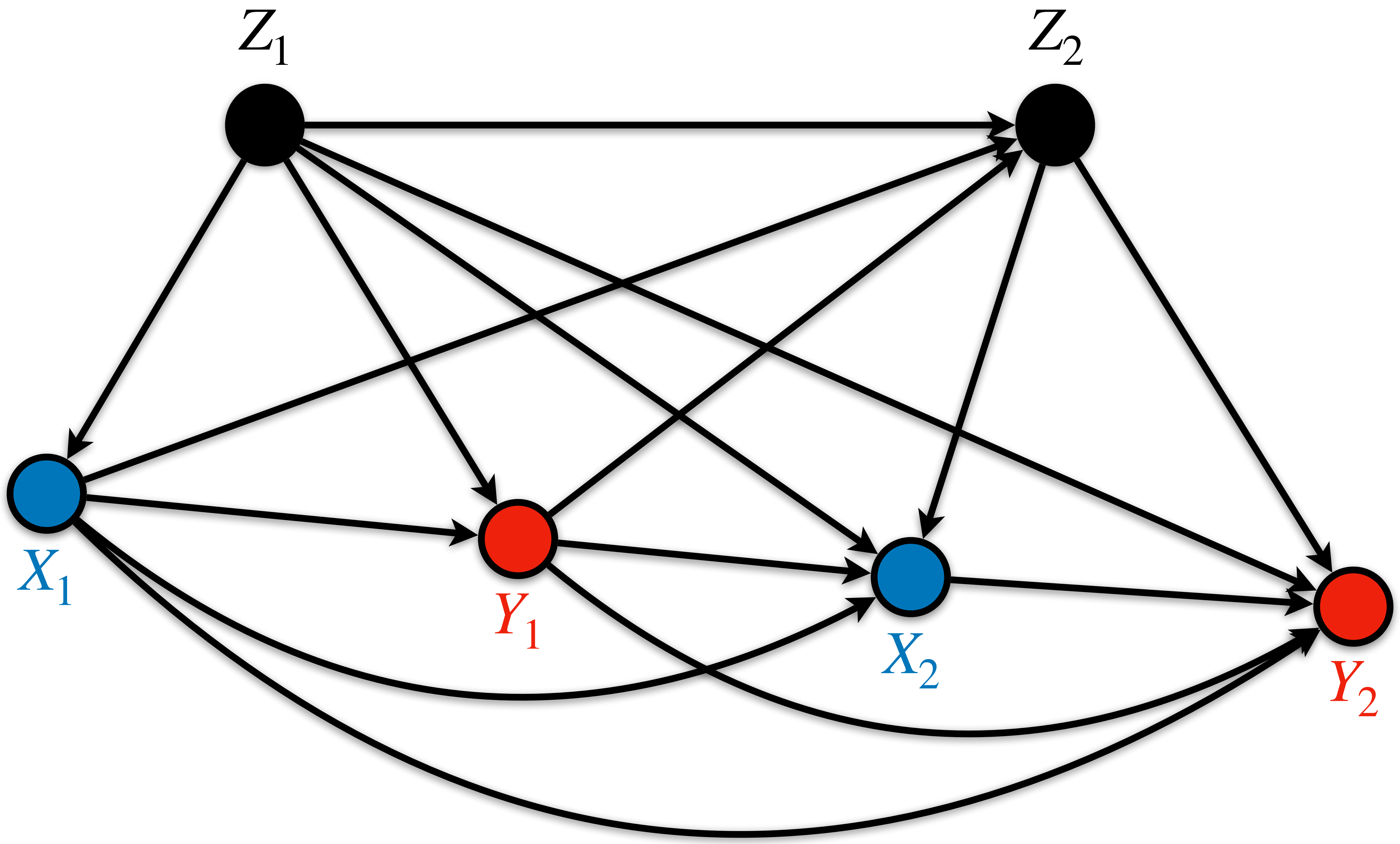

Multi-outcome Sequential BD (mSBD)

A seq. $\mathbf{Z} = (\mathbf{Z}_1, \cdots, \mathbf{Z}_m)$ satisfies the mSBD if, for $i = 1, \cdots, m$, $\mathbf{Z}_i$ satisfies the BD relative to $(\mathbf{X}_i, \mathbf{Y}^{\geq i})$ conditioning on prev. vectors.

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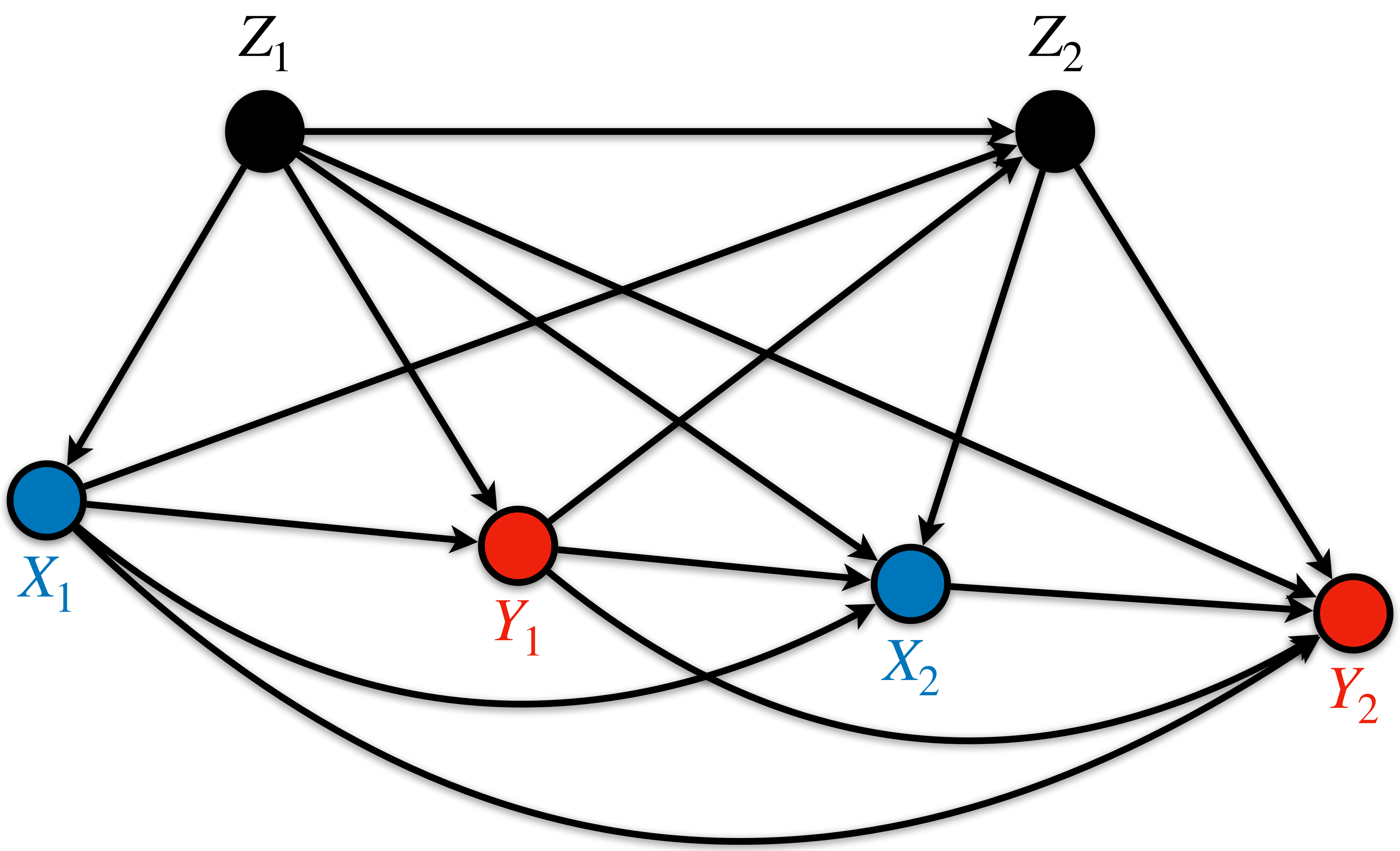

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"mSBD adjustment"

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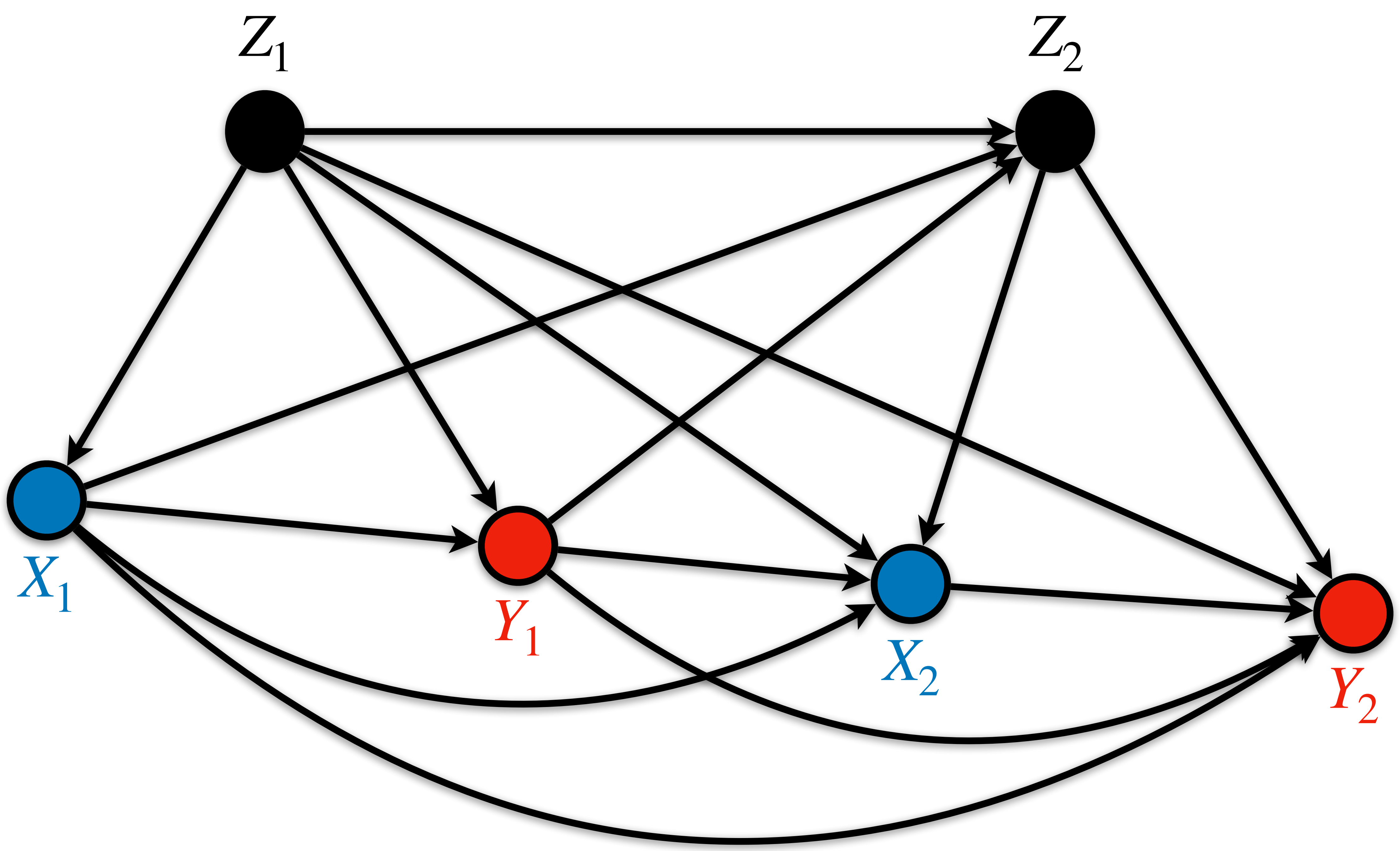

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"mSBD adjustment"

* I'll use "BD" for simplicity, but all results extend to mSBD (as shown in the thesis).

Background: Robust Estimator for BD

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❶ $\text{BD}(\mu,\pi) = \mathbb{E}[\mu \times \pi]$, where $\mu(XC) \triangleq \mathbb{E}[Y \mid X, C]$ and $\pi(XC) \triangleq \dfrac{\mathbb{I}_x(X)}{P(X \mid C)}$

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One-step/Debiased ML estimator (Robins and Rotnitzk, 95; Band and Robins; 2005, van der Laan and Rubin 2006, van der Laan and Gruber 2012, Chernozhukov et al., 2018))

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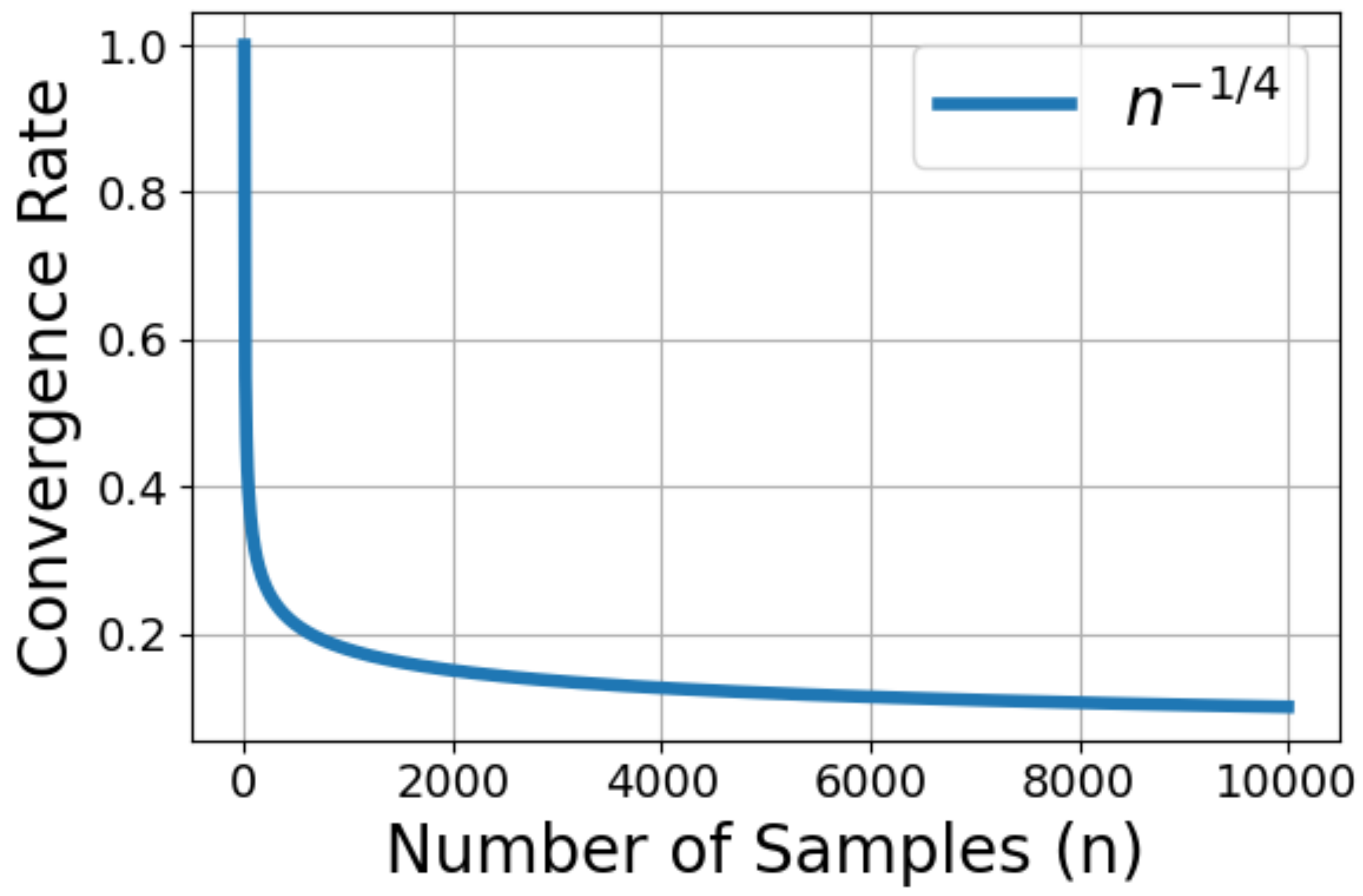

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$n^{-1/4}$ $n^{-1/4}$

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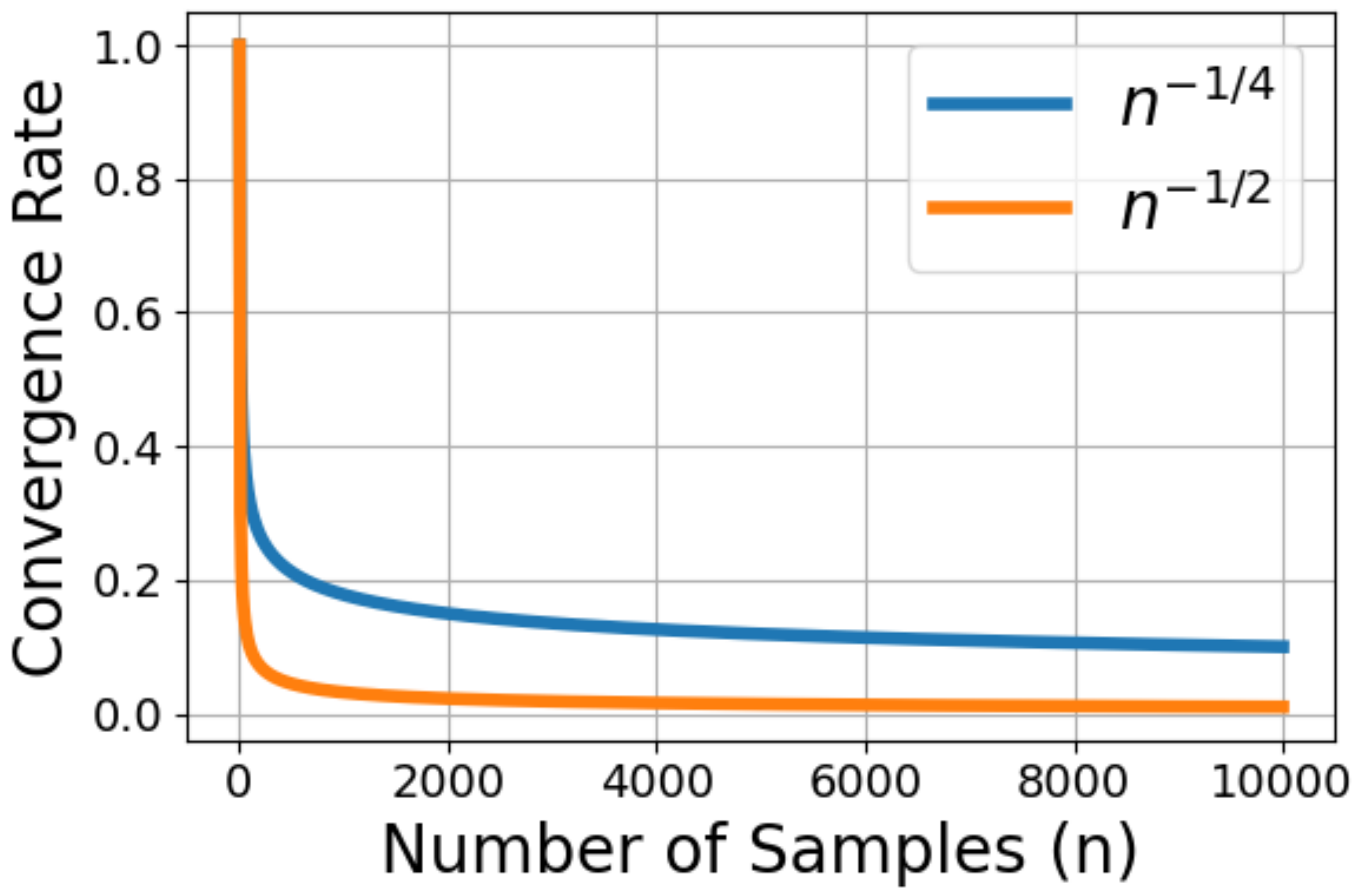

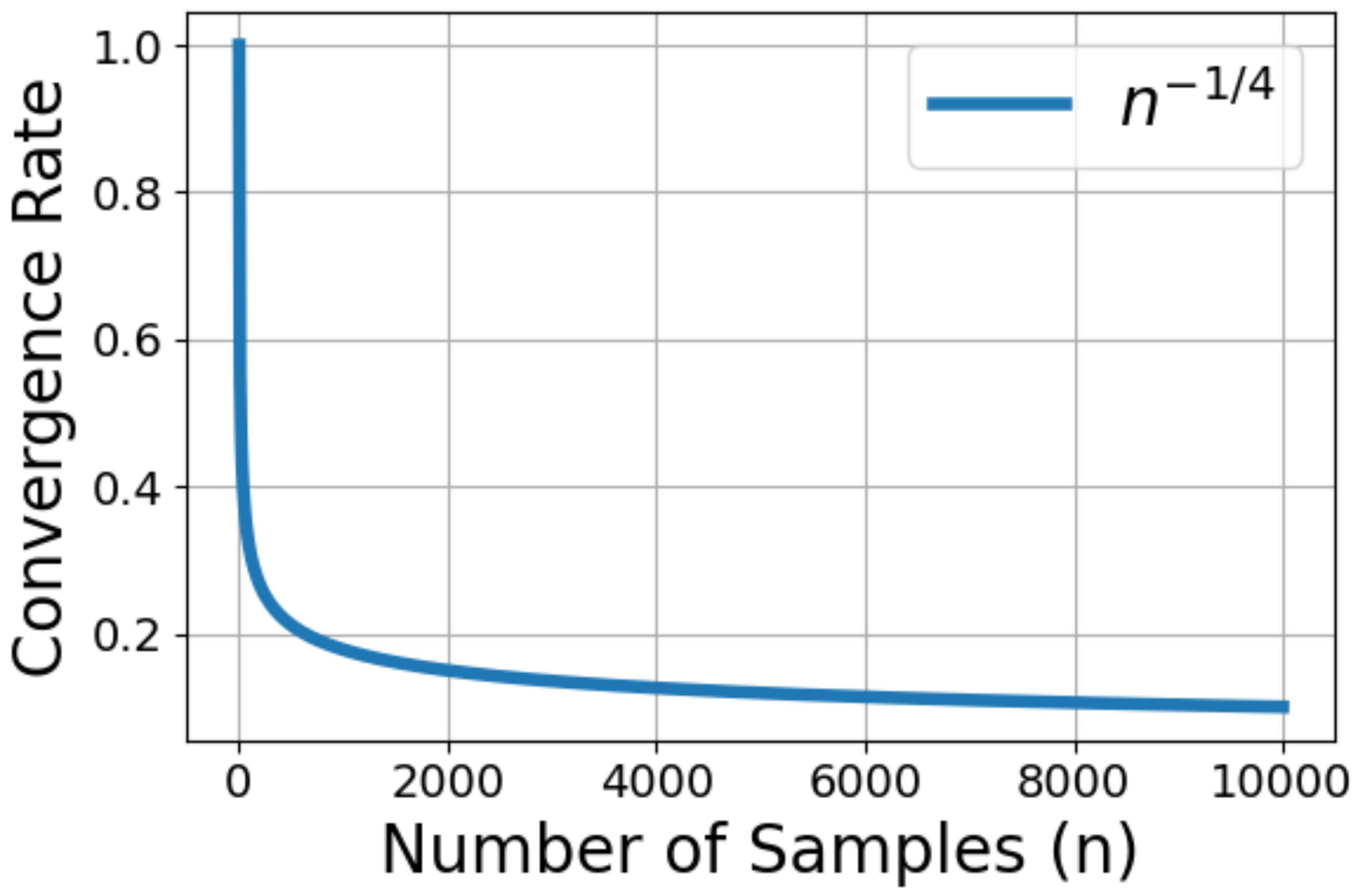

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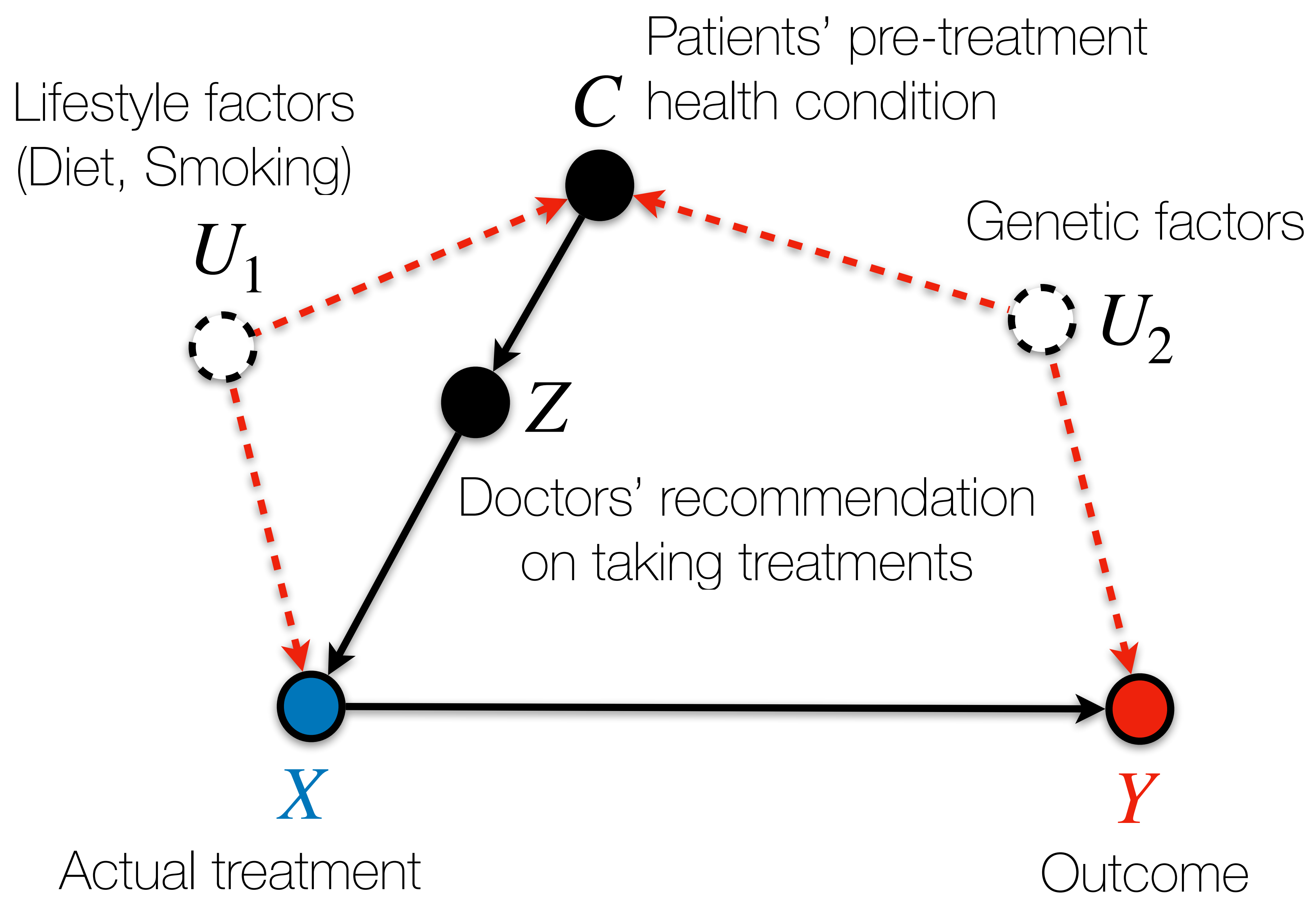

$$\mathbb{E}[Y \mid do(x)]$$

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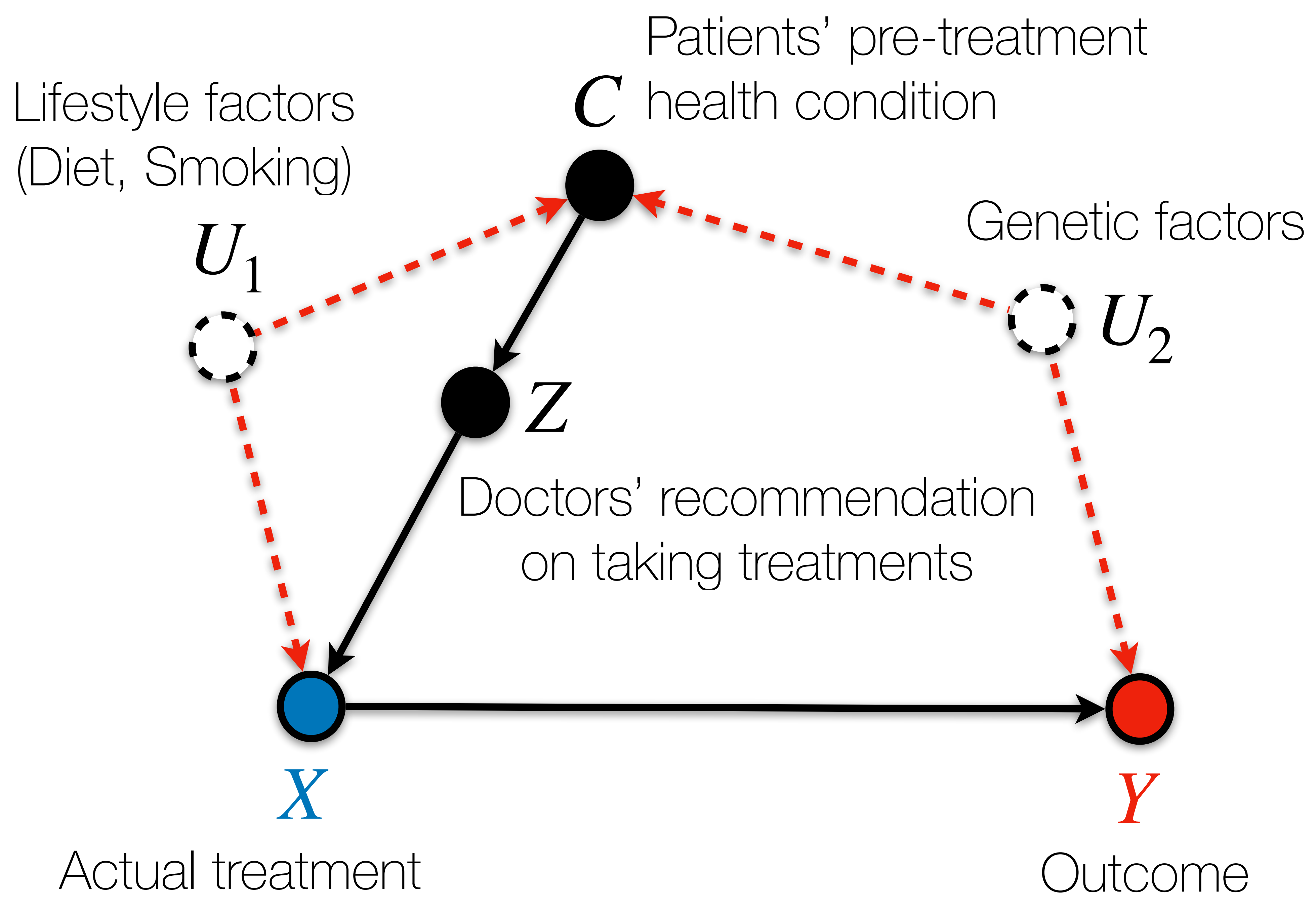

$\mathbb{E}[Y \mid do(x)]$

Identification

$$\mathbb{E}[Y \mid do(x)] = \frac{\sum_c \mathbb{E}[Y \mid x, z, c] P(x \mid z, c) P(c)}{\sum_c P(x \mid z, c) P(c)}$$

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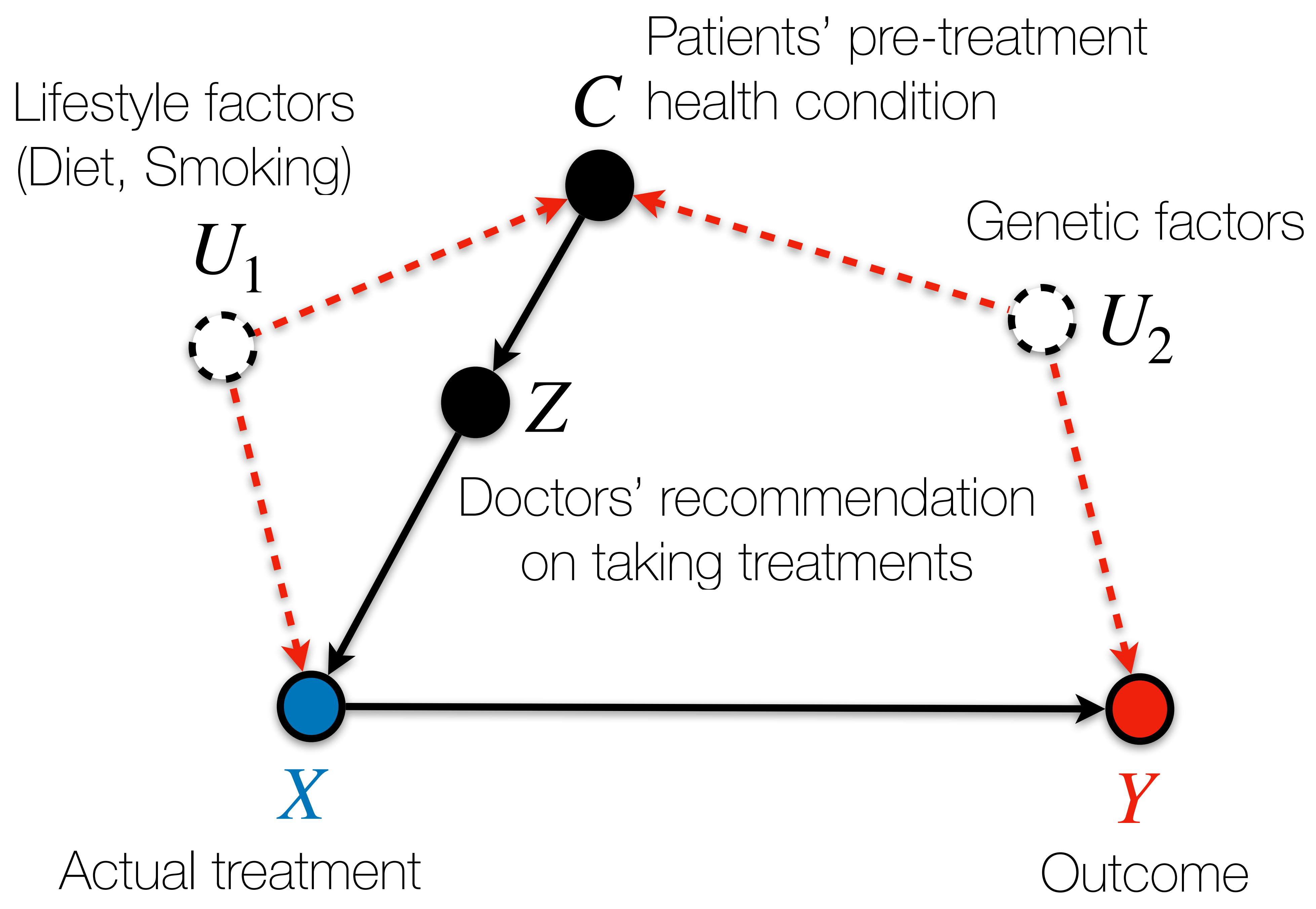

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Estimation

$$\widehat{\mathbb{E}[Y \mid do(x)]} = \ ?$$

Gap bw Identification & Estimation

Data	Scenario	Identification	Estimation
$D \sim P$ Observational	Back-door (BD)		
	Non-BD		

Gap bw Identification & Estimation

Data	Scenario	Identification	Estimation
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If $\mathbb{E}[Y \mid \mathrm{do}(x)]$ is expressible as **a function of BDs** (i.e., $\mathbb{E}[Y \mid \mathrm{do}(x)] = f(\{\text{BD}\})$),

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by strategically **combining robust BD estimators.**

Background: Causal Effect Identification

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Identification (Algo 1)

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- to reach to causal distribution $P(Y \mid \mathrm{do}(X))$
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"$P(Y \mid \mathrm{do}(X))$ is a function of $P(\mathbf{V})$ via factorizations & marginalizations"

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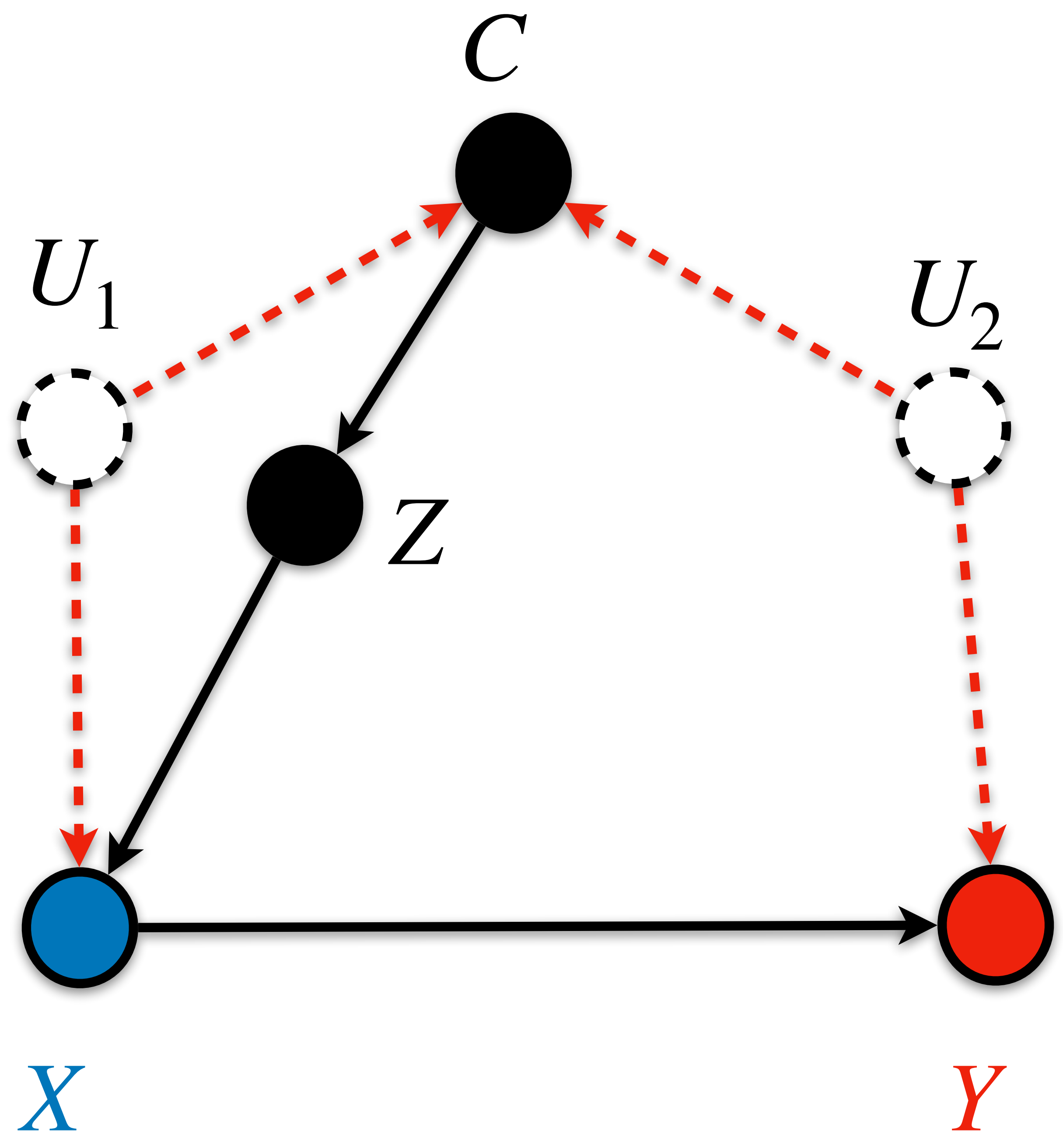

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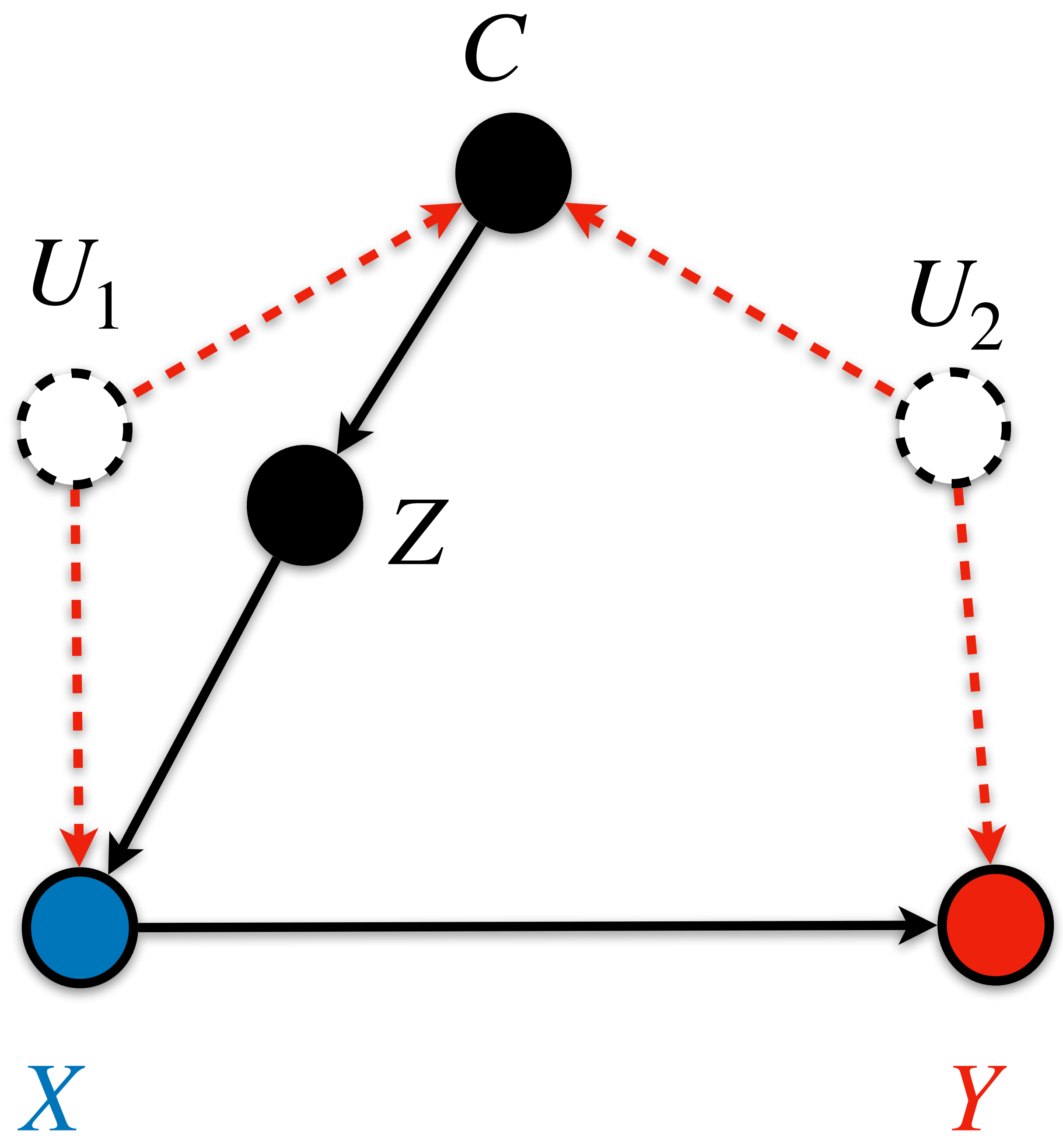

$P(CZXY)$

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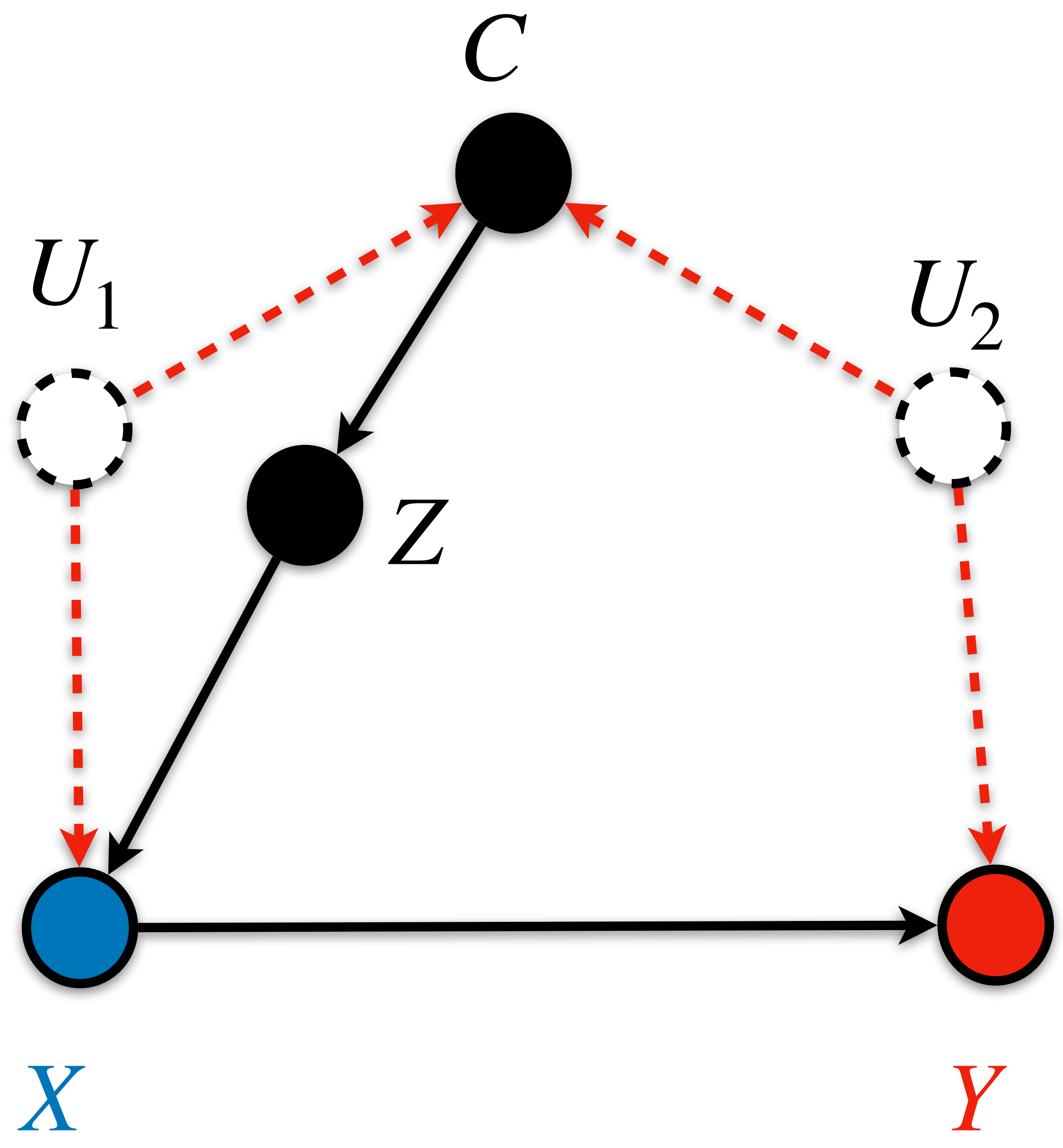

$$P(CZXY) \xrightarrow{\text{Factorization}} P_{\text{do}(Z)}(CXY)$$

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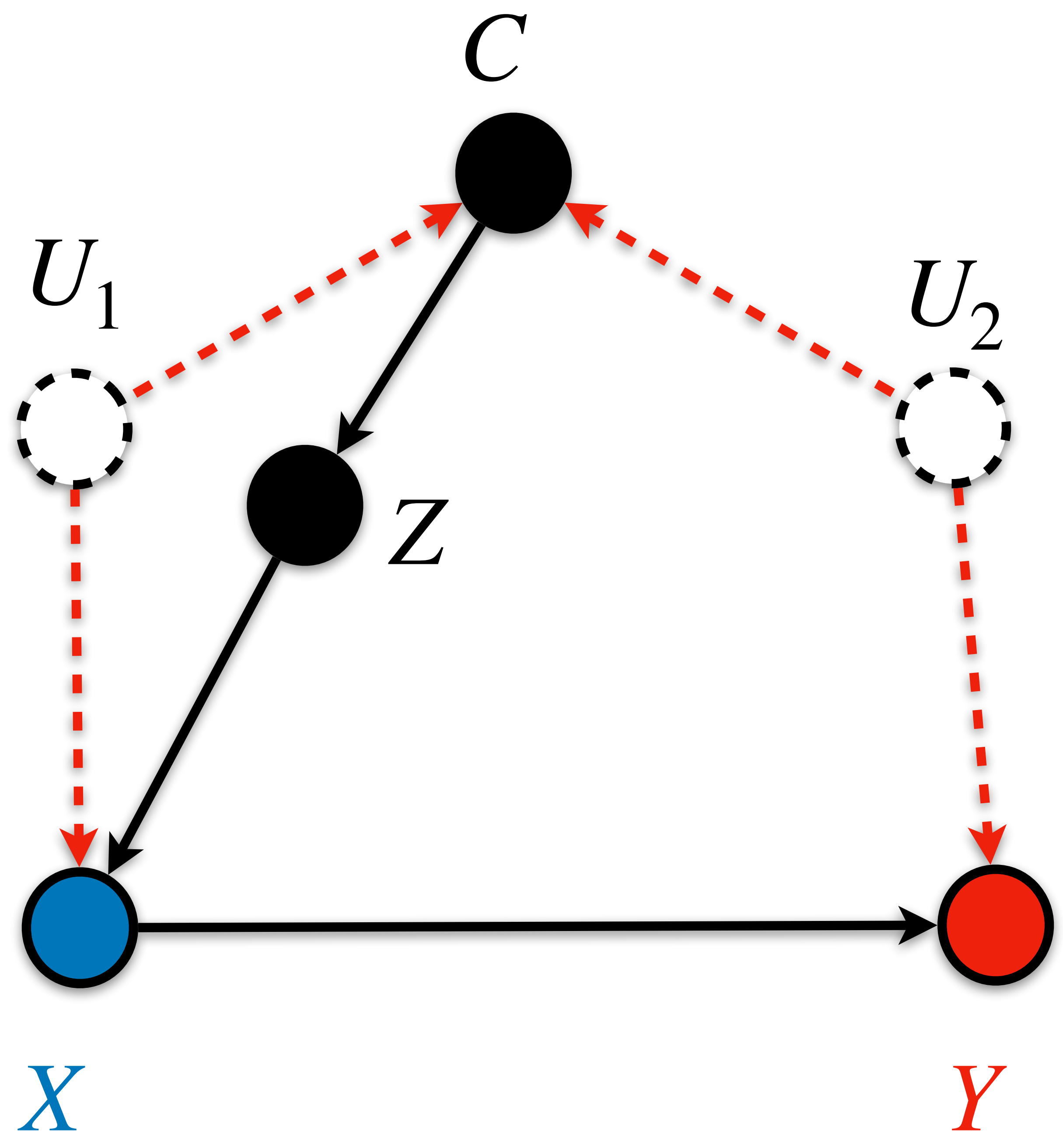

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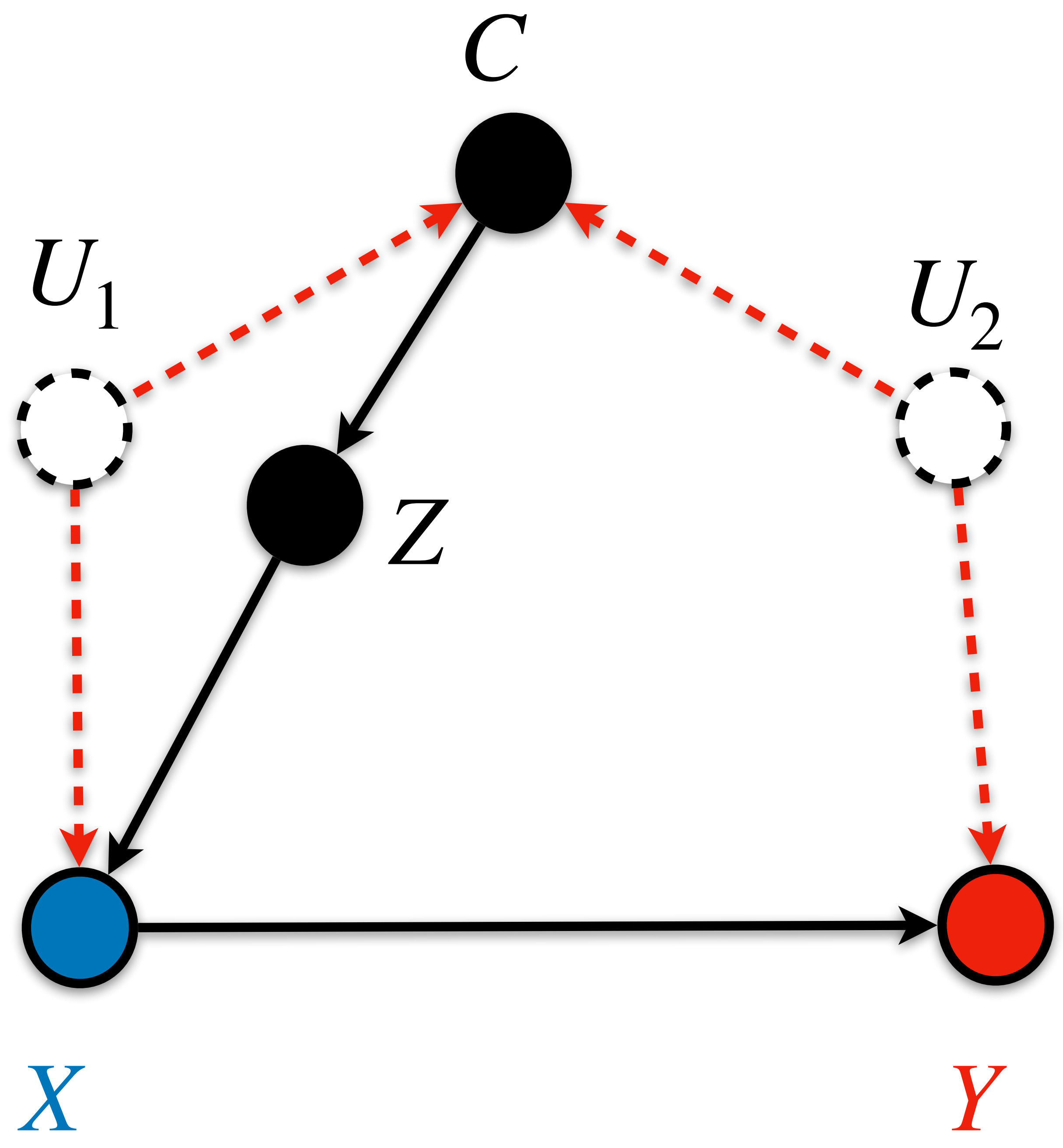

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$P(Y \mid \text{do}(X))$: $P_{\text{do}(Z)}(Y \mid X) = \dfrac{\sum_c P(c)P(XY \mid Zc)}{\sum_c P(c)P(X \mid Zc)}$

My Approach: 3-Step

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So far,

- *BDs (or mSBDs) can be estimated sample-efficiently using robust estimators*
- The computation tree for the effect identification is composed of *interventional distributions as intermediate nodes.*

My Approach: 3-Step

To connect BD & Identification,

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1. **Check** if each interventional distribution on the tree is expressible as BD

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3. **Construct** robust estimators by using robust BD estimators

Complete Criterion for mSBD Adjustment

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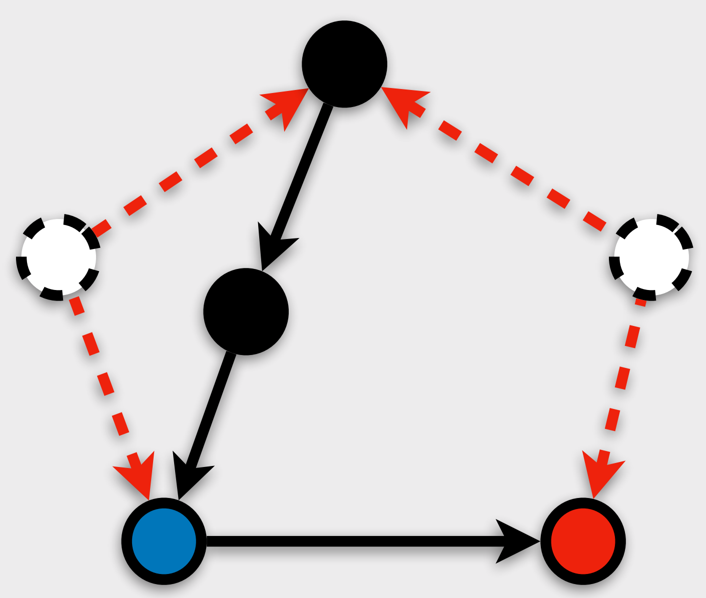
Graph



Interventional Distribution

$P_{\mathrm{do}(a)}(B)$

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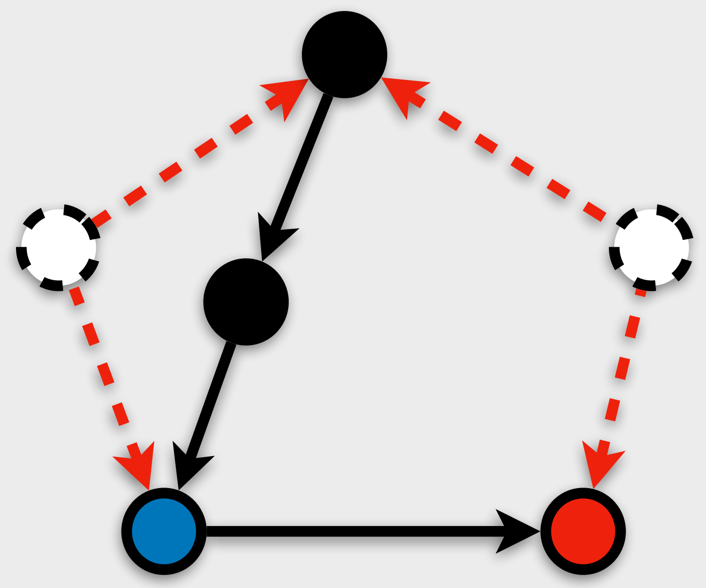

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Sequential Adjustment Criterion

(Sec. 3.2.1)

Check if $P_{\mathrm{do}(a)}(B)$ is expressed as BD

Complete Criterion for mSBD Adjustment

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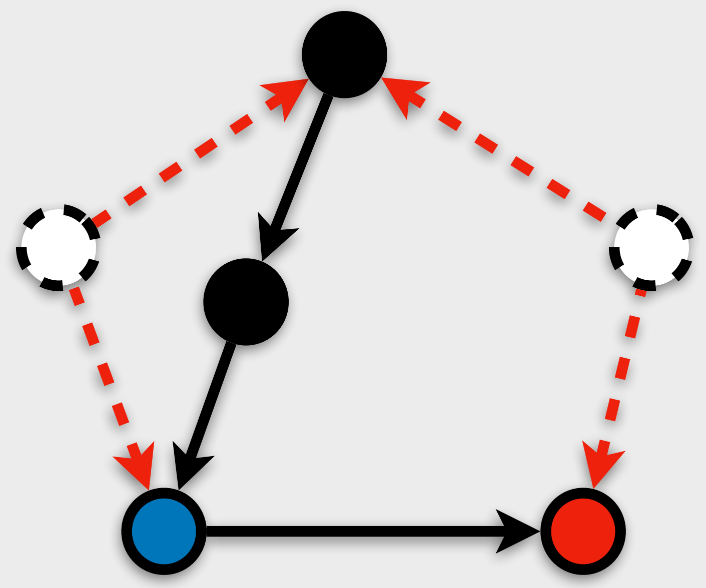

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Sequential Adjustment Criterion

(Sec. 3.2.1)

Check if $P_{\mathrm{do}(a)}(B)$ is expressed as BD

→ $P_{\mathrm{do}(a)}(B) = \mathrm{BD}(\mu, \pi)$ if BD-expressible

Motivation: Incompleteness of BD/mSBD (Sec 3.2)

$\exists$ examples s.t. $P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}))$ is BD adjustment even if BD criterion fails.

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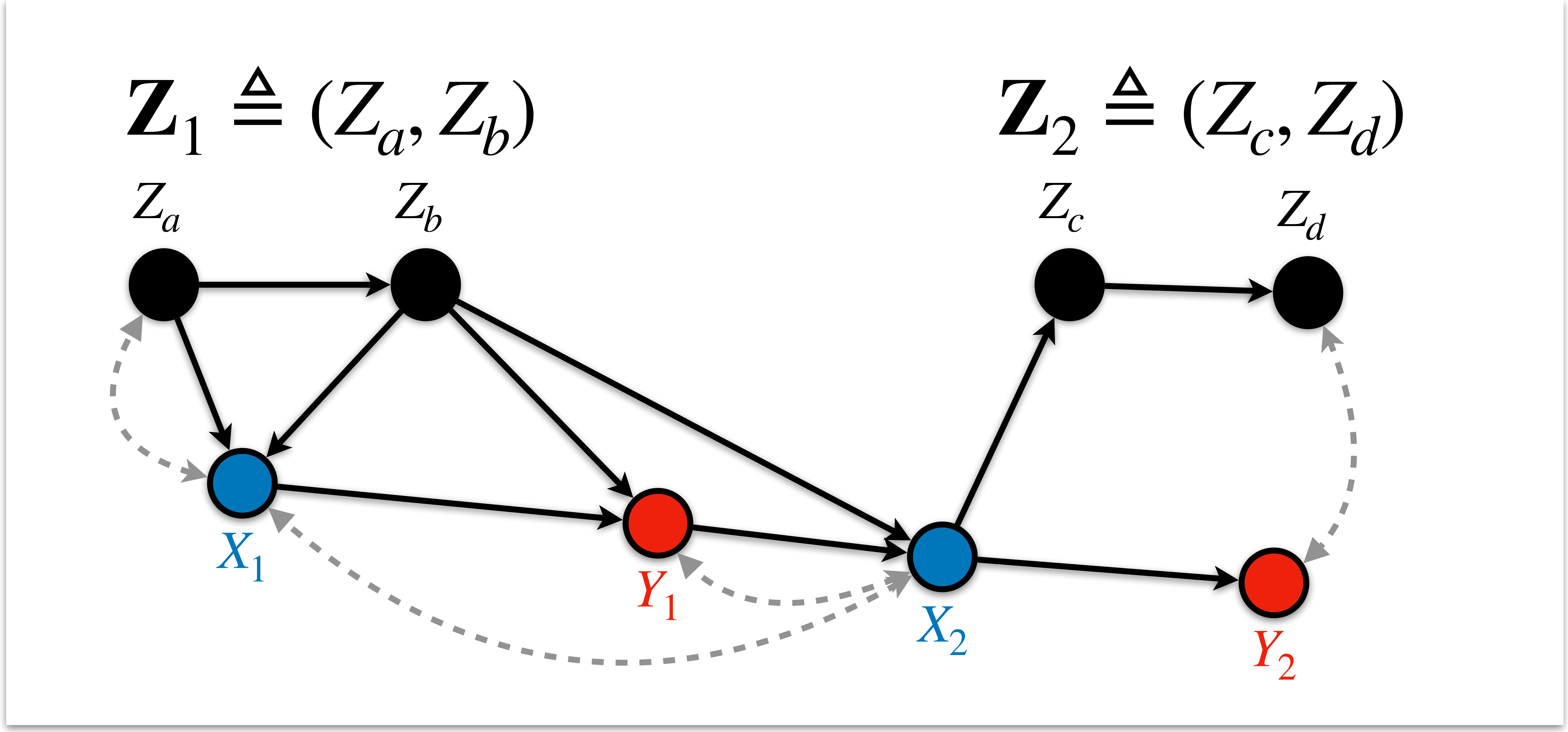

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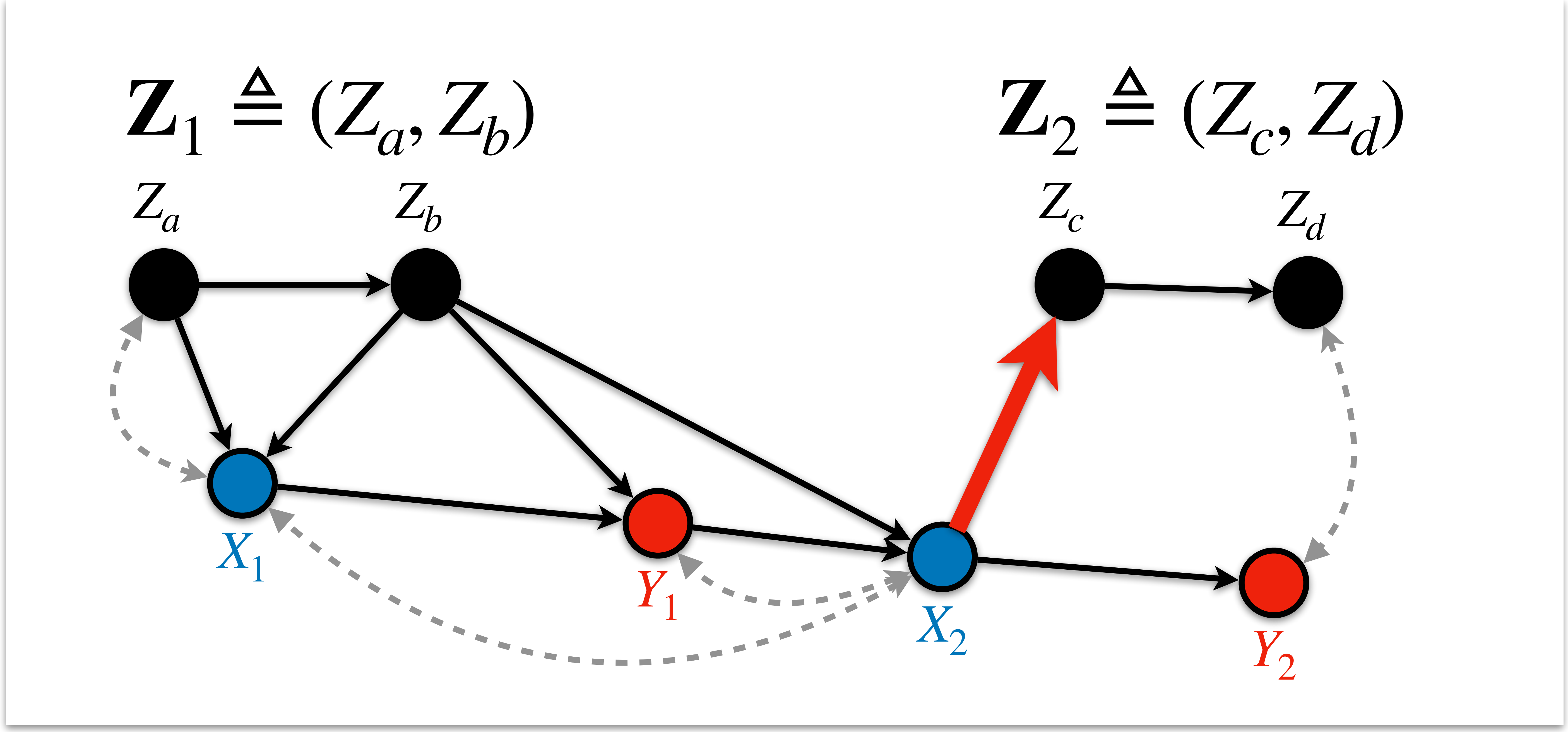

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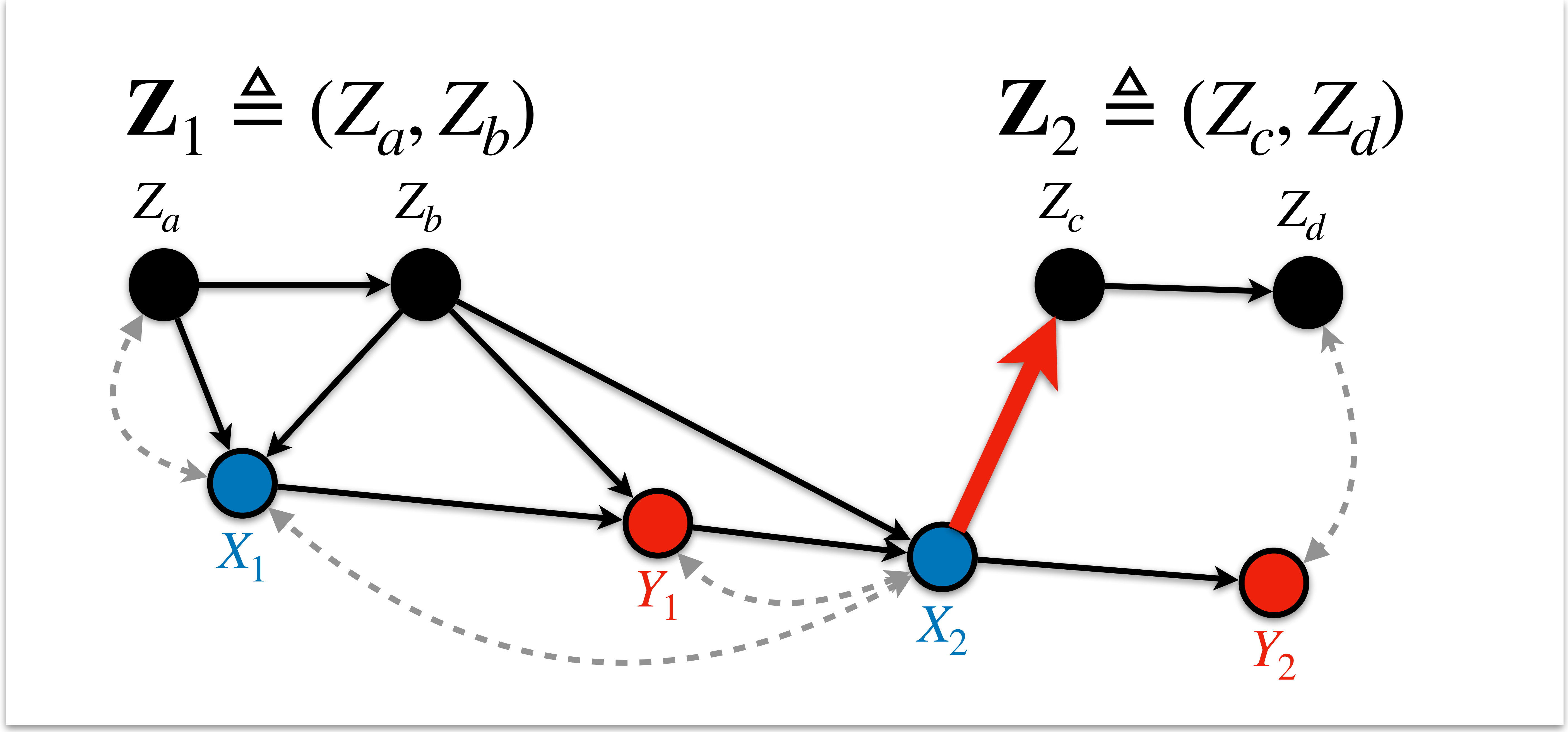

❶ $\mathbf{Z}$ doesn't satisfy the mSBD criterion

"mSBD adjustment"

❷ $P(y_1 y_2 \mid do(x_1 x_2)) = \sum_{\mathbf{z}_1 \mathbf{z}_2} \overbrace{P(y_2 \mid \text{prev}_1, \mathbf{z}_2 x_2) P(y_1 \mathbf{z}_2 \mid \mathbf{z}_1 x_1) P(\mathbf{z}_1)}^{\text{"mSBD adjustment"}}$

Complete Seq. Adjustment Criterion (Sec 3.2)

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A seq. $\mathbf{Z} = (\mathbf{Z}_1, \cdots, \mathbf{Z}_m)$ satisfies the SAC if, for $i = 1, \cdots, m$, $\mathbf{Z}_i \cup \mathbf{prev}_{i-1}$ satisfies the *adjustment criterion* relative to $(\mathbf{X}_i, \mathbf{Y}^{\geq i})$

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Complete criterion for the BD adjustment [Shpitser et al., 2010, van der Zander et al., 2014)

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$\Leftrightarrow$

[Theorem 10] Completeness

$P(\mathbf{y} \mid \text{do}(\mathbf{x}))$ is given as mSBD.

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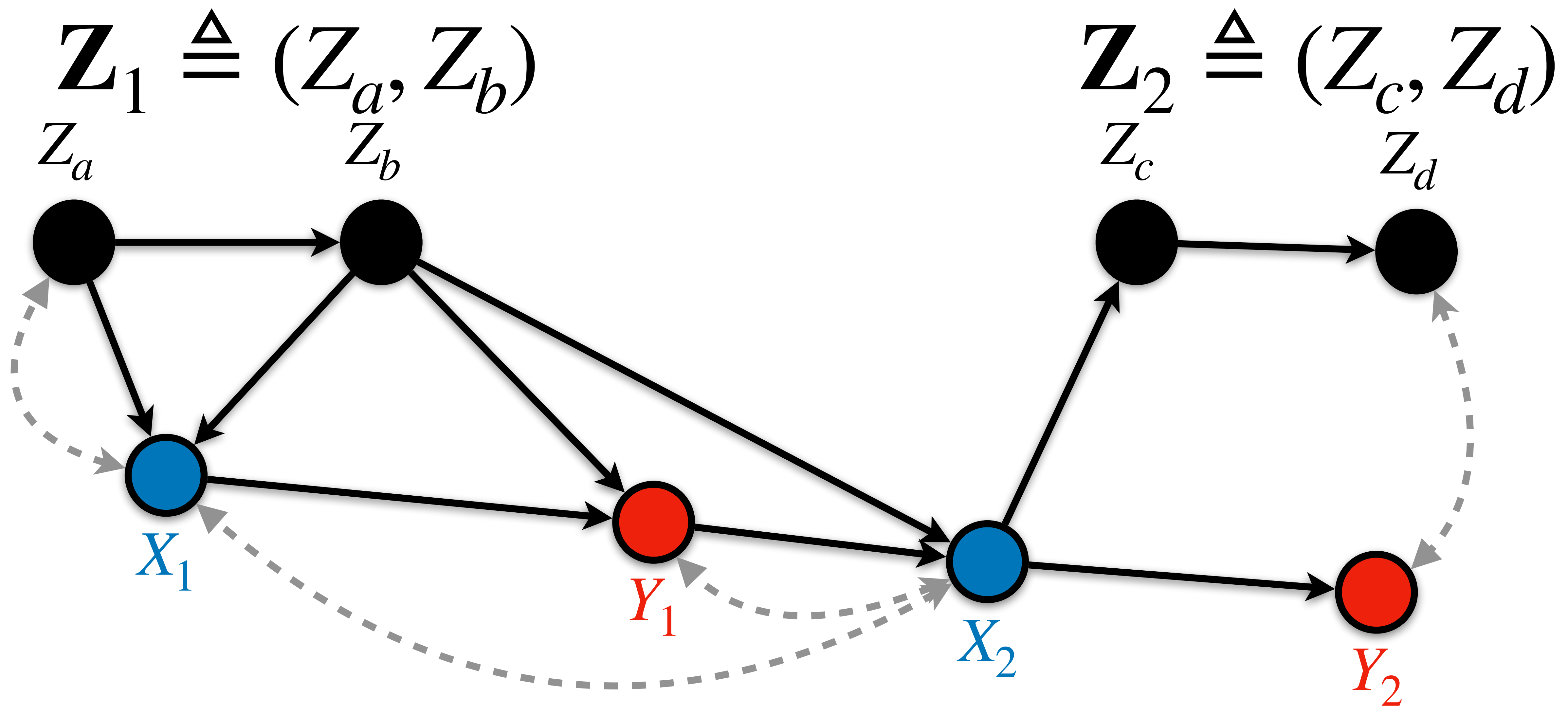
$\Leftrightarrow$

[Theorem 10] Completeness

$P(\mathbf{y} \mid do(\mathbf{x}))$ is given as mSBD.



X mSBD fails

Complete Seq. Adjustment Criterion (Sec 3.2)

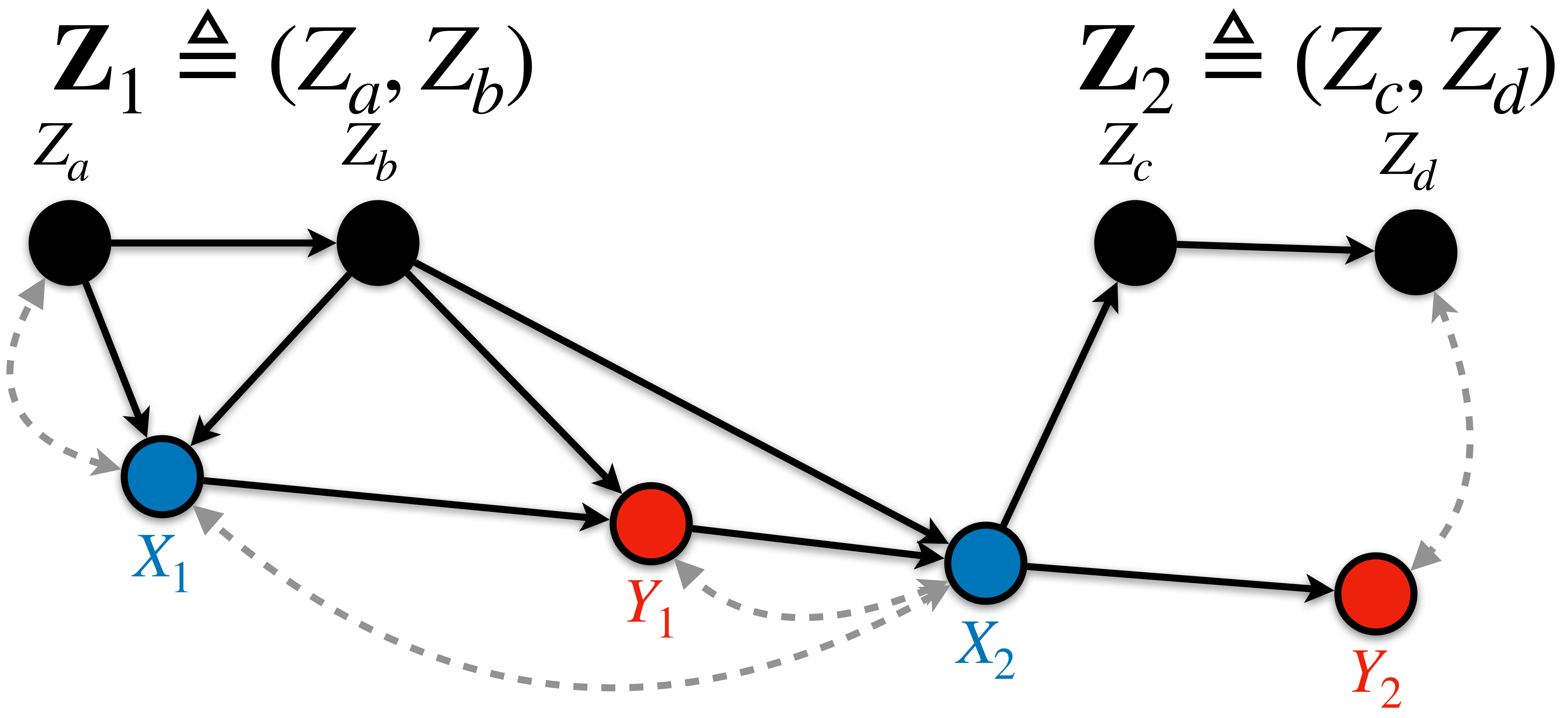
[Def. 29] Sequential Adjustment Criterion (SAC)

A seq. $\mathbf{Z} = (\mathbf{Z}_1, \cdots, \mathbf{Z}_m)$ satisfies the SAC if, for $i = 1, \cdots, m$, $\mathbf{Z}_i \cup \mathbf{prev}_{i-1}$ satisfies the *adjustment criterion* relative to $(\mathbf{X}_i, \mathbf{Y}^{\geq i})$

$\Leftrightarrow$

[Theorem 10] Completeness

$P(\mathbf{y} \mid do(\mathbf{x}))$ is given as mSBD.



✗ mSBD fails

✔ SAC holds

Estimating Causal Effects in 3-Steps

Estimating Causal Effects in 3-Steps

2 **Express** causal effects as a function of BD

Estimating Causal Effects in 3-Steps

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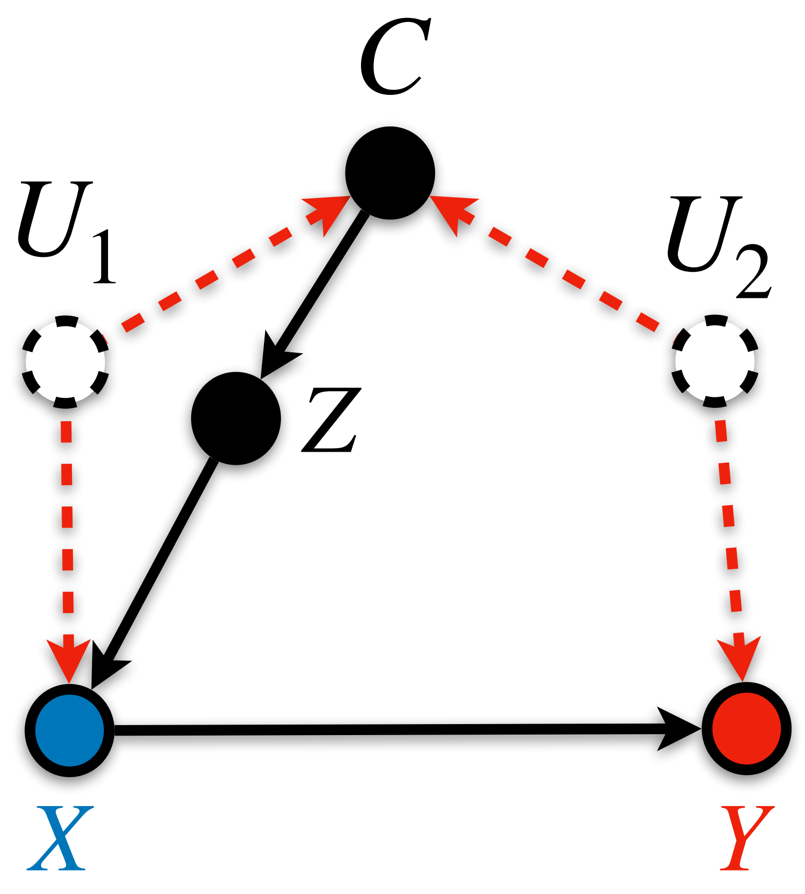


$$P(CZXY) \xrightarrow{\text{Factorization}} P_{\text{do}(Z)}(CXY) \xrightarrow{\text{Marginalization}} P_{\text{do}(Z)}(XY) \xrightarrow{\text{Factorization}} P(Y \mid \text{do}(X))$$

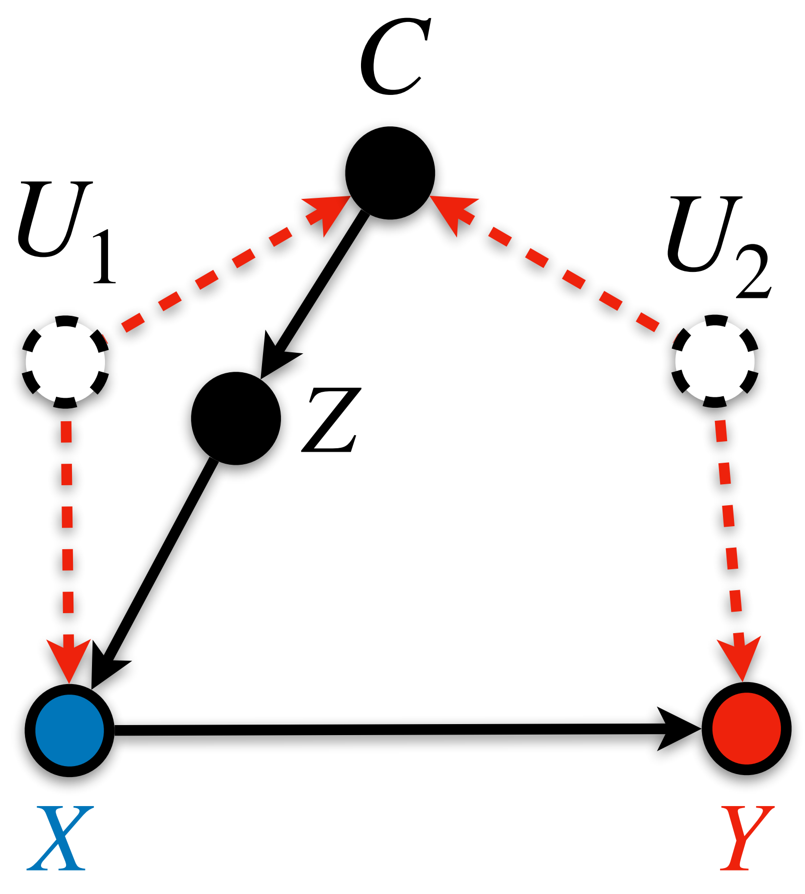
$P_{\text{do}(Z)}(CXY)$: $P(C)P(XY \mid ZC)$

$P_{\text{do}(Z)}(XY)$: $\sum_c P(c)P(XY \mid Zc)$

$P(Y \mid \text{do}(X))$: $\dfrac{\sum_c P(c)P(XY \mid Zc)}{\sum_c P(c)P(X \mid Zc)}$

Estimating Causal Effects in 3-Steps

2 **Express** causal effects as a function of BD

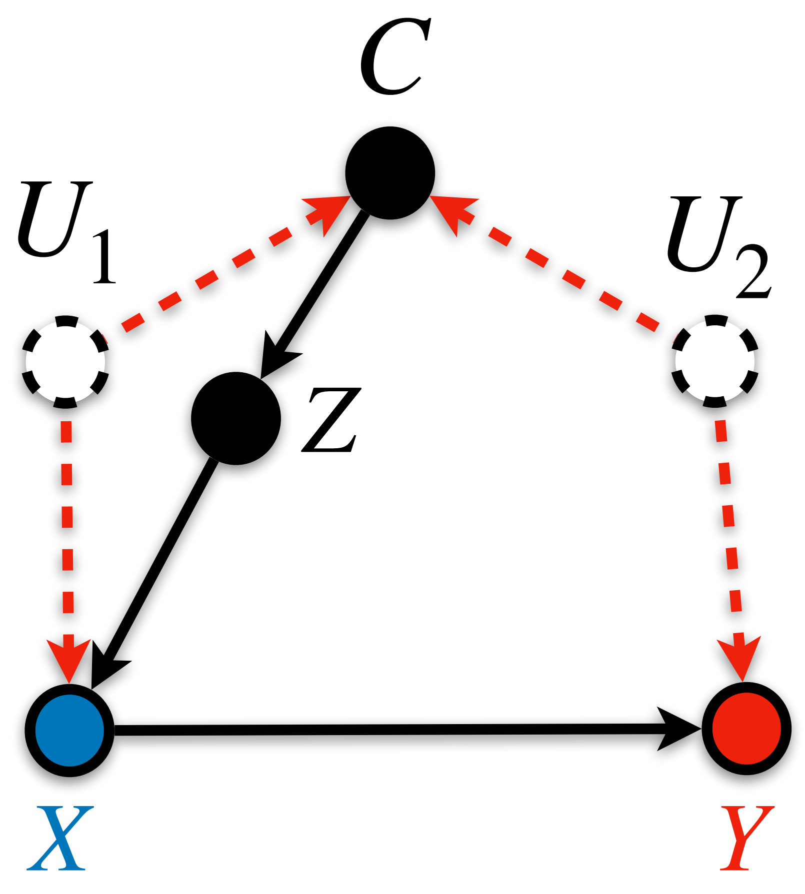


$$P(CZXY) \xrightarrow{\text{Factorization}} P_{\text{do}(Z)}(CXY) \xrightarrow{\text{Marginalization}} P_{\text{do}(Z)}(XY) \xrightarrow{\text{Factorization}} P(Y \mid \text{do}(X))$$

SAC

Estimating Causal Effects in 3-Steps

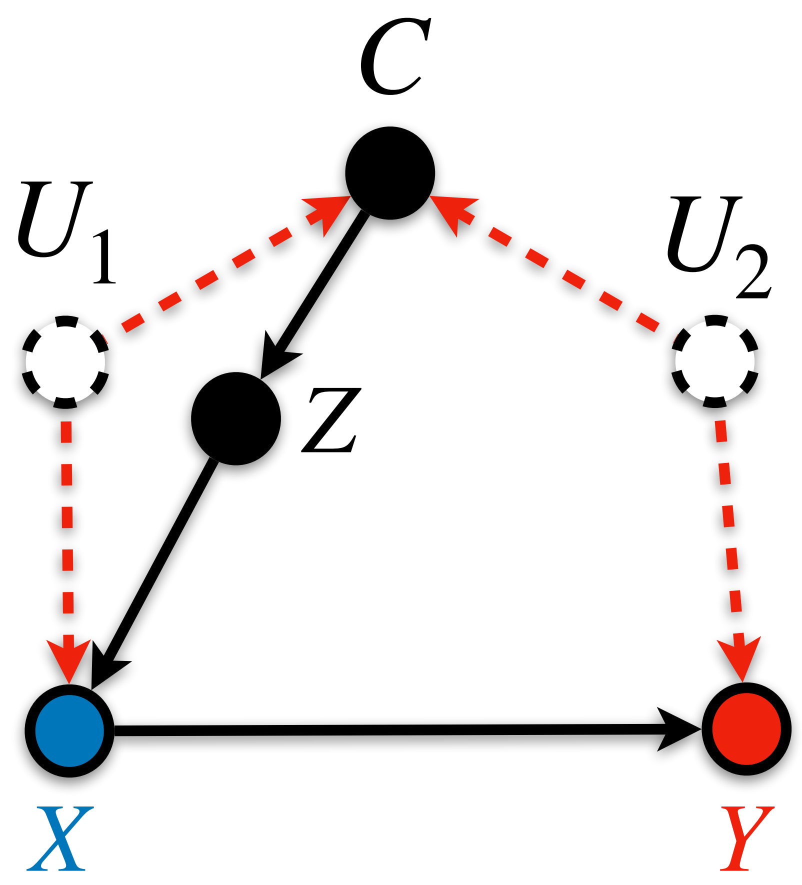
2 **Express** causal effects as a function of BD



$$P(CZXY) \xrightarrow{\text{Factorization}} P_{\text{do}(Z)}(CXY) \xrightarrow{\text{Marginalization}} P_{\text{do}(Z)}(XY) \xrightarrow{\text{Factorization}} P(Y \mid \text{do}(X))$$

Estimating Causal Effects in 3-Steps

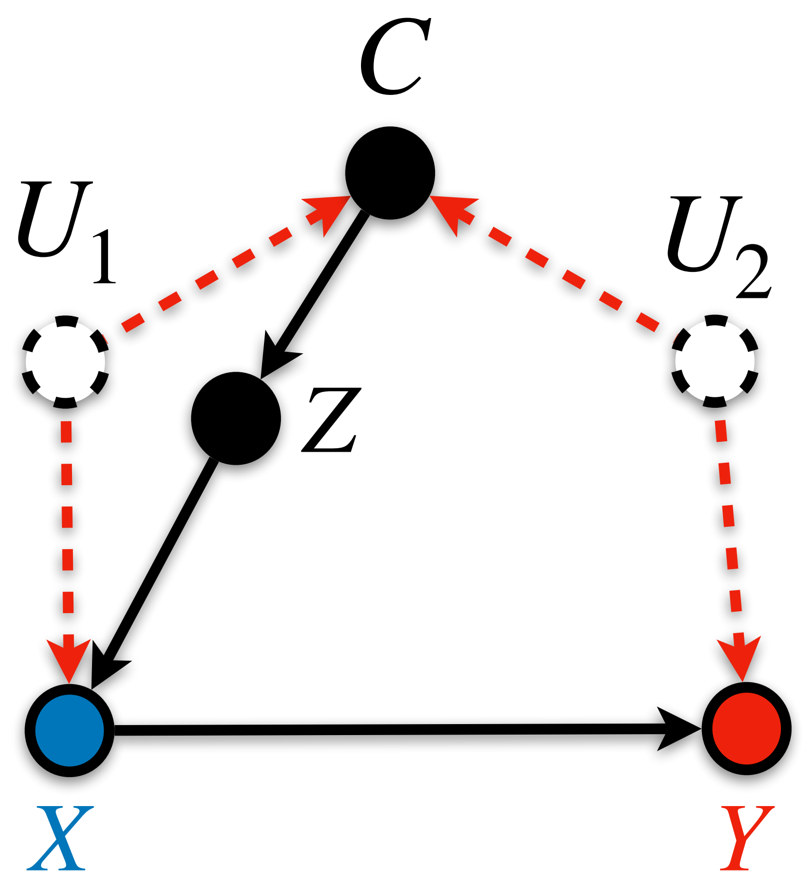
2 **Express** causal effects as a function of BD



$\text{BD}_1(\mu, \pi) \xrightarrow{\text{Factorization}} P(Y \mid \text{do}(X))$

Estimating Causal Effects in 3-Steps

2 **Express** causal effects as a function of BD



$$= \frac{\text{BD}_1(\mu, \pi)}{\text{BD}_2(\mu, \pi)}$$

$\text{BD}_1(\mu, \pi) \xrightarrow{\text{Factorization}} P(Y \mid \text{do}(X))$

Estimating Causal Effects in 3-Steps

2 **Express** causal effects as a function of BD

Theorem 14

The followings are equivalent:

1. $P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}))$ is identifiable from $(\mathcal{G}, P)$
2. $P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}))$ is expressible as a ***function of BDs*** through AdmissibleID（Algo 4）

DML-ID: Estimator for Identifiable Causal Effects

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3 **Construct** robust estimators by combining DML-BD

DML-ID: Estimator for Identifiable Causal Effects

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$$\mathbb{E}[Y \mid do(\mathbf{x})] = f(\{\text{BD}(\mu_1, \pi_1), \text{BD}(\mu_2, \pi_2), \cdots, \text{BD}(\mu_m, \pi_m)\})$$

DML-ID: Estimator for Identifiable Causal Effects

3 **Construct** robust estimators by combining DML-BD

$$\mathbb{E}[Y \mid do(\mathbf{x})] = f(\{BD(\mu_1, \pi_1), BD(\mu_2, \pi_2), \cdots, BD(\mu_m, \pi_m)\})$$

$$\widehat{\mathbb{E}[Y \mid do(\mathbf{x})]}$$

"DML-ID" (Def 36)

DML-ID: Estimator for Identifiable Causal Effects

3 **Construct** robust estimators by combining DML-BD

$$\mathbb{E}[Y \mid do(\mathbf{x})] = f(\{\text{BD}(\mu_1, \pi_1), \text{BD}(\mu_2, \pi_2), \cdots, \text{BD}(\mu_m, \pi_m)\})$$

$$\widehat{\mathbb{E}[Y \mid do(\mathbf{x})]} \triangleq f(\{ \qquad \})$$

"DML-ID" (Def 36)

DML-ID: Estimator for Identifiable Causal Effects

3 **Construct** robust estimators by combining DML-BD

$$\mathbb{E}[Y \mid do(\mathbf{x})] = f(\{\text{BD}(\mu_1, \pi_1), \text{BD}(\mu_2, \pi_2), \cdots, \text{BD}(\mu_m, \pi_m)\})$$

DML-BD ↓ DML-BD ↓ ⋯ DML-BD ↓

$$\widehat{\mathbb{E}[Y \mid do(\mathbf{x})]} \triangleq f(\{\widehat{\text{BD}}(\mu_1, \pi_1), \widehat{\text{BD}}(\mu_2, \pi_2), \ldots, \widehat{\text{BD}}(\mu_m, \pi_m)\})$$

"DML-ID" (Def 36)

Robustness of DML-ID

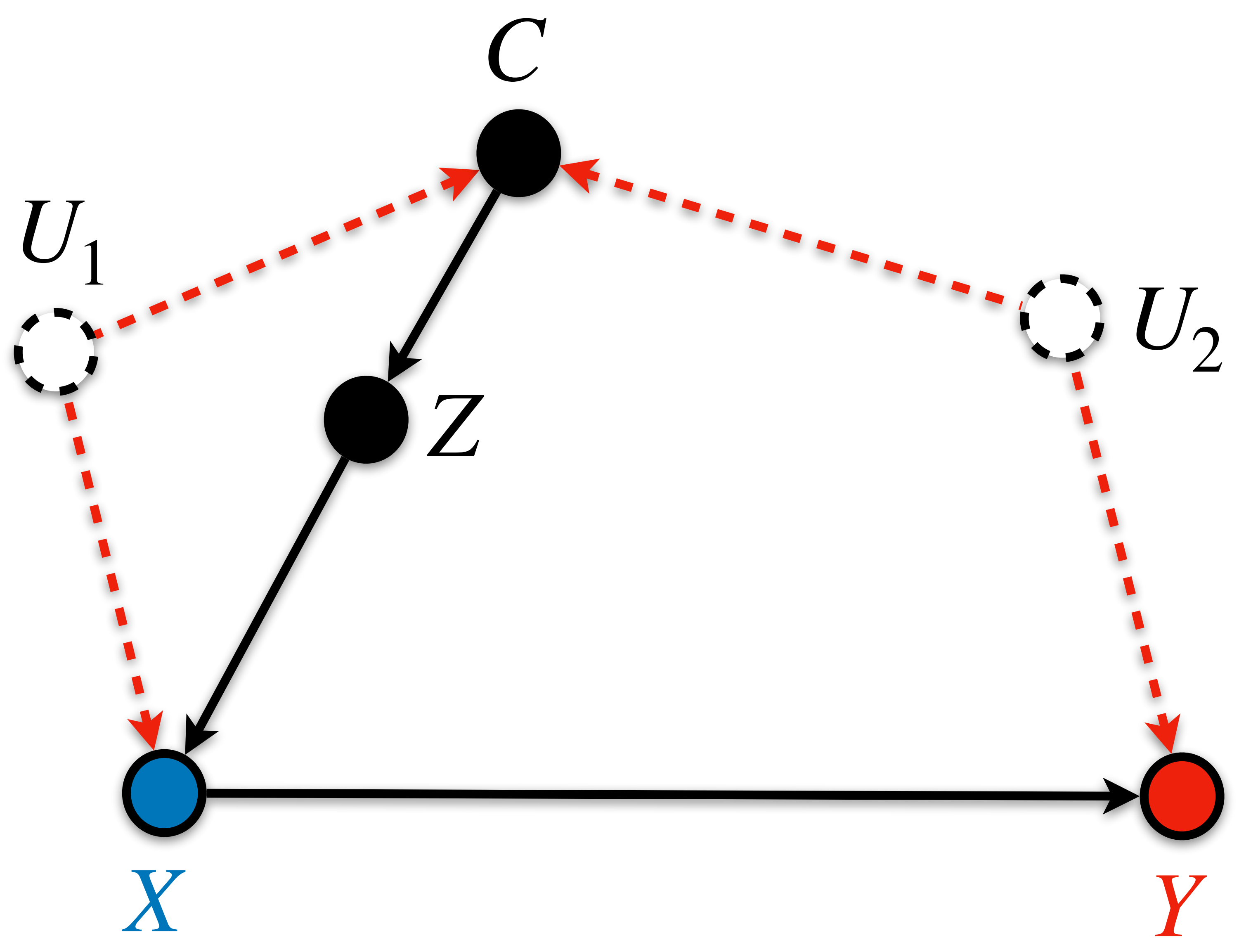
Theorem 16

$$\text{Error}(\text{DML-ID}, \mathbb{E}[Y \mid do(x)]) = \sum_{i=1}^{m} \text{Error}(\hat{\mu}_i, \mu_i) \times \text{Error}(\hat{\pi}_i, \pi_i)$$

- **Double Robustness:** $\text{Error} = 0$ if either $\hat{\mu}_i = \mu_i$ or $\hat{\pi}_i = \pi_i$ for all $i = 1, \cdots, m$.
- **Fast Convergence:** $\text{Error} \to 0$ *fast* even when $\hat{\mu}_i \to \mu_i$ and $\hat{\pi}_i \to \pi_i$ *slow*.

DML-ID - Simulation (Sec. 3.5)

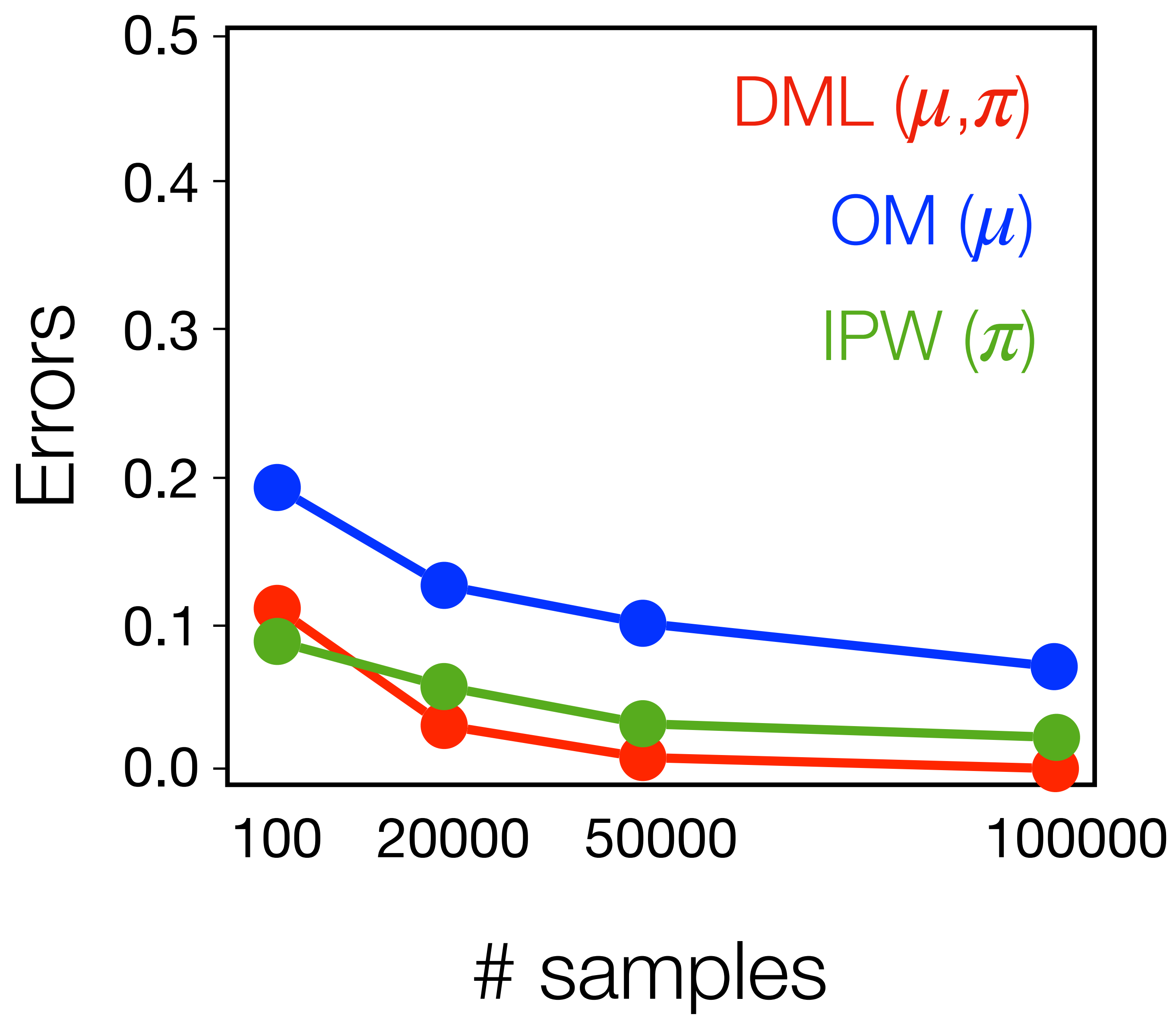
DML-ID - Simulation (Sec. 3.5)



DML-ID - Simulation (Sec. 3.5)

Fast Convergence

$(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\pi}}) \to (\boldsymbol{\mu}_0, \boldsymbol{\pi}_0)$ slowly



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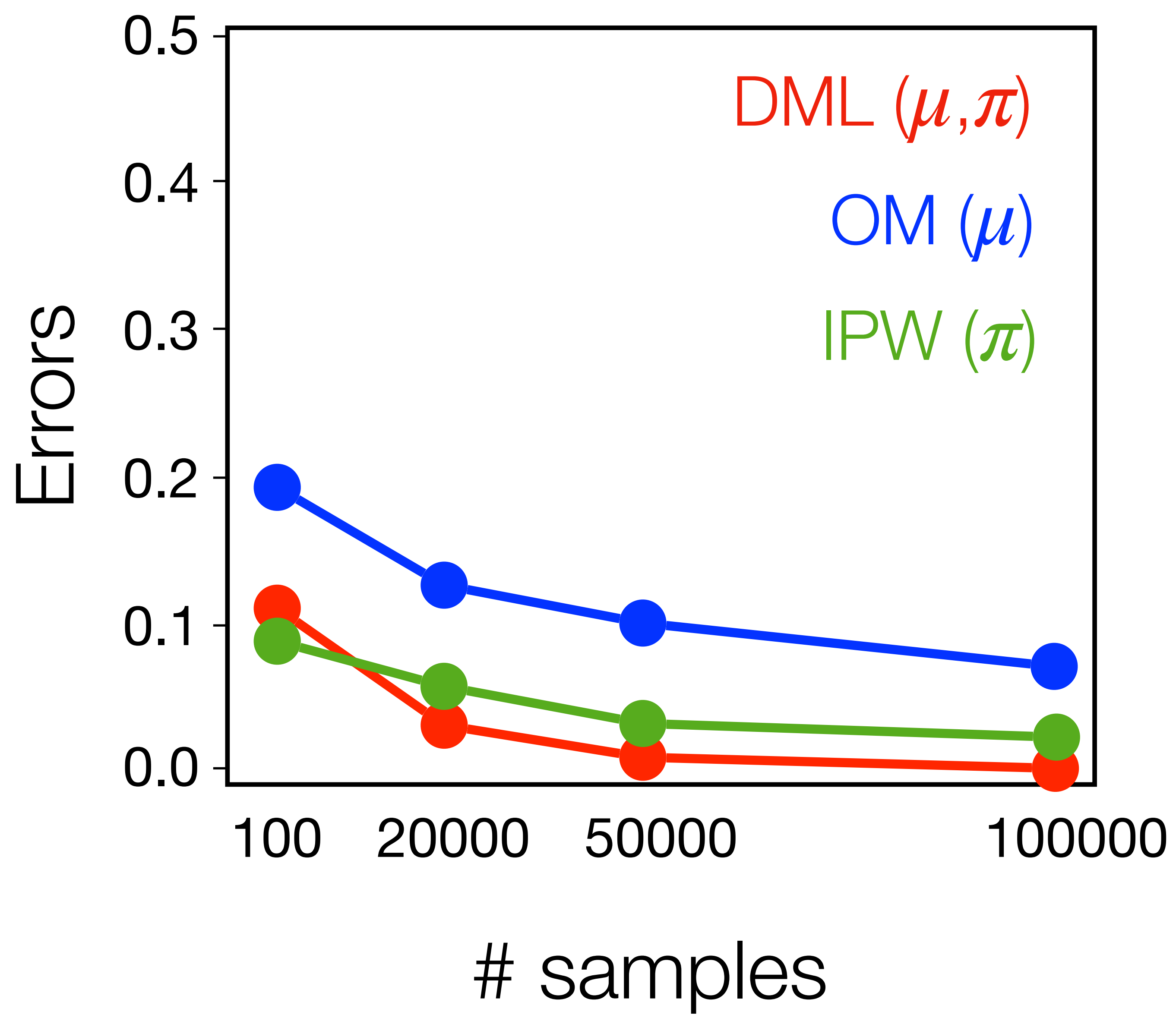
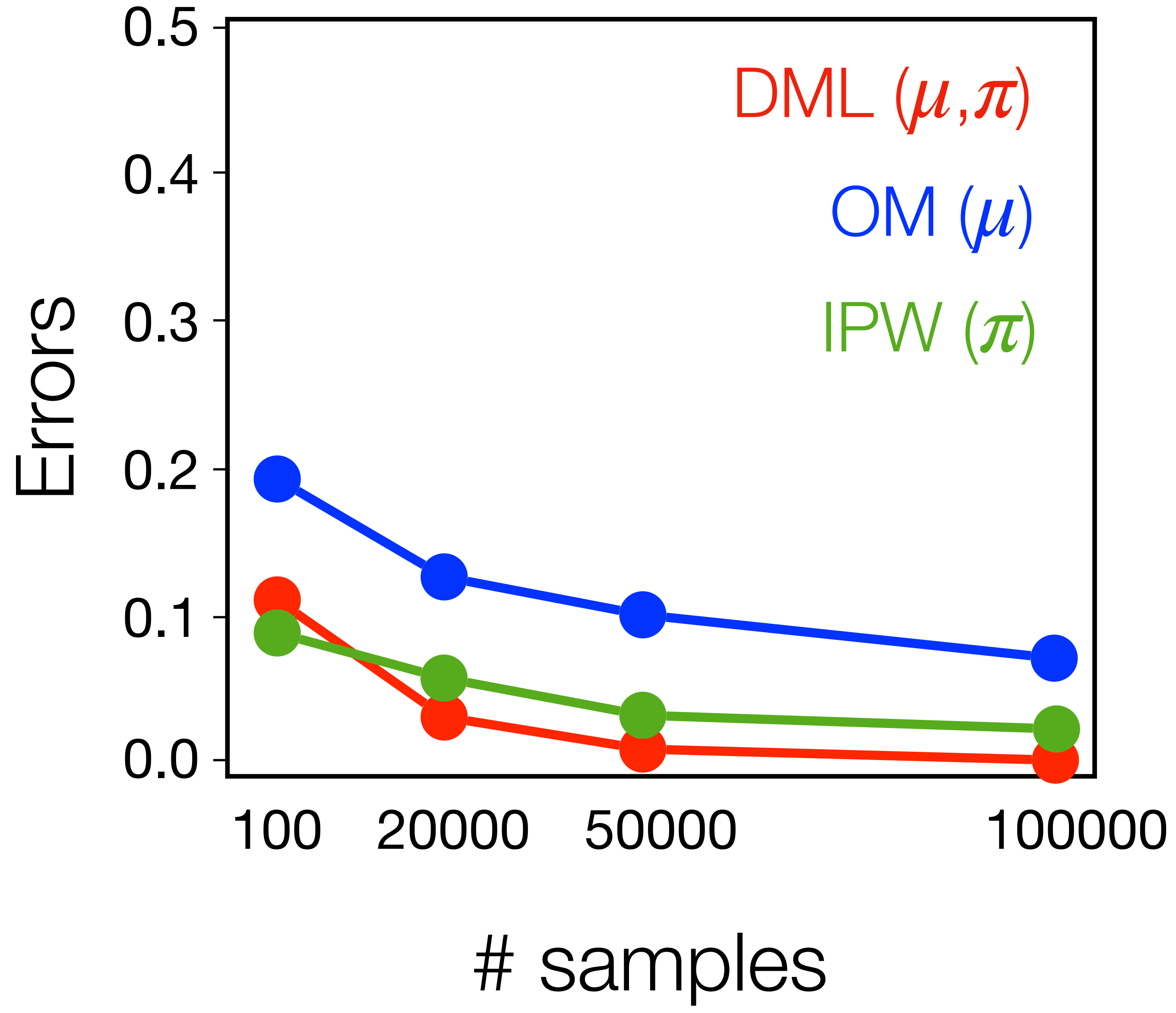


DML-ID converges fast, even when $(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\pi}})$ converge slowly

DML-ID - Simulation (Sec. 3.5)

Fast Convergence

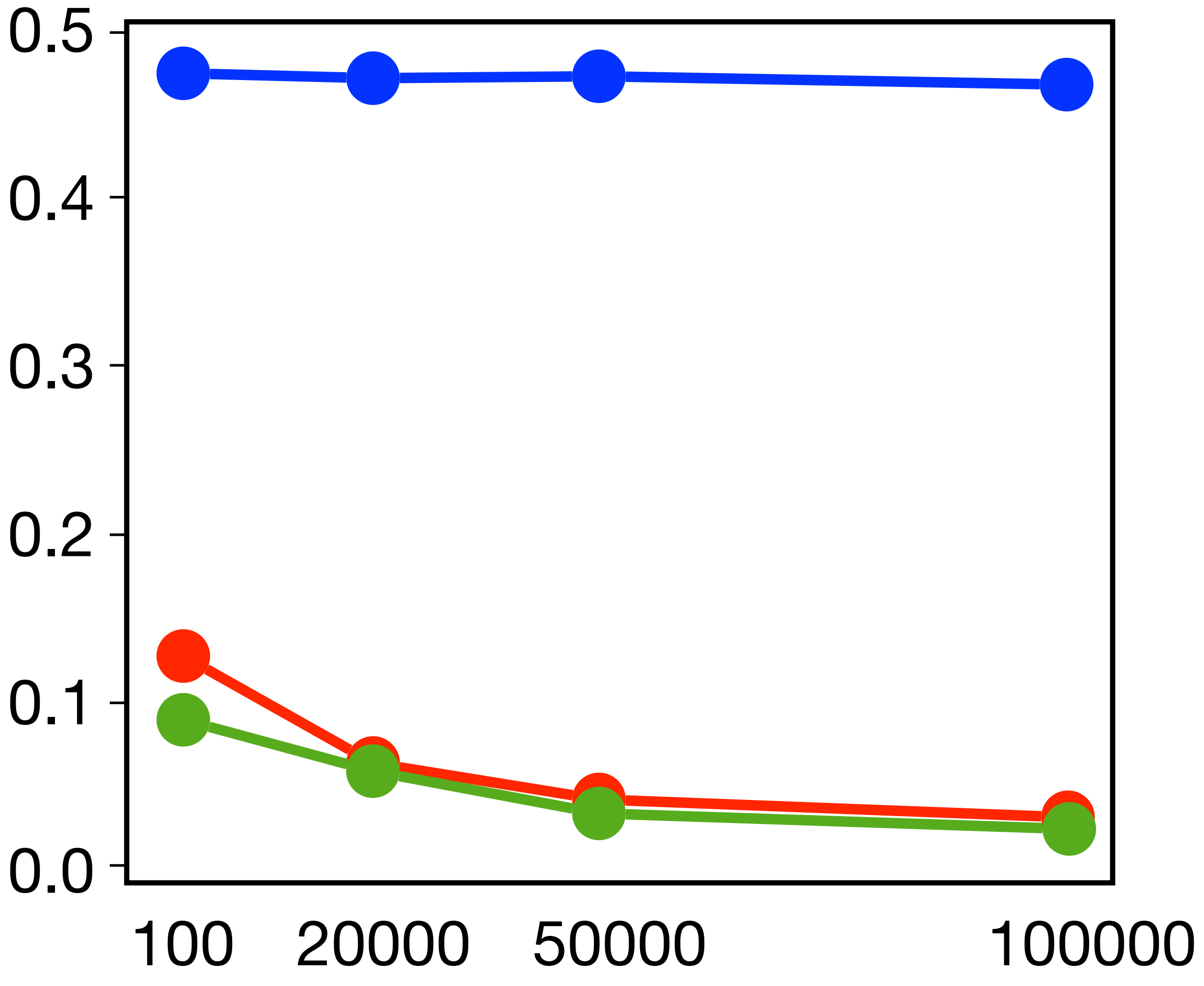
$(\hat{\mu},\hat{\pi}) \rightarrow (\mu_0, \pi_0)$ slowly



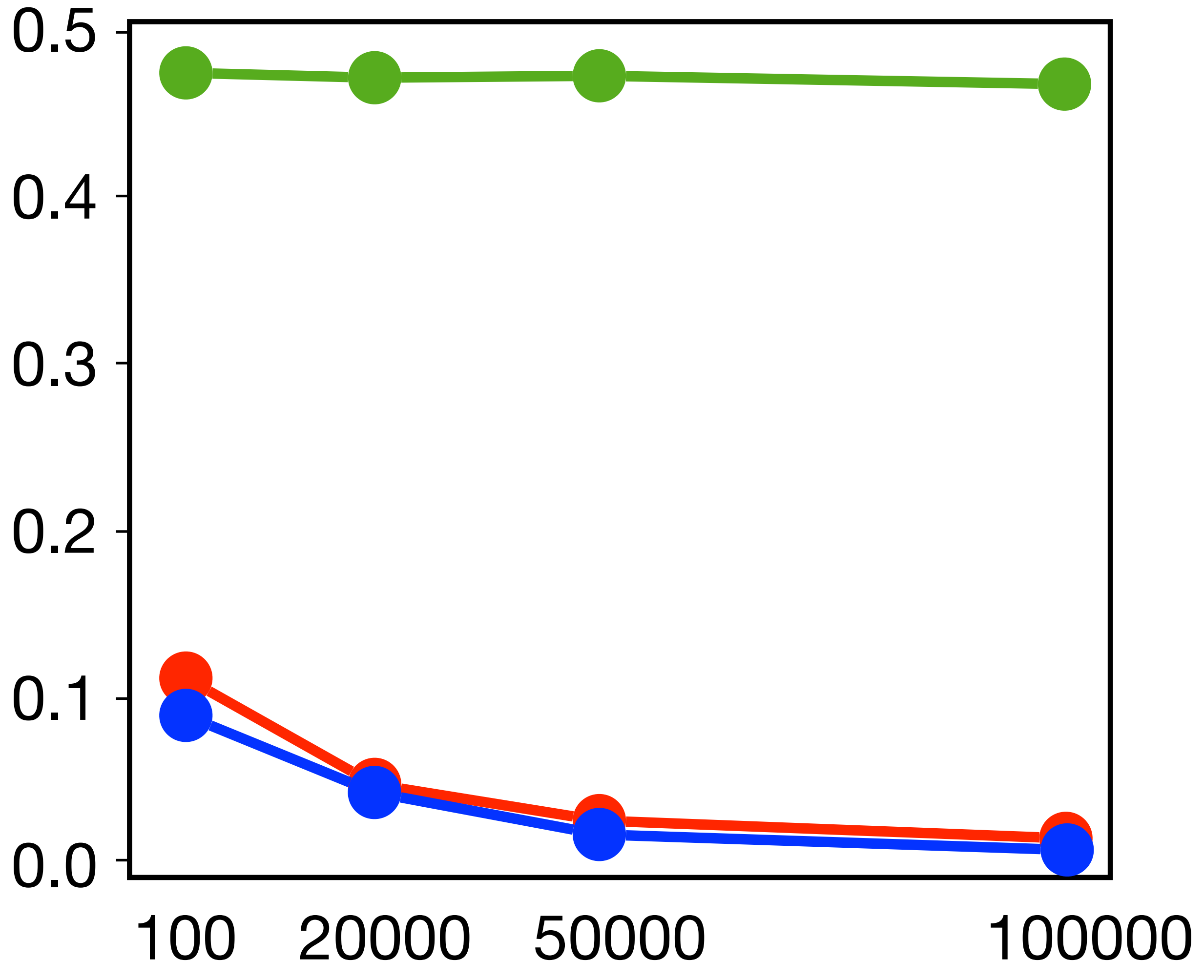
DML-ID converges fast, even when $(\hat{\mu},\hat{\pi})$ converge slowly

Double Robustness

$\hat{\mu}$ misspecified ($\hat{\mu} \neq \mu$)



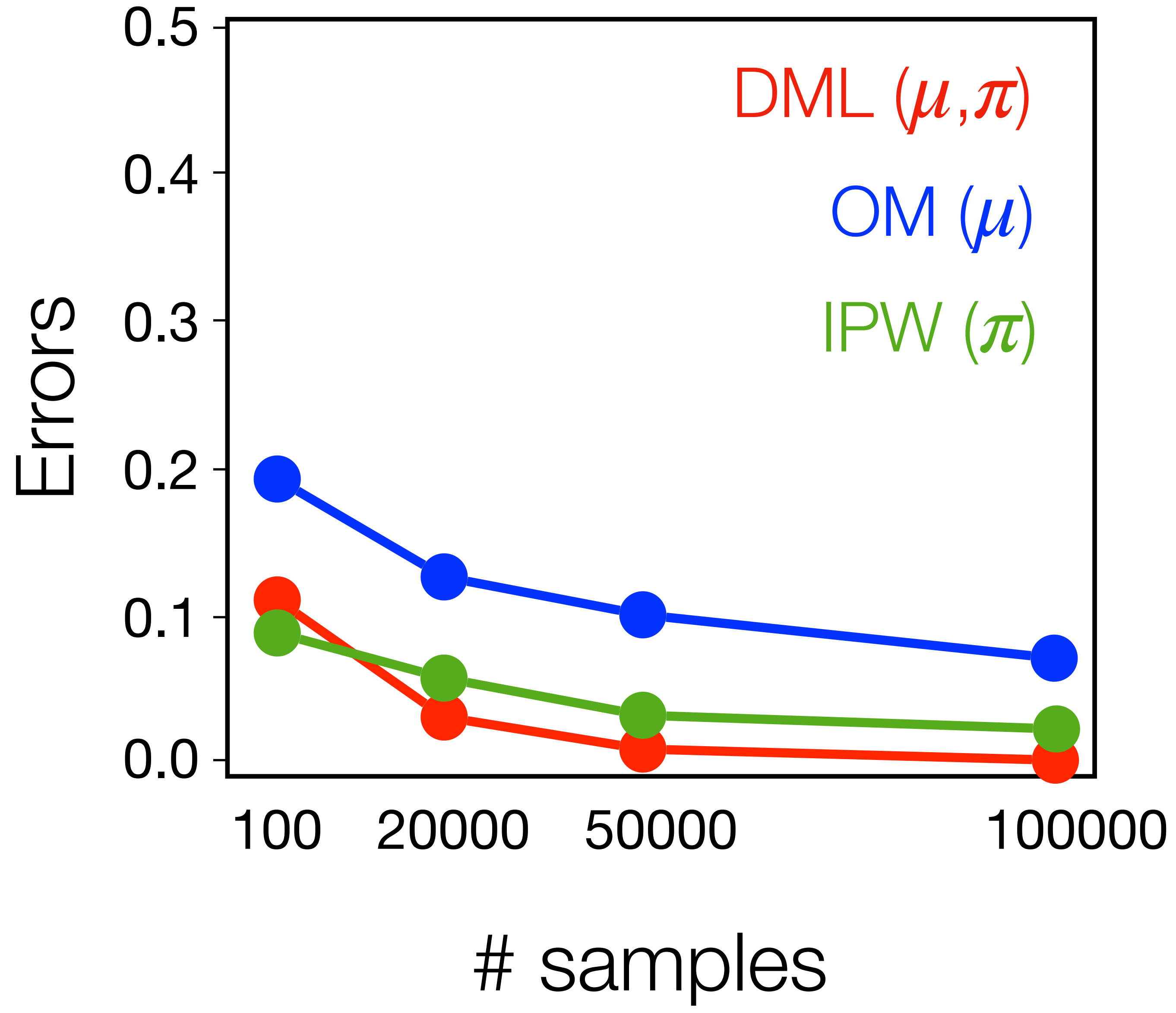
$\hat{\pi}$ misspecified ($\hat{\pi} \neq \pi$)



DML-ID - Simulation (Sec. 3.5)

Fast Convergence

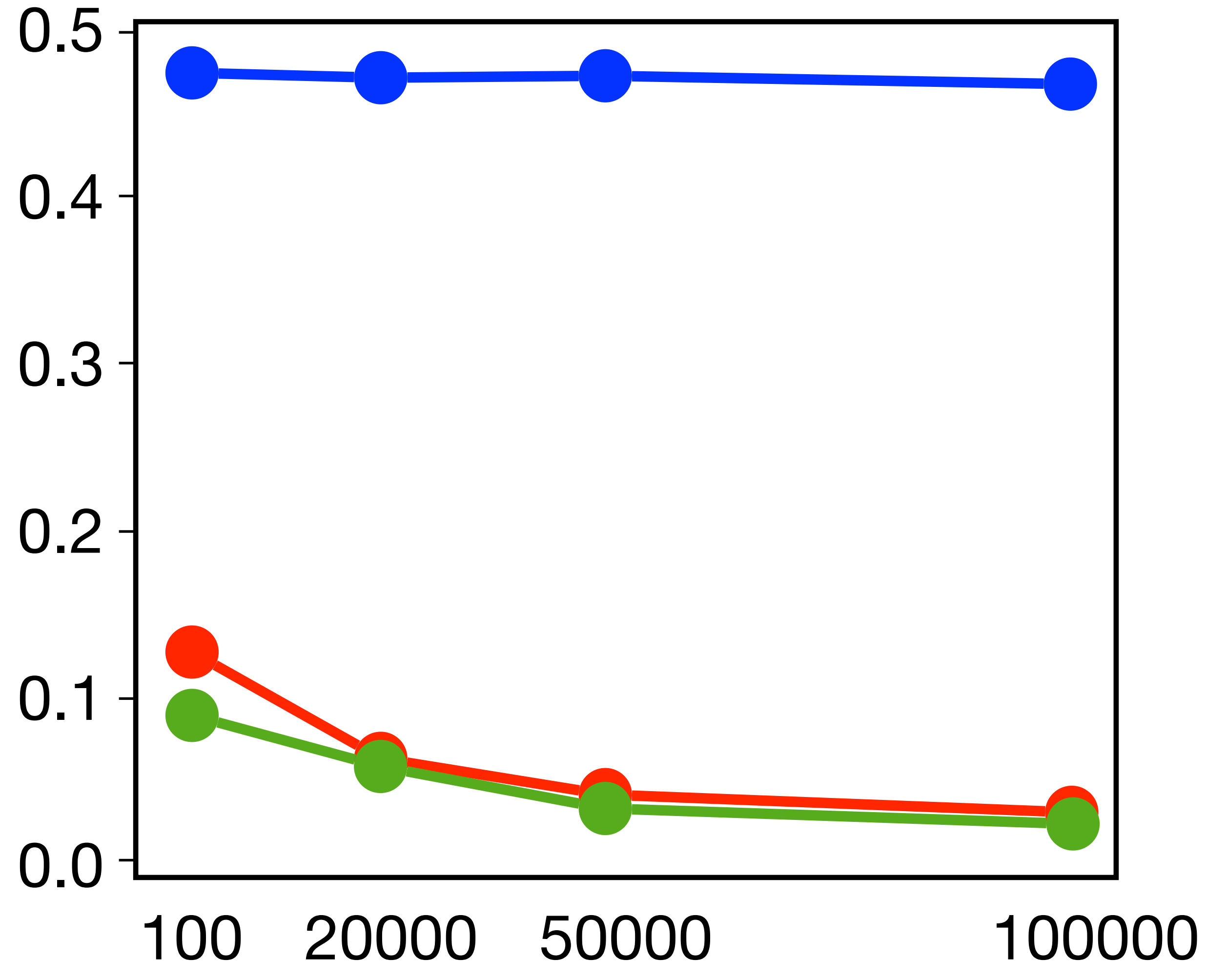
$(\hat{\boldsymbol{\mu}},\hat{\boldsymbol{\pi}}) \rightarrow (\boldsymbol{\mu}_0, \boldsymbol{\pi}_0)$ slowly



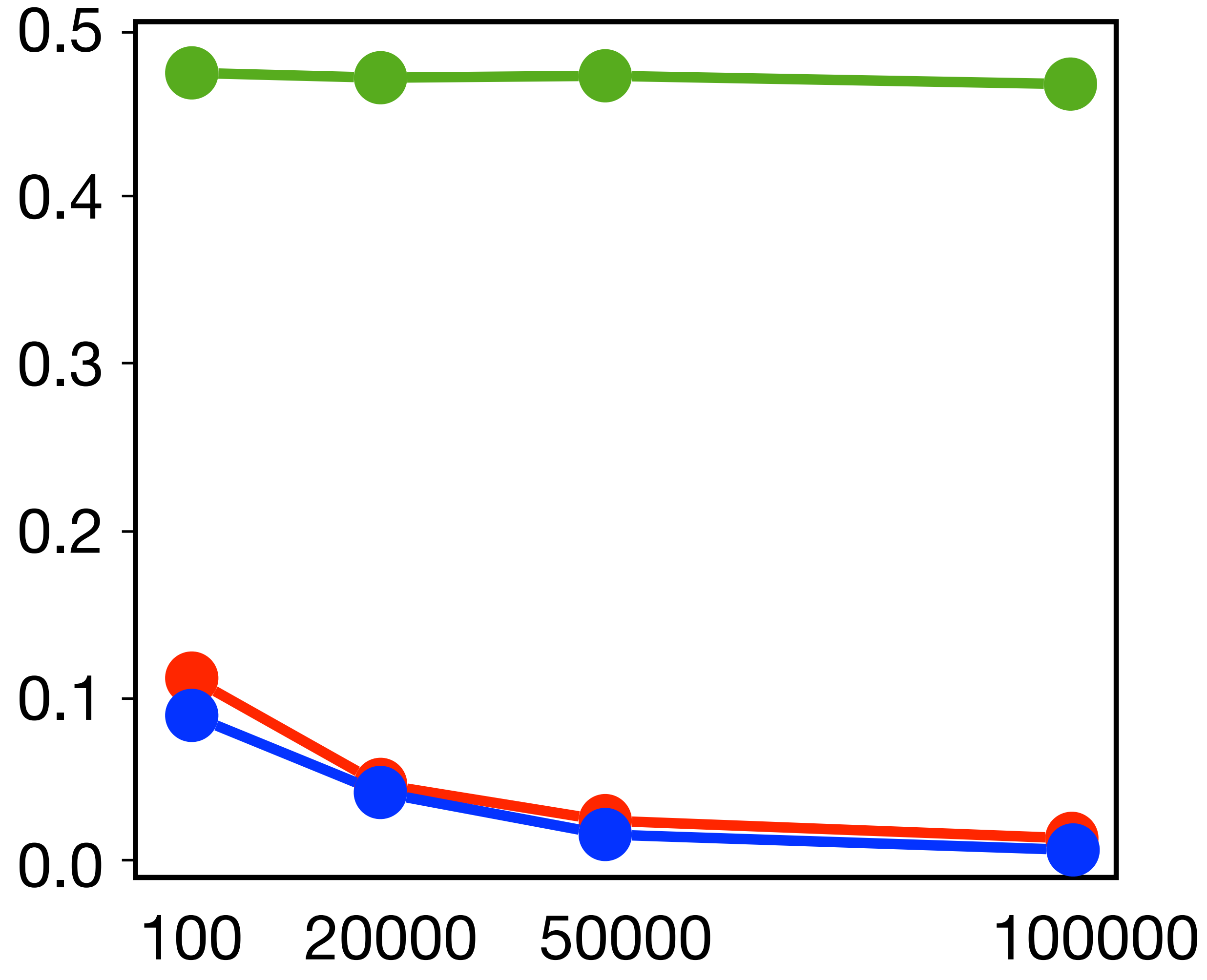
DML-ID converges fast, even when $(\hat{\boldsymbol{\mu}},\hat{\boldsymbol{\pi}})$ converge slowly

Double Robustness

$\hat{\boldsymbol{\mu}}$ misspecified ($\hat{\boldsymbol{\mu}}\neq\boldsymbol{\mu}$)



$\hat{\boldsymbol{\pi}}$ misspecified ($\hat{\boldsymbol{\pi}}\neq\boldsymbol{\pi}$)



DML-ID converges to the true causal effect even when $\hat{\boldsymbol{\mu}}$ or $\hat{\boldsymbol{\pi}}$ are misspecified.

DML-ID - Random (Sec. 3.5)

DML-ID - Random (Sec. 3.5)

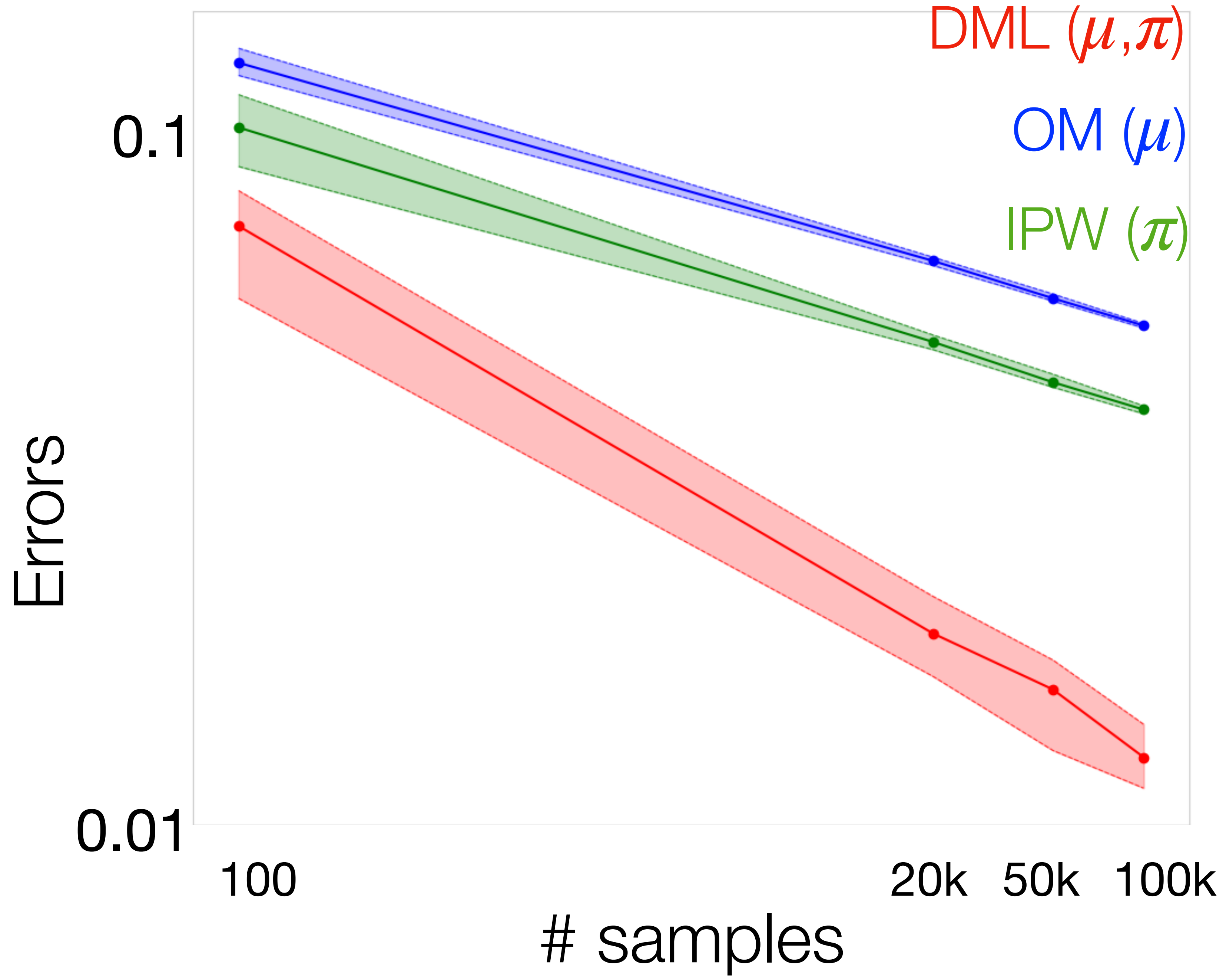
Performed simulations for 100 random graphs.

DML-ID - Random (Sec. 3.5)

Performed simulations for 100 random graphs.

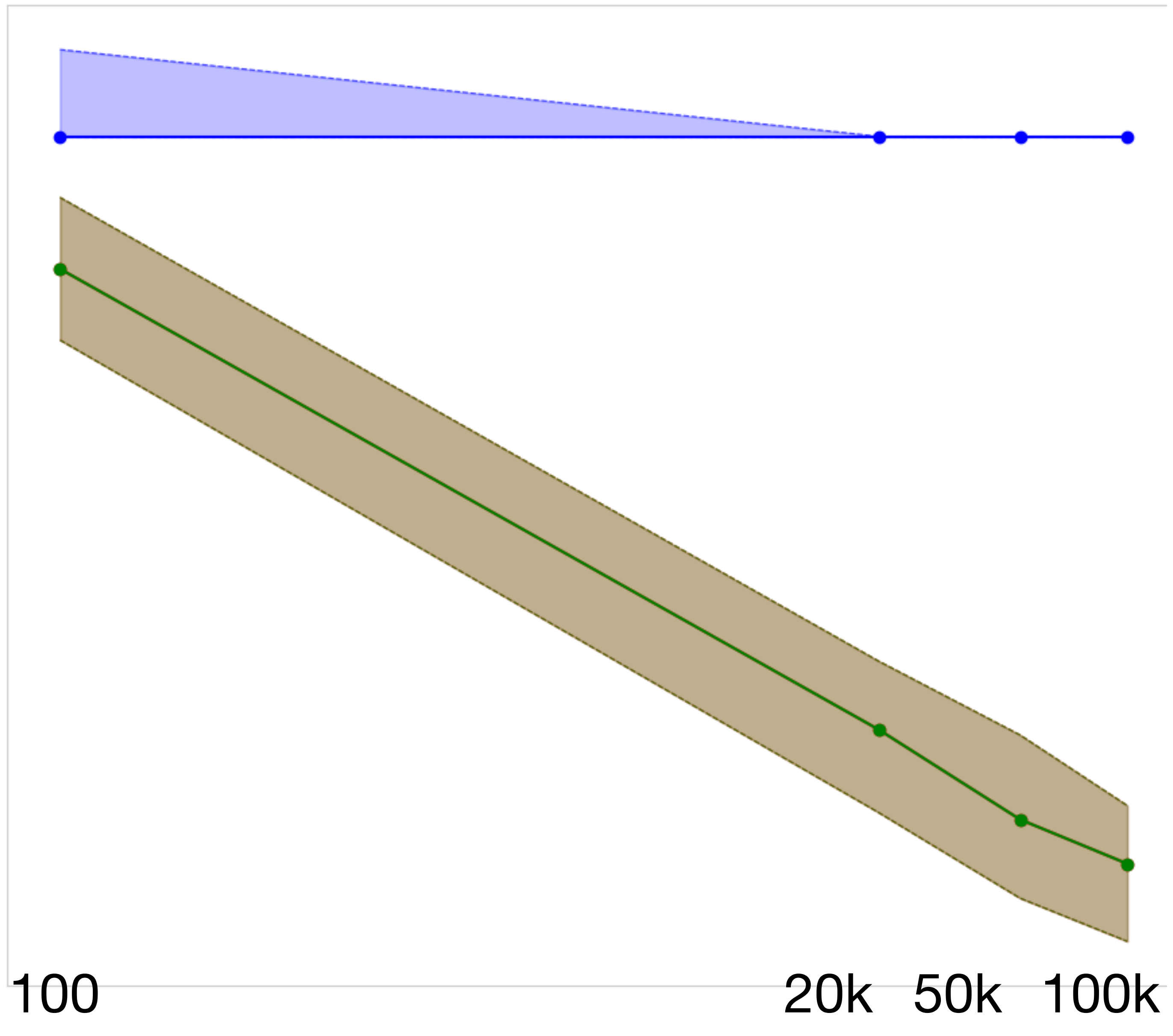
Fast Convergence

$(\hat{\mu},\hat{\pi}) \to (\mu_0, \pi_0)$ slowly

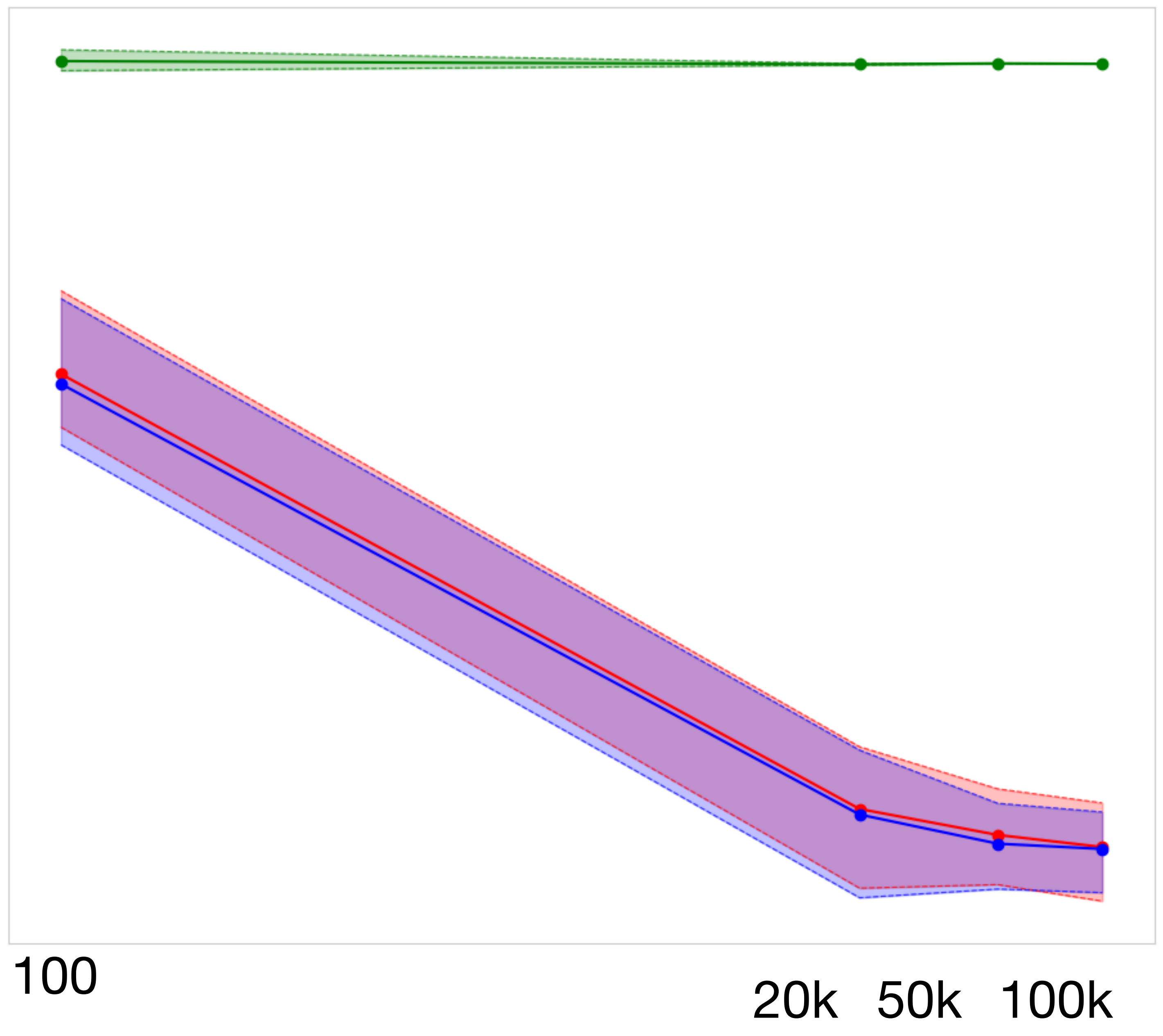


Double Robustness

$\hat{\mu}$ misspecified ($\hat{\mu} \neq \mu$)



$\hat{\pi}$ misspecified ($\hat{\pi} \neq \pi$)



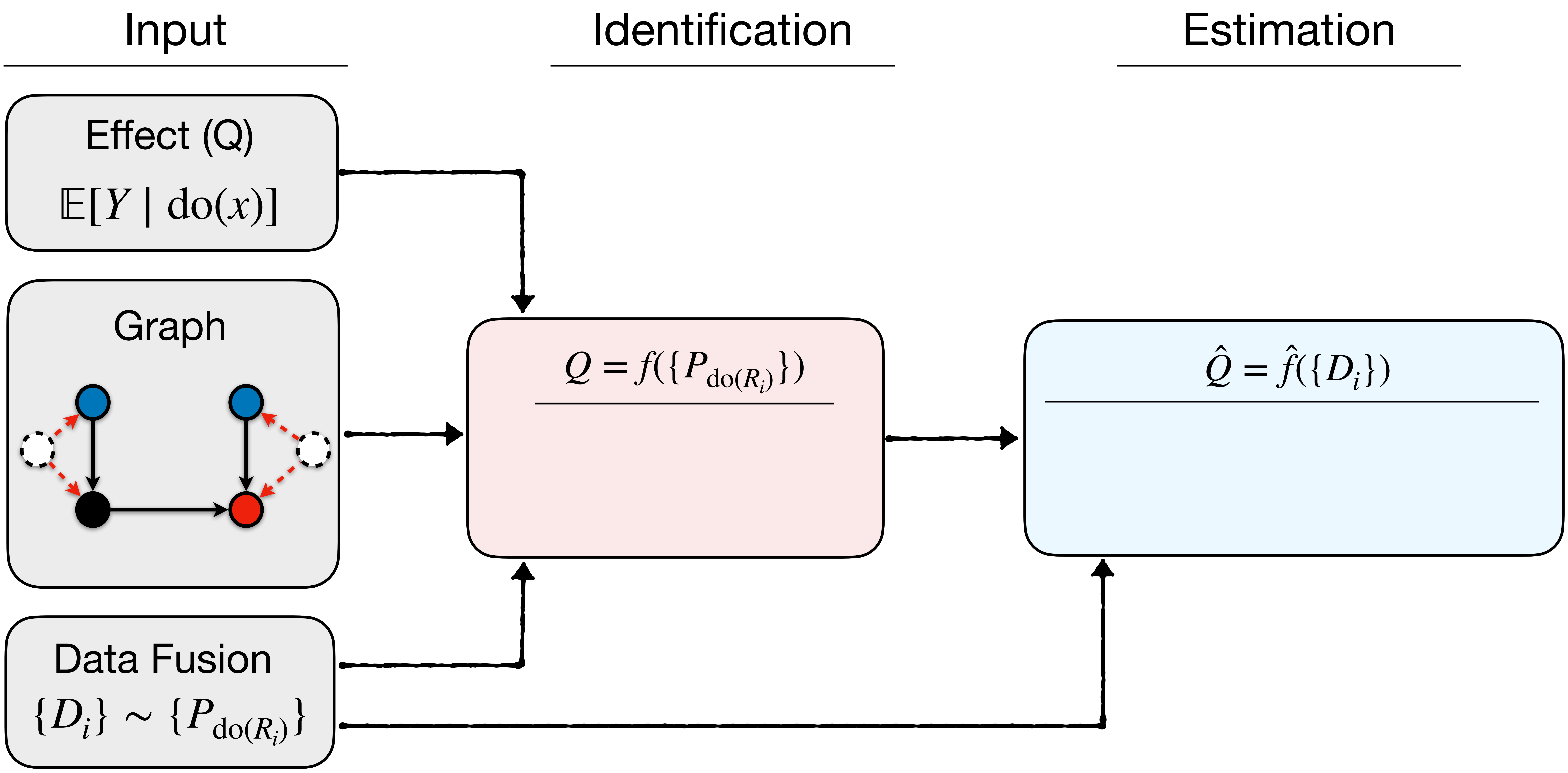
Talk Outline

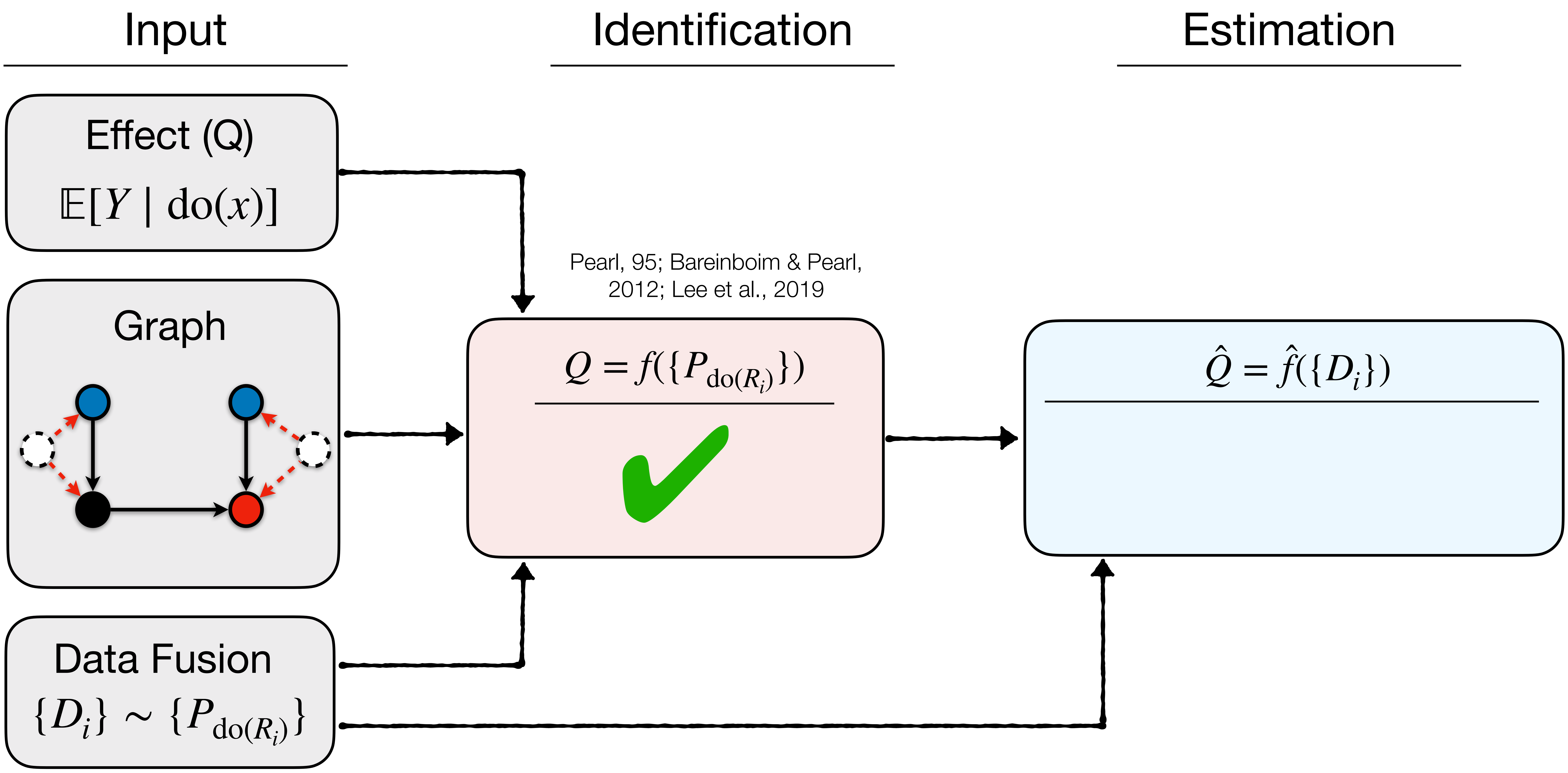
1. **Ch.3** Estimating causal effects from observations
2. **Ch.4** Estimating causal effects from data fusion
3. **Ch.5** Unified causal effect estimation method
4. Conclusion

Talk Outline

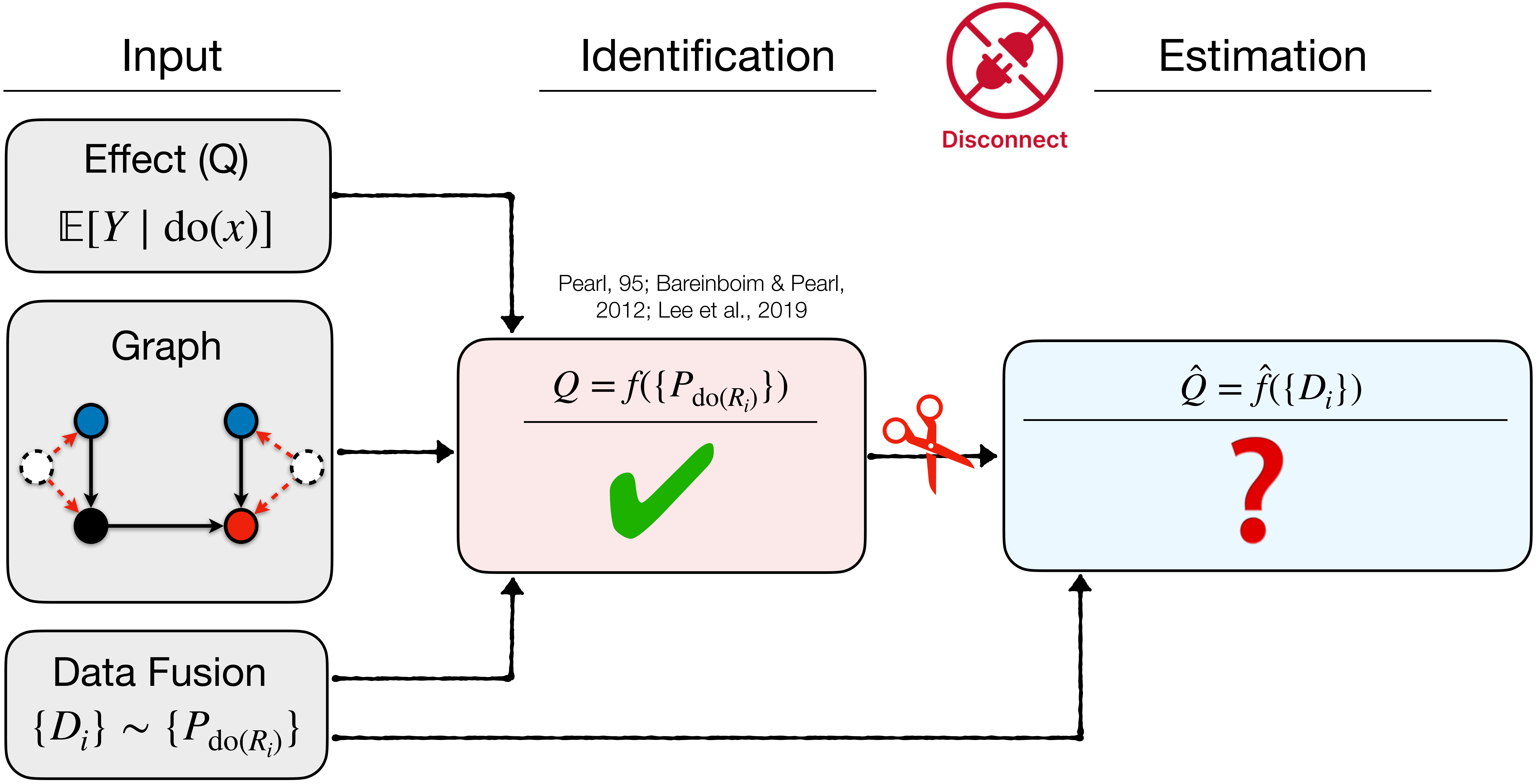
✔

➡ ❷ **Ch.4** Estimating causal effects from data fusion

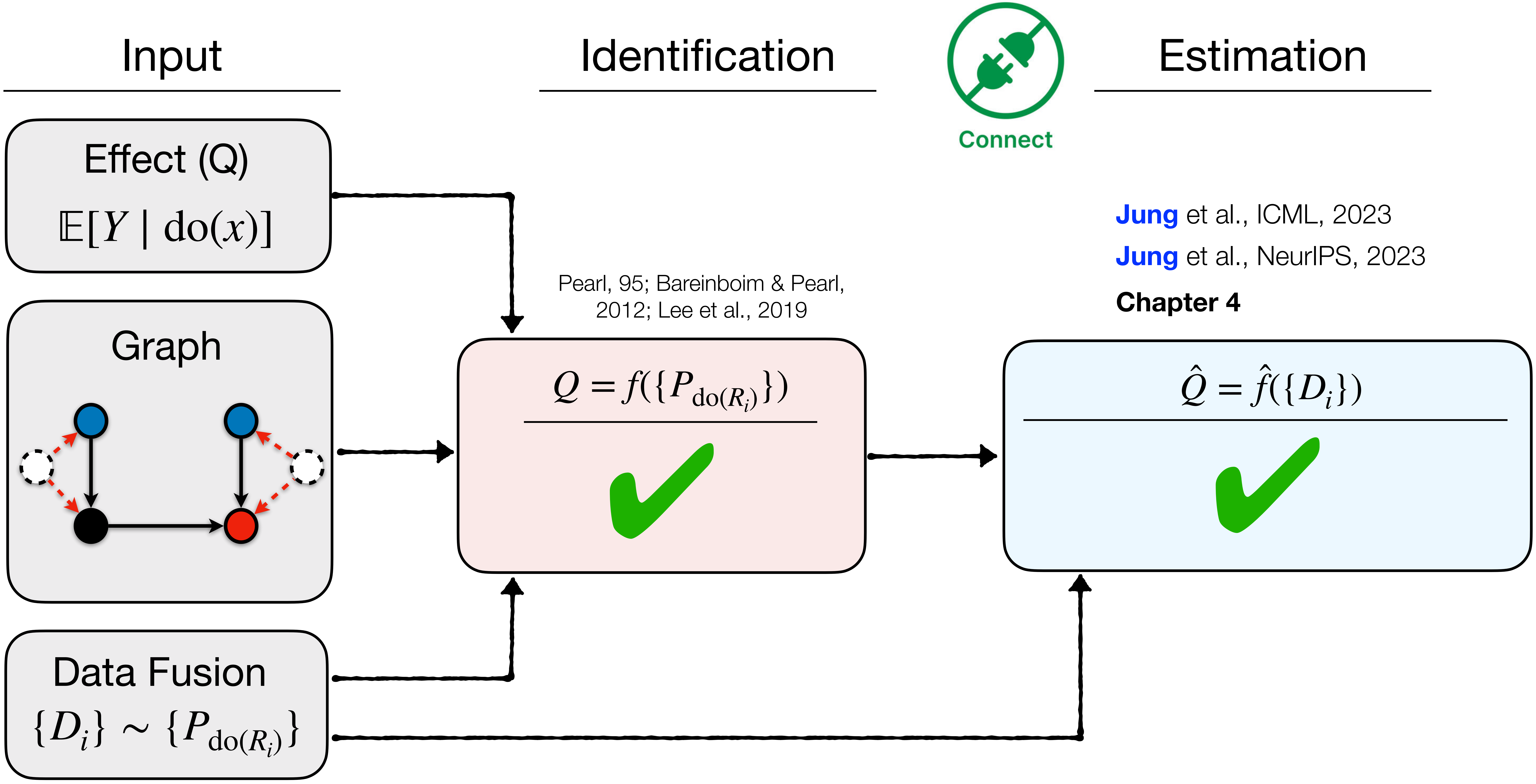

Input
Identification
Estimation
Effect (Q)
$\mathbb{E}[Y \mid do(x)]$
Graph
Data Fusion
$\{D_i\} \sim \{P_{do(R_i)}\}$
$Q = f(\{P_{do(R_i)}\})$
$\hat{Q} = \hat{f}(\{D_i\})$



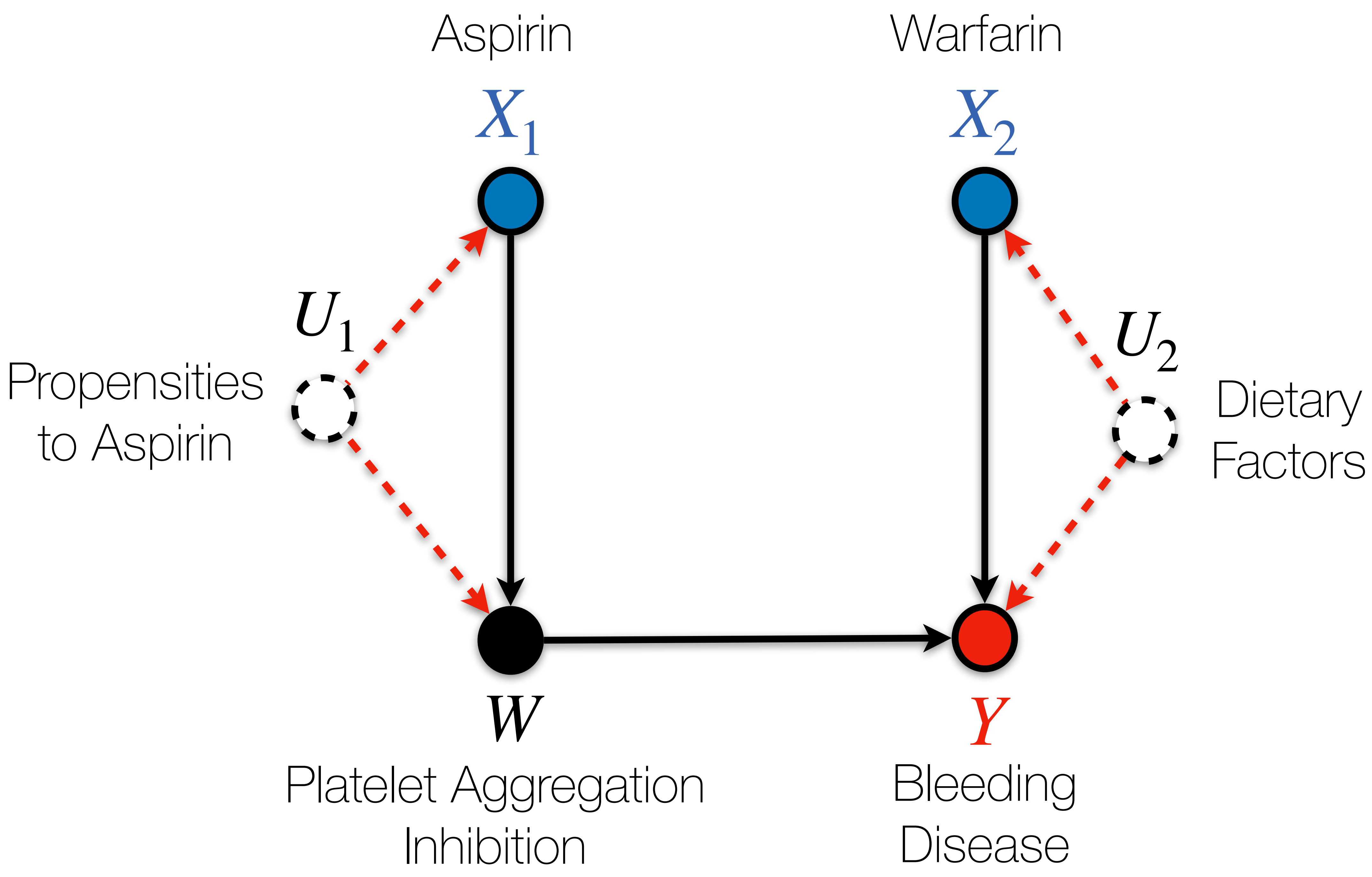
Input
Identification
Estimation
Effect (Q)
$\mathbb{E}[Y \mid \mathrm{do}(x)]$
Graph
Data Fusion
$\{D_i\} \sim \{P_{\mathrm{do}(R_i)}\}$
Pearl, 95; Bareinboim & Pearl, 2012; Lee et al., 2019
$Q = f(\{P_{\mathrm{do}(R_i)}\})$
$\hat{Q} = \hat{f}(\{D_i\})$



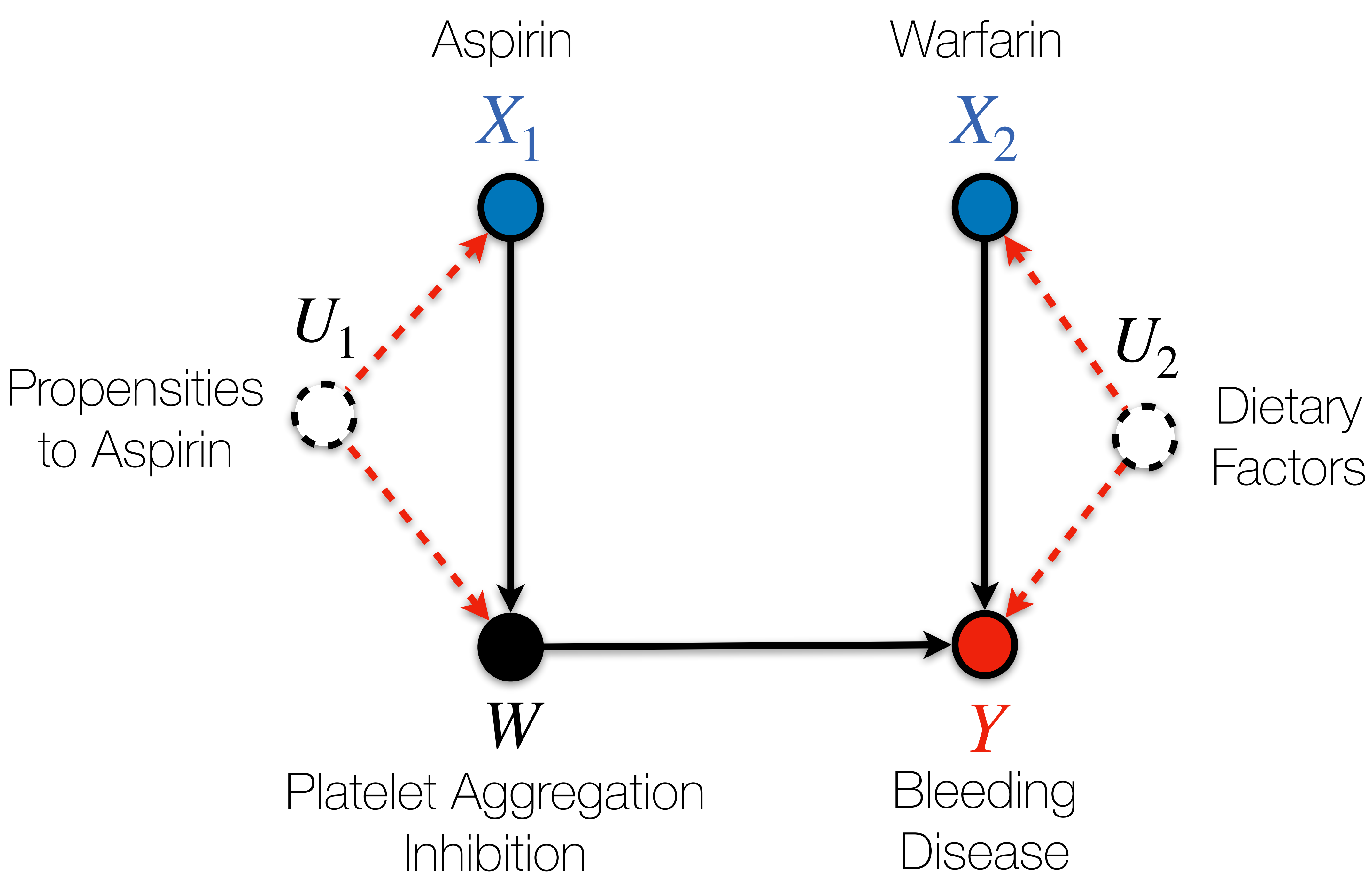
Input
Identification
Disconnect
Estimation
Effect (Q)
$\mathbb{E}[Y \mid do(x)]$
Graph
Data Fusion
$\{D_i\} \sim \{P_{do(R_i)}\}$
Pearl, 95; Bareinboim & Pearl, 2012; Lee et al., 2019
$Q = f(\{P_{do(R_i)}\})$
$\hat{Q} = \hat{f}(\{D_i\})$


Input
Identification
Connect
Estimation
Effect (Q)
$\mathbb{E}[Y \mid do(x)]$
Graph
Data Fusion
$\{D_i\} \sim \{P_{do(R_i)}\}$
Pearl, 95; Bareinboim & Pearl, 2012; Lee et al., 2019
$Q = f(\{P_{do(R_i)}\})$
Jung et al., ICML, 2023
Jung et al., NeurIPS, 2023
Chapter 4
$\hat{Q} = \hat{f}(\{D_i\})$

Motivation: Joint Treatment Effect Estimation

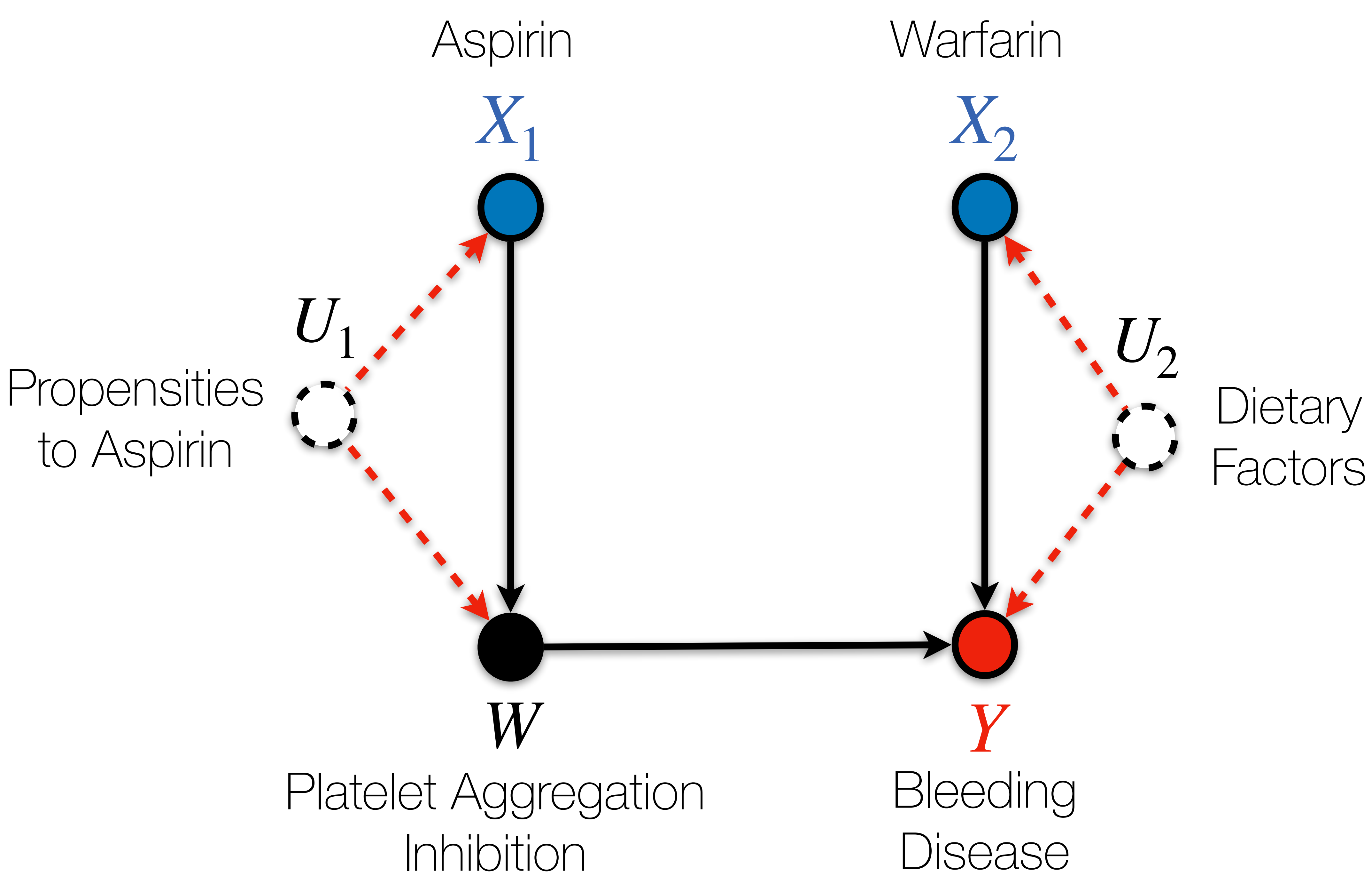


Motivation: Joint Treatment Effect Estimation



Challenges for Estimating $\mathbb{E}[Y \mid \mathrm{do}(x_1, x_2)]$

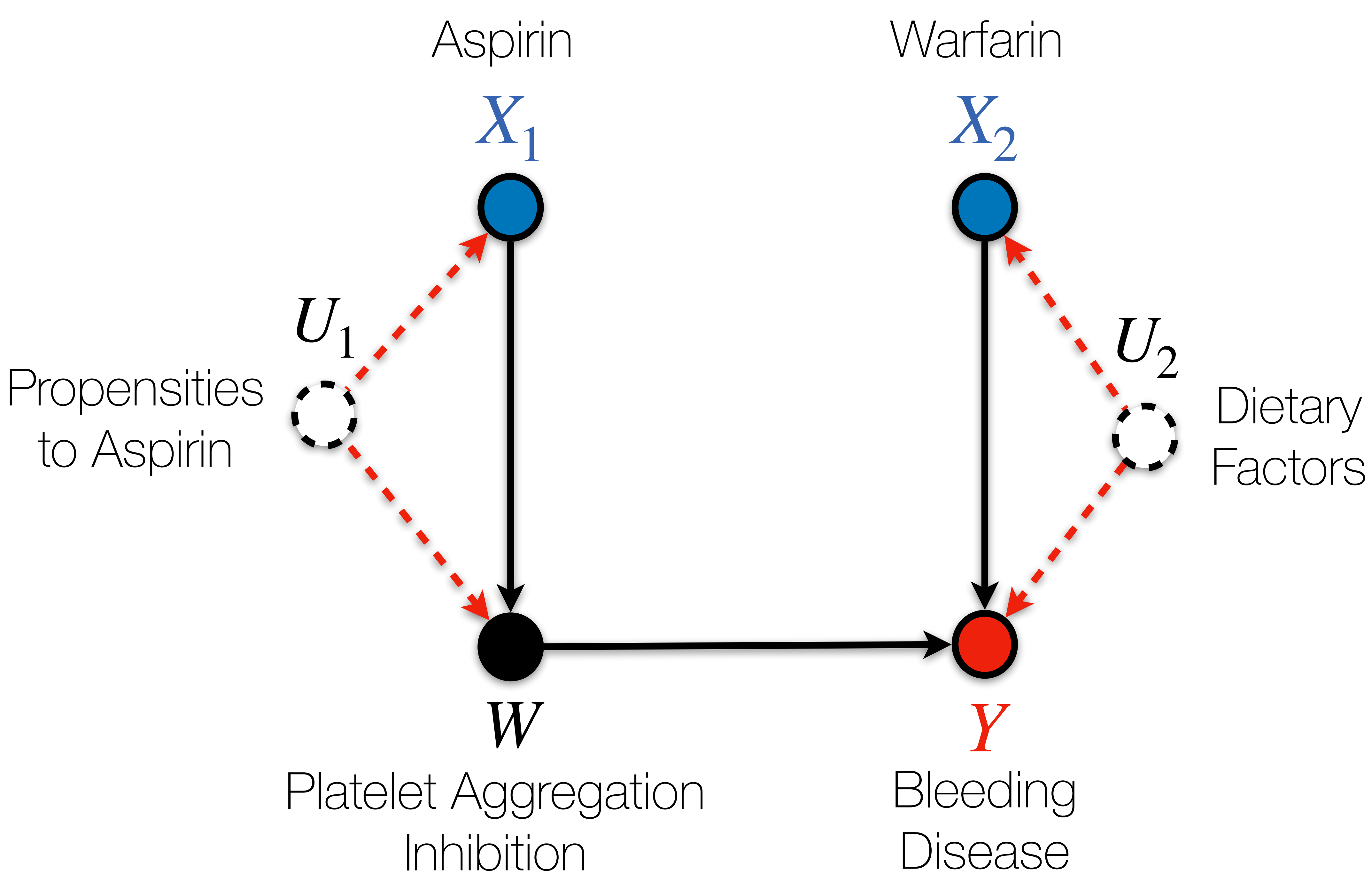
Motivation: Joint Treatment Effect Estimation



Challenges for Estimating $\mathbb{E}[Y \mid \mathrm{do}(x_1, x_2)]$

- BD is not applicable

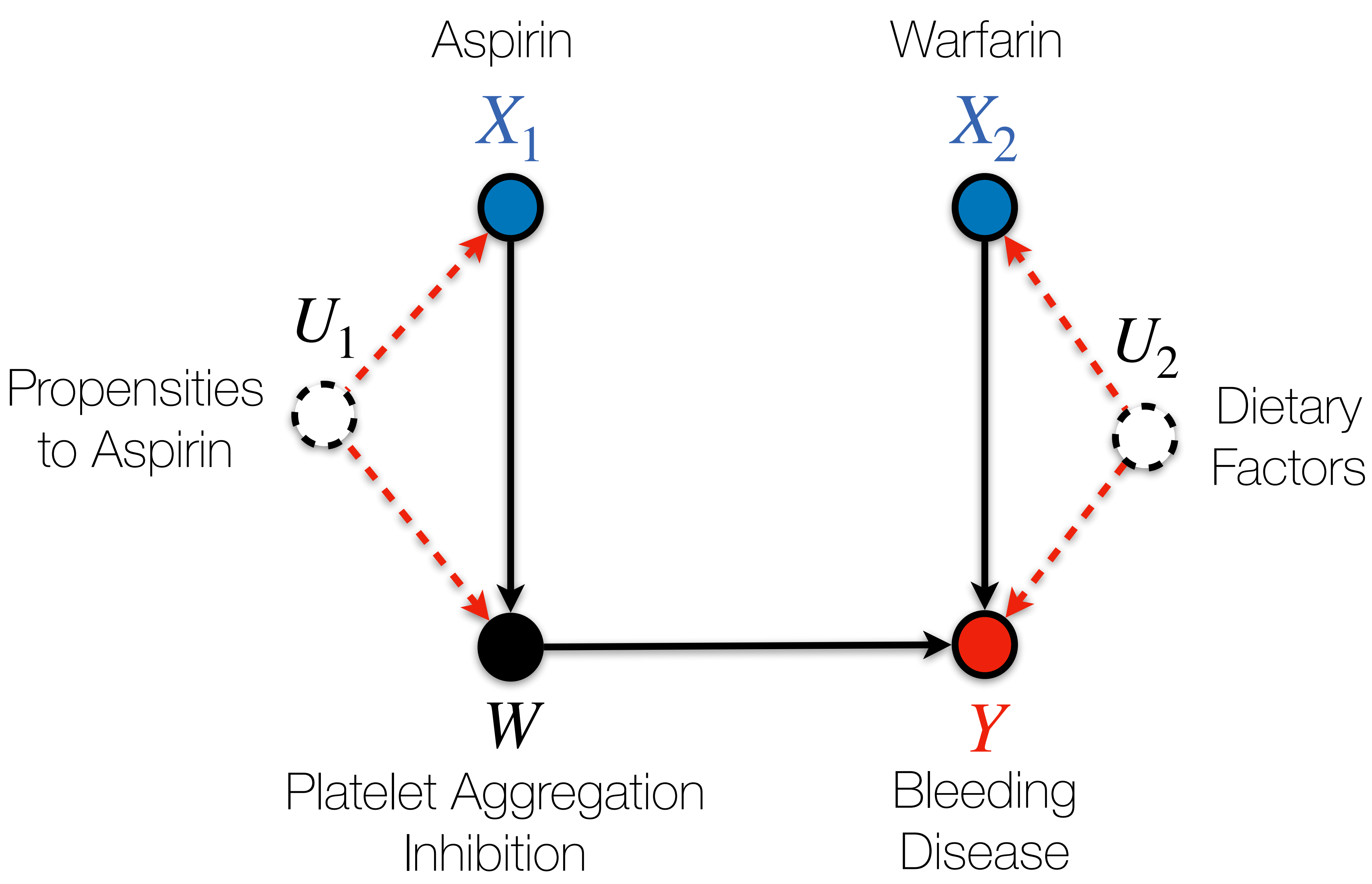
Motivation: Joint Treatment Effect Estimation



Challenges for Estimating $\mathbb{E}[Y \mid \mathrm{do}(x_1, x_2)]$

- BD is not applicable
- Not identifiable from observations $P(\mathbf{V})$.

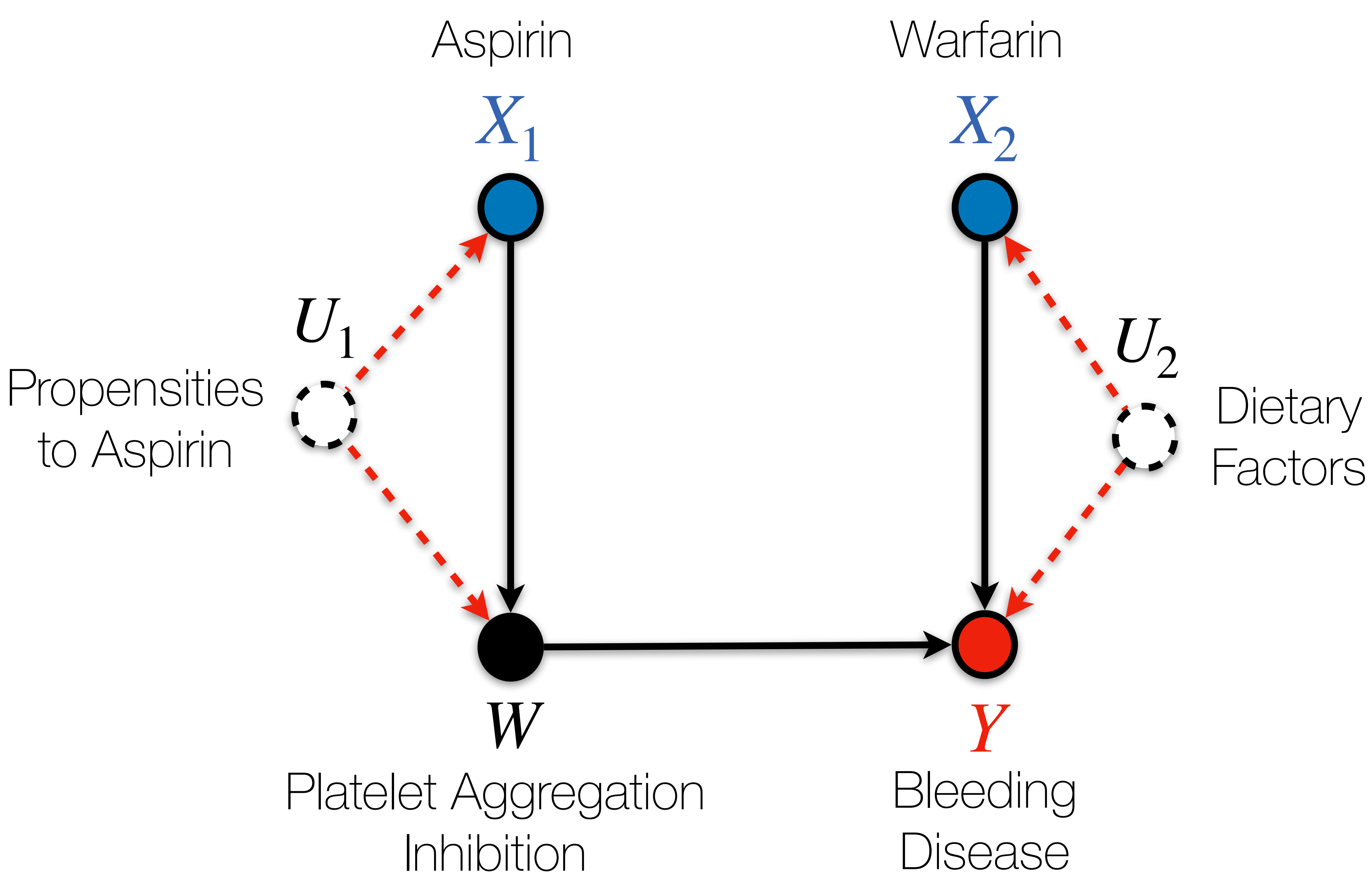
Motivation: Joint Treatment Effect Estimation



Challenges for Estimating $\mathbb{E}[Y \mid \text{do}(x_1, x_2)]$

- BD is not applicable
- Not identifiable from observations $P(\mathbf{V})$.
- Can't run experiments $\text{do}(x_1, x_2)$ due to drug-interactions

Motivation: Joint Treatment Effect Estimation



Challenges for Estimating $\mathbb{E}[Y \mid \mathrm{do}(x_1, x_2)]$

- BD is not applicable
- Not identifiable from observations $P(\mathbf{V})$.
- Can't run experiments $\mathrm{do}(x_1, x_2)$ due to drug-interactions

Can $\mathbb{E}[Y \mid \mathrm{do}(x_1, x_2)]$ be estimated from two trials $P_{\mathrm{do}(x_1)}(\mathbf{V})$ and $P_{\mathrm{do}(x_2)}(\mathbf{V})$?

Joint Treatment Effect Identification

(Def. 39) BD Criterion for Joint Treatment Effect (BD^+))

A set $\mathbf{Z}$ satisfies the *BD criterion from marginal experiments* $P_{\text{do}(\mathbf{x}_1)}$ *and* $P_{\text{do}(\mathbf{x}_2)}$ relative to the outcome $\mathbf{Y}$ for the *joint treatment effect* $(\mathbf{X}_1, \mathbf{X}_2)$ in $\mathcal{G}$ if

1. $\mathbf{Z}$ is not a descendent of $\mathbf{X}_2$ in $\mathcal{G}$; and
2. $\mathbf{Z}$ blocks every spurious path between $\mathbf{X}_1$ and $\mathbf{Y}$ in the experiment $\text{do}(\mathbf{X}_2)$

Joint Treatment Effect Identification

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(Theorem 17)

$$\mathbb{E}[Y \mid \text{do}(\mathbf{x}_1, \mathbf{x}_2)] = \sum_{\mathbf{z}} \mathbb{E}_{\text{do}(\mathbf{x}_2)}[Y \mid \mathbf{x}_1, \mathbf{z}] P_{\text{do}(\mathbf{x}_1)}(\mathbf{z})$$

Joint Treatment Effect Identification

(Def. 39) BD Criterion for Joint Treatment Effect (BD$^+$)

A set $\mathbf{Z}$ satisfies the *BD criterion from marginal experiments* $P_{\text{do}(\mathbf{x}_1)}$ *and* $P_{\text{do}(\mathbf{x}_2)}$ relative to the outcome $\mathbf{Y}$ for the *joint treatment effect* $(\mathbf{X}_1, \mathbf{X}_2)$ in $\mathcal{G}$ if

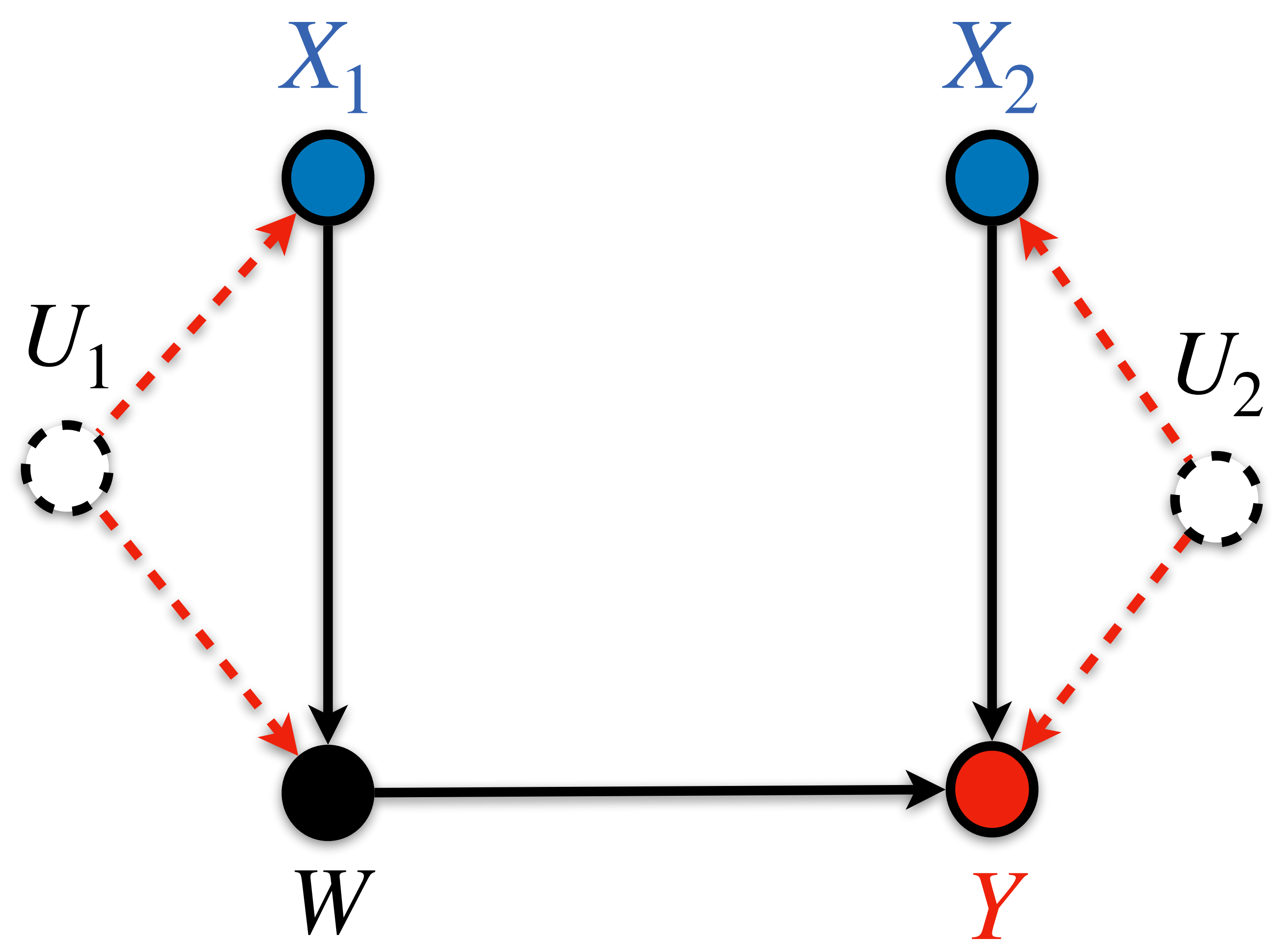
1. $\mathbf{Z}$ is not a descendent of $\mathbf{X}_2$ in $\mathcal{G}$; and
2. $\mathbf{Z}$ blocks every spurious path between $\mathbf{X}_1$ and $\mathbf{Y}$ in the experiment $\text{do}(\mathbf{X}_2)$

(Theorem 17)

$$\mathbb{E}[Y \mid \text{do}(\mathbf{x}_1, \mathbf{x}_2)] = \sum_{\mathbf{z}} \underbrace{\mathbb{E}_{\text{do}(\mathbf{x}_2)}[Y \mid \mathbf{x}_1, \mathbf{z}]}_{\text{Trial on } X_2} \underbrace{P_{\text{do}(\mathbf{x}_1)}(\mathbf{z})}_{\text{Trial on } X_1}$$

Example of BD+

1. $\mathbf{Z} = \{W\}$ is not a descendent of $\mathbf{X}_2$ in $\mathcal{G}$; and
2. $\mathbf{Z} = \{W\}$ blocks every spurious path between $\mathbf{X}_1$ and $\mathbf{Y}$ in the experiment $\text{do}(\mathbf{X}_2)$



$$\mathbb{E}[Y \mid \text{do}(x_1, x_2)] = \sum_w \underbrace{\mathbb{E}_{\text{do}(x_2)}[Y \mid x_1, w]}_{\text{Trial on } X_2} \underbrace{P_{\text{do}(x_1)}(w)}_{\text{Trial on } X_1}$$

Parametrization of BD+ (Sec. 4.2.2)

$$\mathbb{E}[Y \mid \mathrm{do}(\mathbf{x}_1, \mathbf{x}_2)] = \sum_{\mathbf{z}} \mathbb{E}_{\mathrm{do}(\mathbf{x}_2)}[Y \mid \mathbf{x}_1, \mathbf{z}] P_{\mathrm{do}(x_1)}(\mathbf{z})$$

Parametrization of BD+ (Sec. 4.2.2)

$$\mathbb{E}[Y \mid do(\mathbf{x}_1, \mathbf{x}_2)] = \sum_{\mathbf{z}} \mathbb{E}_{do(\mathbf{x}_2)}[Y \mid \mathbf{x}_1, \mathbf{z}] P_{do(x_1)}(\mathbf{z})$$

$$\mu(\mathbf{X}_1, \mathbf{Z}) \triangleq \mathbb{E}_{do(\mathbf{x}_2)}[Y \mid \mathbf{X}_1, \mathbf{Z}]$$

Parametrization of BD+ (Sec. 4.2.2)

$$\mathbb{E}[Y \mid do(\mathbf{x}_1, \mathbf{x}_2)] = \sum_{\mathbf{z}} \mathbb{E}_{do(\mathbf{x}_2)}[Y \mid \mathbf{x}_1, \mathbf{z}] P_{do(x_1)}(\mathbf{z})$$

$$\mu(\mathbf{X}_1, \mathbf{Z}) \triangleq \mathbb{E}_{do(\mathbf{x}_2)}[Y \mid \mathbf{X}_1, \mathbf{Z}]$$

$$\mathbb{E}_{do(\mathbf{x}_1)}[\mu(\mathbf{x}_1, \mathbf{Z})]$$

$$= \sum_{\mathbf{z}} \mu(\mathbf{x}_1, \mathbf{z}) P_{do(\mathbf{x}_1)}(\mathbf{z})$$

$$= \mathbb{E}[Y \mid do(\mathbf{x}_1, \mathbf{x}_2)]$$

Parametrization of BD+ (Sec. 4.2.2)

$$\mathbb{E}[Y \mid do(\mathbf{x}_1, \mathbf{x}_2)] = \sum_{\mathbf{z}} \mathbb{E}_{do(\mathbf{x}_2)}[Y \mid \mathbf{x}_1, \mathbf{z}] P_{do(x_1)}(\mathbf{z})$$

$$\mu(\mathbf{X}_1, \mathbf{Z}) \triangleq \mathbb{E}_{do(\mathbf{x}_2)}[Y \mid \mathbf{X}_1, \mathbf{Z}]$$

$$\mathbb{E}_{do(\mathbf{x}_1)}[\mu(\mathbf{x}_1, \mathbf{Z})]$$

$$= \sum_{\mathbf{z}} \mu(\mathbf{x}_1, \mathbf{z}) P_{do(\mathbf{x}_1)}(\mathbf{z})$$

$$= \mathbb{E}[Y \mid do(\mathbf{x}_1, \mathbf{x}_2)]$$

$\pi(\mathbf{X}_1, \mathbf{Z})$: Solution of

$$\mathbb{E}_{do(\mathbf{x}_2)}[\pi(\mathbf{X}_1 \mathbf{Z}) \times \mu(\mathbf{X}_1 \mathbf{Z})] = \mathbb{E}_{do(\mathbf{x}_1)}[\mu(\mathbf{x}_1, \mathbf{Z})]$$

Parametrization of BD+ (Sec. 4.2.2)

$$\mathbb{E}[Y \mid do(\mathbf{x}_1, \mathbf{x}_2)] = \sum_{\mathbf{z}} \mathbb{E}_{do(\mathbf{x}_2)}[Y \mid \mathbf{x}_1, \mathbf{z}] P_{do(x_1)}(\mathbf{z})$$

$$\mu(\mathbf{X}_1, \mathbf{Z}) \triangleq \mathbb{E}_{do(\mathbf{x}_2)}[Y \mid \mathbf{X}_1, \mathbf{Z}]$$

$$\mathbb{E}_{do(\mathbf{x}_1)}[\mu(\mathbf{x}_1, \mathbf{Z})]$$

$$= \sum_{\mathbf{z}} \mu(\mathbf{x}_1, \mathbf{z}) P_{do(\mathbf{x}_1)}(\mathbf{z})$$

$$= \mathbb{E}[Y \mid do(\mathbf{x}_1, \mathbf{x}_2)]$$

$\pi(\mathbf{X}_1, \mathbf{Z})$: Solution of

$$\mathbb{E}_{do(\mathbf{x}_2)}[\pi(\mathbf{X}_1 \mathbf{Z}) \times \mu(\mathbf{X}_1 \mathbf{Z})] = \mathbb{E}_{do(\mathbf{x}_1)}[\mu(\mathbf{x}_1, \mathbf{Z})]$$

$$\mathbb{E}_{do(\mathbf{x}_2)}[\pi(\mathbf{X}_1 \mathbf{Z}) \times Y]$$

$$= \mathbb{E}_{do(\mathbf{x}_2)}[\pi(\mathbf{X}_1 \mathbf{Z}) \times \mu(\mathbf{X}_1 \mathbf{Z})]$$

$$= \mathbb{E}_{do(\mathbf{x}_1)}[\mu(\mathbf{x}_1, \mathbf{Z})]$$

$$= \mathbb{E}[Y \mid do(\mathbf{x}_1, \mathbf{x}_2)]$$

Doubly Robust Estimator for BD+

$$\mathbb{E}[Y \mid do(\mathbf{x}_1, \mathbf{x}_2)] = \text{BD}^{+}(\mu, \pi) \triangleq \mathbb{E}_{do(x_2)}[\mu \times \pi]$$

Doubly Robust Estimator for BD+

$$\mathbb{E}[Y \mid do(\mathbf{x}_1, \mathbf{x}_2)] = \text{BD}^{+}(\mu, \pi) \triangleq \mathbb{E}_{do(x_2)}[\mu \times \pi]$$

"Double Robustness"

$$?(\hat{\mu}, \hat{\pi}) - \mathbb{E}_{do(x_2)}[\mu \times \pi] = \mathbb{E}_{do(x_2)}[\{\hat{\mu} - \mu\} \times \{\pi - \hat{\pi}\}]$$

Doubly Robust Estimator for BD+

$$\mathbb{E}[Y \mid \text{do}(\mathbf{x}_1, \mathbf{x}_2)] = \text{BD}^{+}(\mu, \pi) \triangleq \mathbb{E}_{\text{do}(\text{x}_2)}[\mu \times \pi]$$

$$?(\hat{\mu}, \hat{\pi}) \;= \mathbb{E}_{\text{do}(x_2)}[\{\hat{\mu} - \mu\} \times \{\pi - \hat{\pi}\}] \;+ \mathbb{E}_{\text{do}(x_2)}[\mu \times \pi]$$

Doubly Robust Estimator for BD+

$$\mathbb{E}[Y \mid do(\mathbf{x}_1, \mathbf{x}_2)] = \text{BD}^+(\mu, \pi) \triangleq \mathbb{E}_{do(x_2)}[\mu \times \pi]$$

$$?(\hat{\mu}, \hat{\pi}) = \mathbb{E}_{do(x_2)}[\{\hat{\mu} - \mu\} \times \{\pi - \hat{\pi}\}] + \mathbb{E}_{do(x_2)}[\mu \times \pi]$$

$$= \mathbb{E}_{do(x_2)}[\hat{\pi}\{\mu - \hat{\mu}\} + \pi\hat{\mu}]$$

Doubly Robust Estimator for BD+

$$\mathbb{E}[Y \mid do(\mathbf{x}_1, \mathbf{x}_2)] = \text{BD}^{+}(\mu,\pi) \triangleq \mathbb{E}_{do(x_2)}[\mu \times \pi]$$

$$? (\hat{\mu},\hat{\pi}) = \mathbb{E}_{do(x_2)}[\{\hat{\mu}-\mu\}\times\{\pi-\hat{\pi}\}] + \mathbb{E}_{do(x_2)}[\mu\times\pi]$$

$$= \mathbb{E}_{do(x_2)}[\hat{\pi}\{\mu-\hat{\mu}\}+\pi\hat{\mu}]$$

$$= \mathbb{E}_{do(x_2)}[\hat{\pi}\{Y-\hat{\mu}\}] + \mathbb{E}_{do(x_1)}[\hat{\mu}(x, C)]$$

Doubly Robust Estimator for BD+

$$\mathbb{E}[Y \mid do(\mathbf{x}_1, \mathbf{x}_2)] = \text{BD}^{+}(\mu,\pi) \triangleq \mathbb{E}_{do(x_2)}[\mu \times \pi]$$

$$\textbf{?}(\hat{\mu},\hat{\pi}) = \mathbb{E}_{do(x_2)}[\{\hat{\mu}-\mu\}\times\{\pi-\hat{\pi}\}] + \mathbb{E}_{do(x_2)}[\mu\times\pi]$$

$$= \mathbb{E}_{do(x_2)}[\hat{\pi}\{\mu-\hat{\mu}\}+\pi\hat{\mu}]$$

$$= \mathbb{E}_{do(x_2)}[\hat{\pi}\{Y-\hat{\mu}\}] + \mathbb{E}_{do(x_1)}[\hat{\mu}(x, C)]$$

DML-BD+ (Def. 46)

$$\widehat{\text{BD}^{+}}(\hat{\mu},\hat{\pi}) \triangleq \mathbb{E}_{do(x_2)}[\hat{\pi}\{Y-\hat{\mu}\}] + \mathbb{E}_{do(x_1)}[\hat{\mu}(x, C)]$$

Robustness of DML-BD+

$$\text{Error}(\text{DML-BD}^+(\hat{\mu},\hat{\pi}),\ \text{BD}^+(\mu,\pi)) = \text{Error}(\hat{\mu},\mu) \times \text{Error}(\hat{\pi},\pi)$$

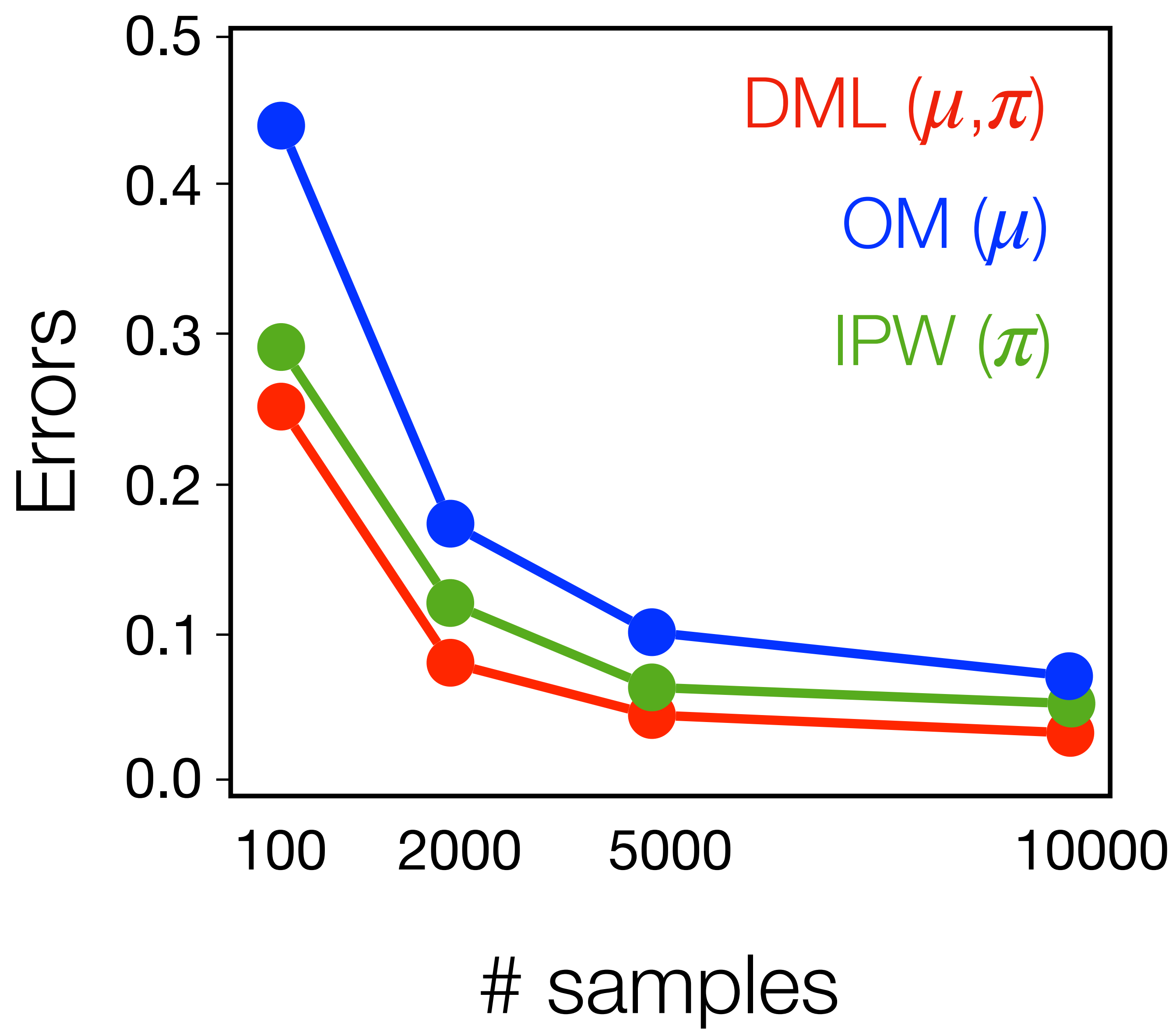
- **Double Robustness:** Error $= 0$ if either $\hat{\mu} = \mu$ or $\hat{\pi} = \pi$
- **Fast Convergence:** Error $\to 0$ *fast* even when $\hat{\mu} \to \mu$ and $\hat{\pi} \to \pi$ *slowly*.

Simulation: DML-BD+

Simulation: DML-BD+

Fast Convergence

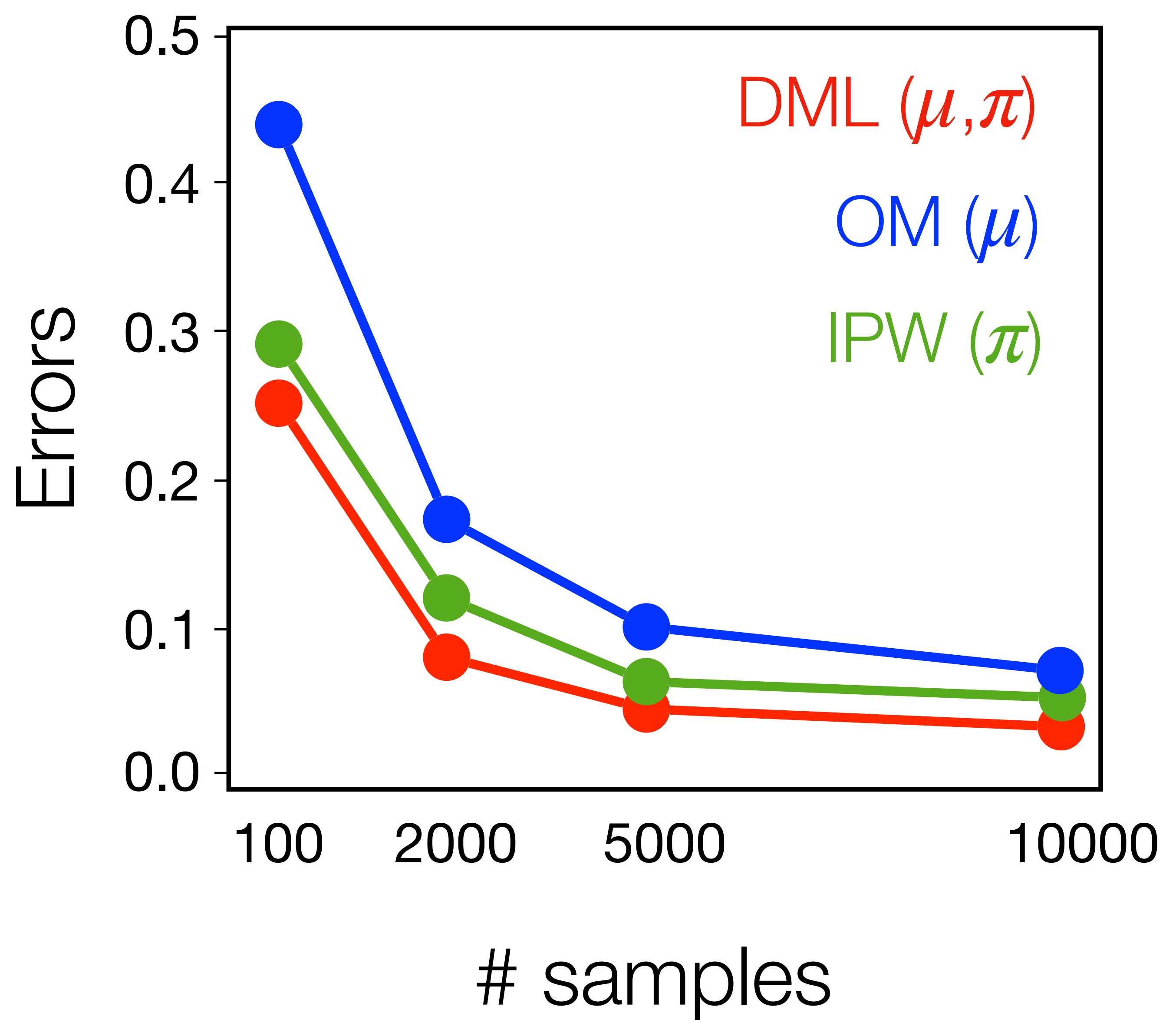
$(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\pi}}) \rightarrow (\boldsymbol{\mu}_0, \boldsymbol{\pi}_0)$ slowly



Simulation: DML-BD+

Fast Convergence

$(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\pi}}) \rightarrow (\boldsymbol{\mu}_0, \boldsymbol{\pi}_0)$ slowly

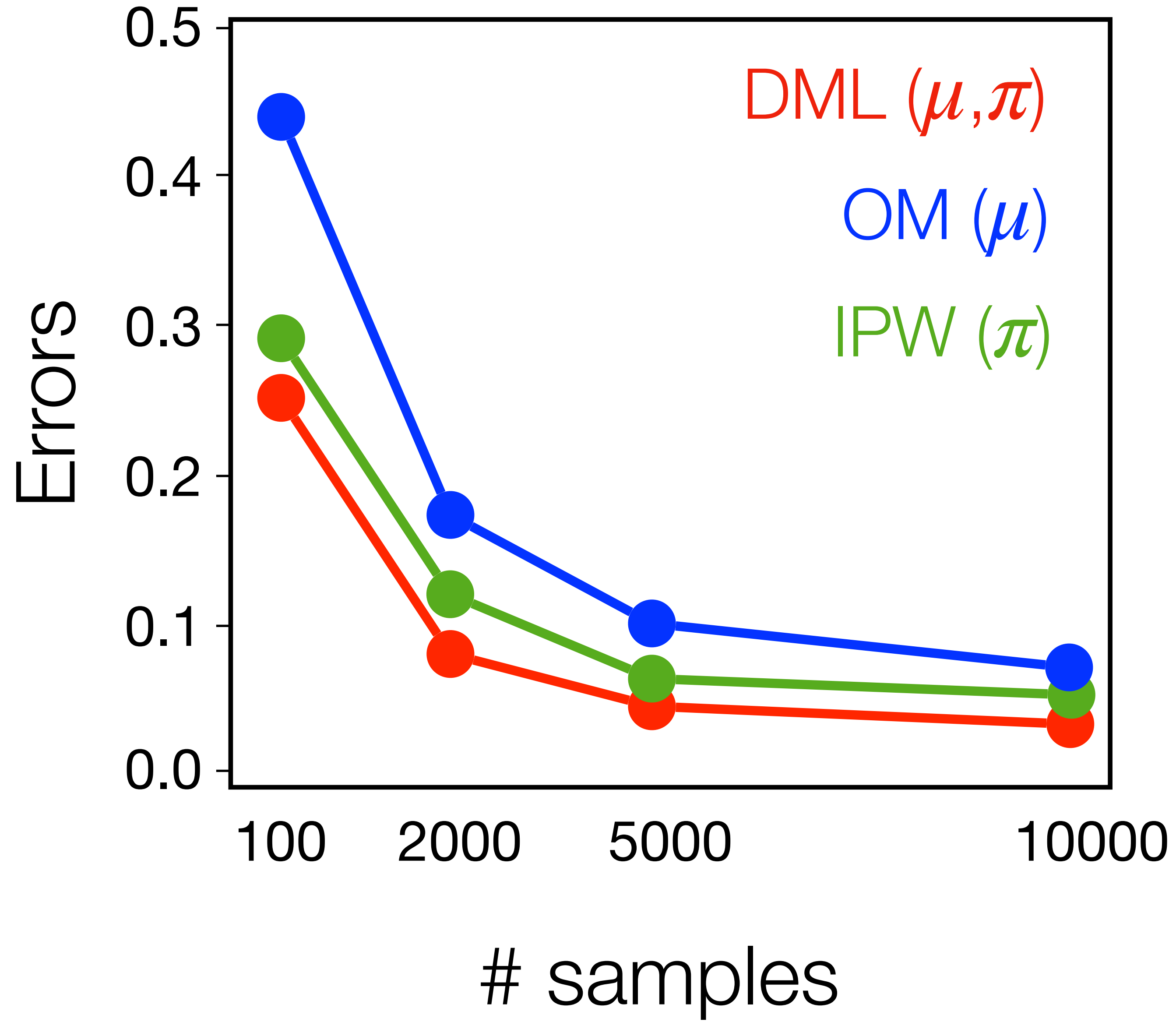


DML-BD+ converges fast, even when $(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\pi}})$ converge slowly

Simulation: DML-BD+

Fast Convergence

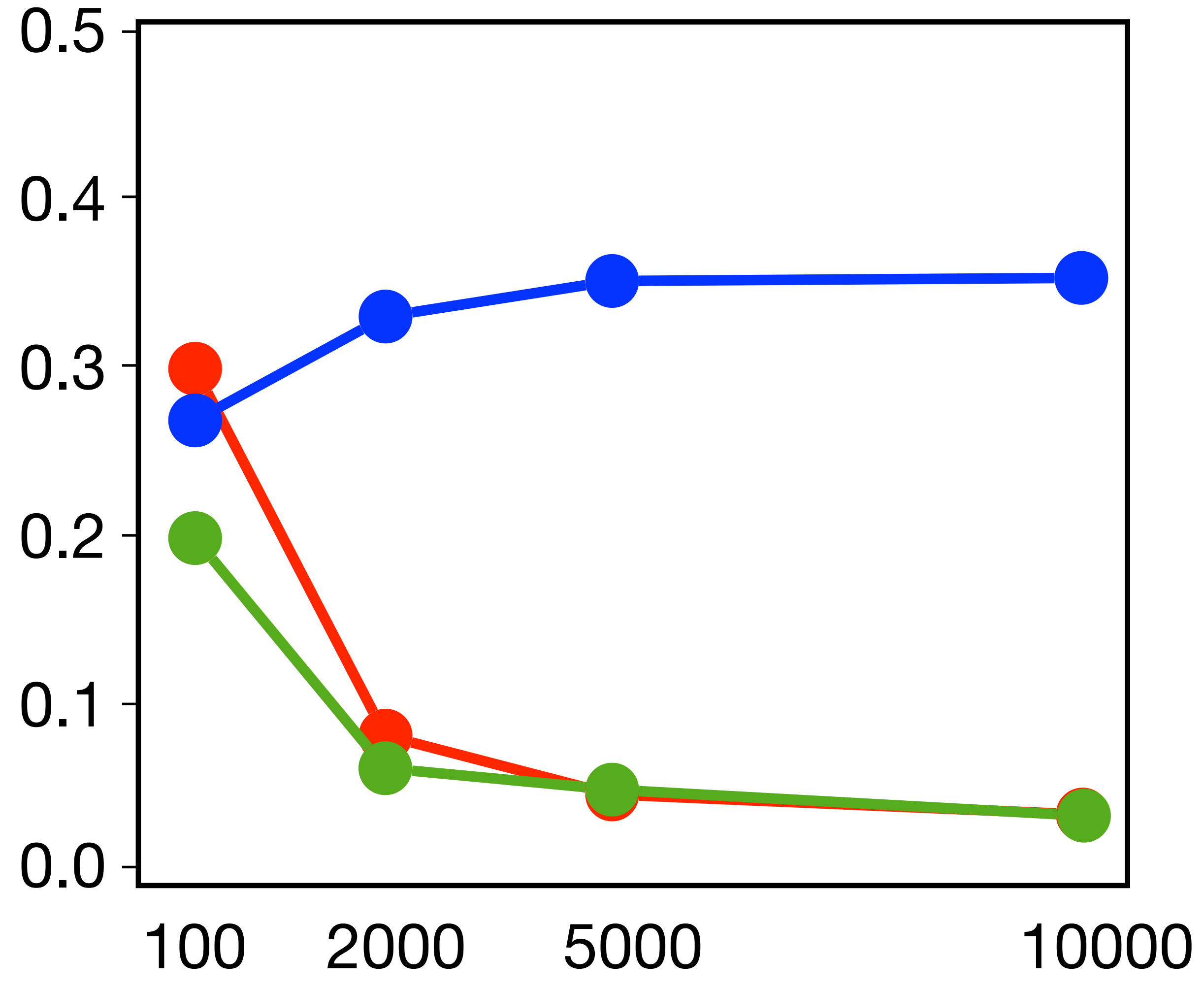
$(\hat{\mu},\hat{\pi}) \rightarrow (\mu_0, \pi_0)$ slowly



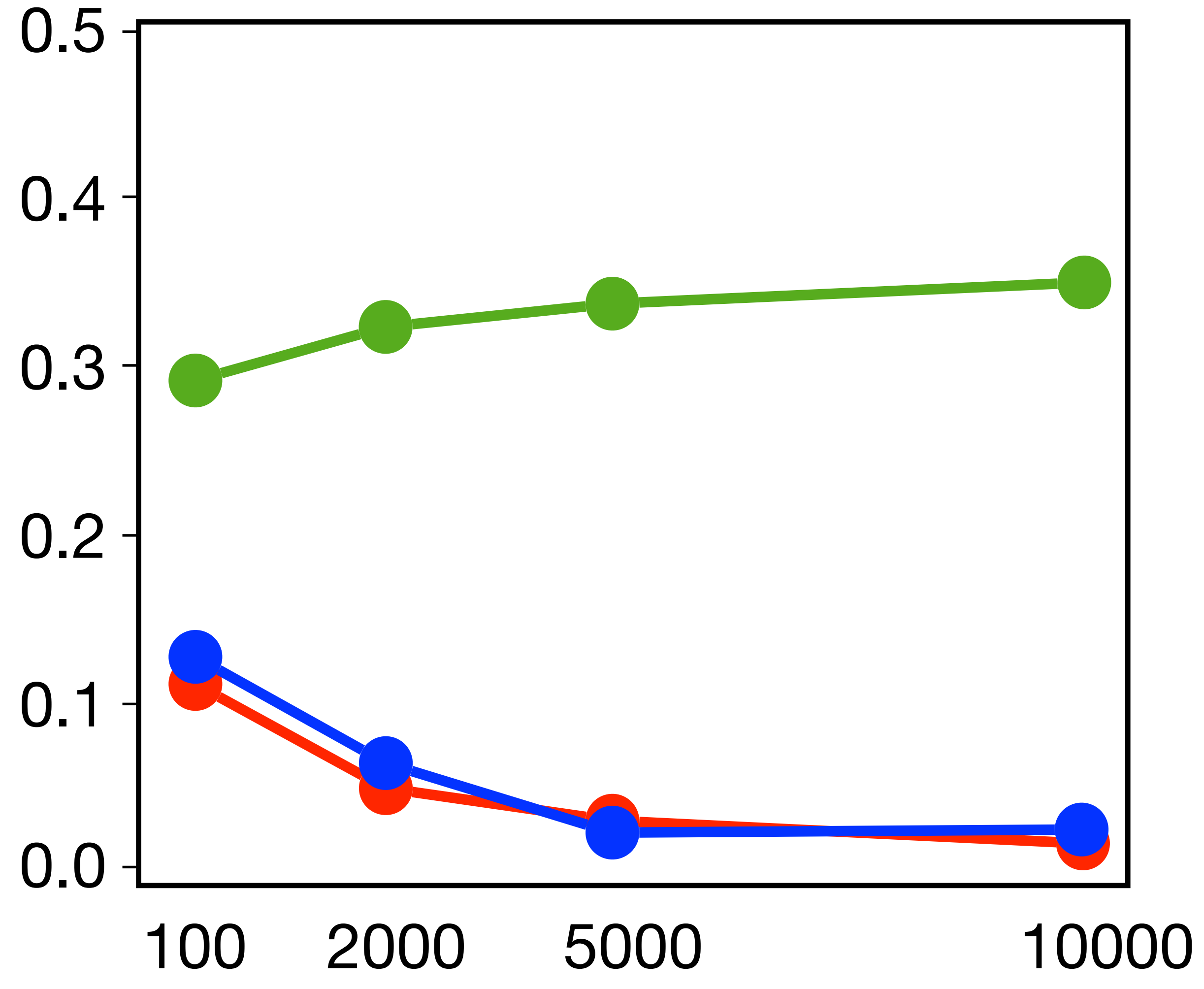
DML-BD+ converges fast, even when $(\hat{\mu},\hat{\pi})$ converge slowly

Double Robustness

$\hat{\mu}$ misspecified ($\hat{\mu} \neq \mu$)



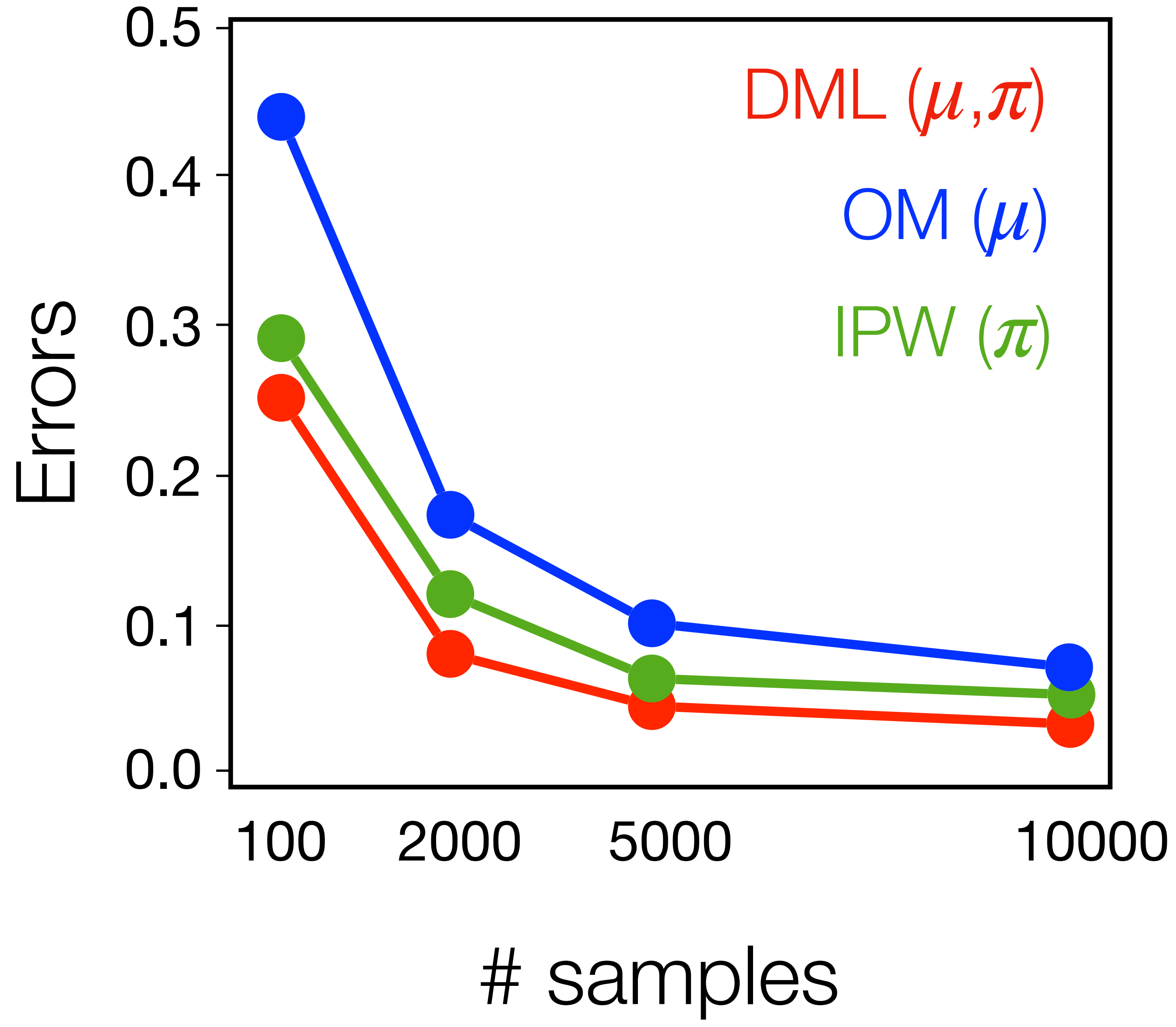
$\hat{\pi}$ misspecified ($\hat{\pi} \neq \pi$)



Simulation: DML-BD+

Fast Convergence

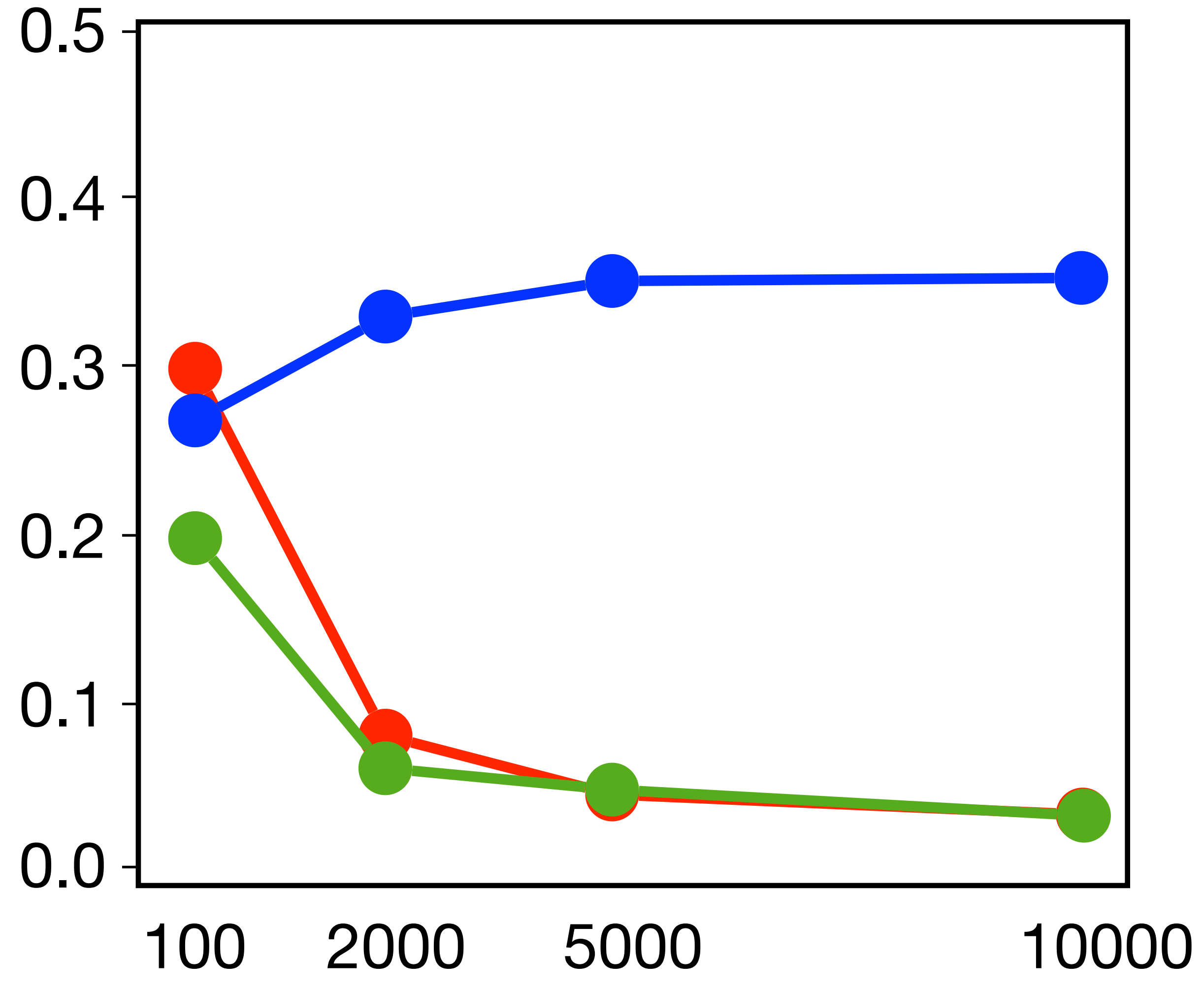
$(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\pi}}) \to (\boldsymbol{\mu}_0, \boldsymbol{\pi}_0)$ slowly



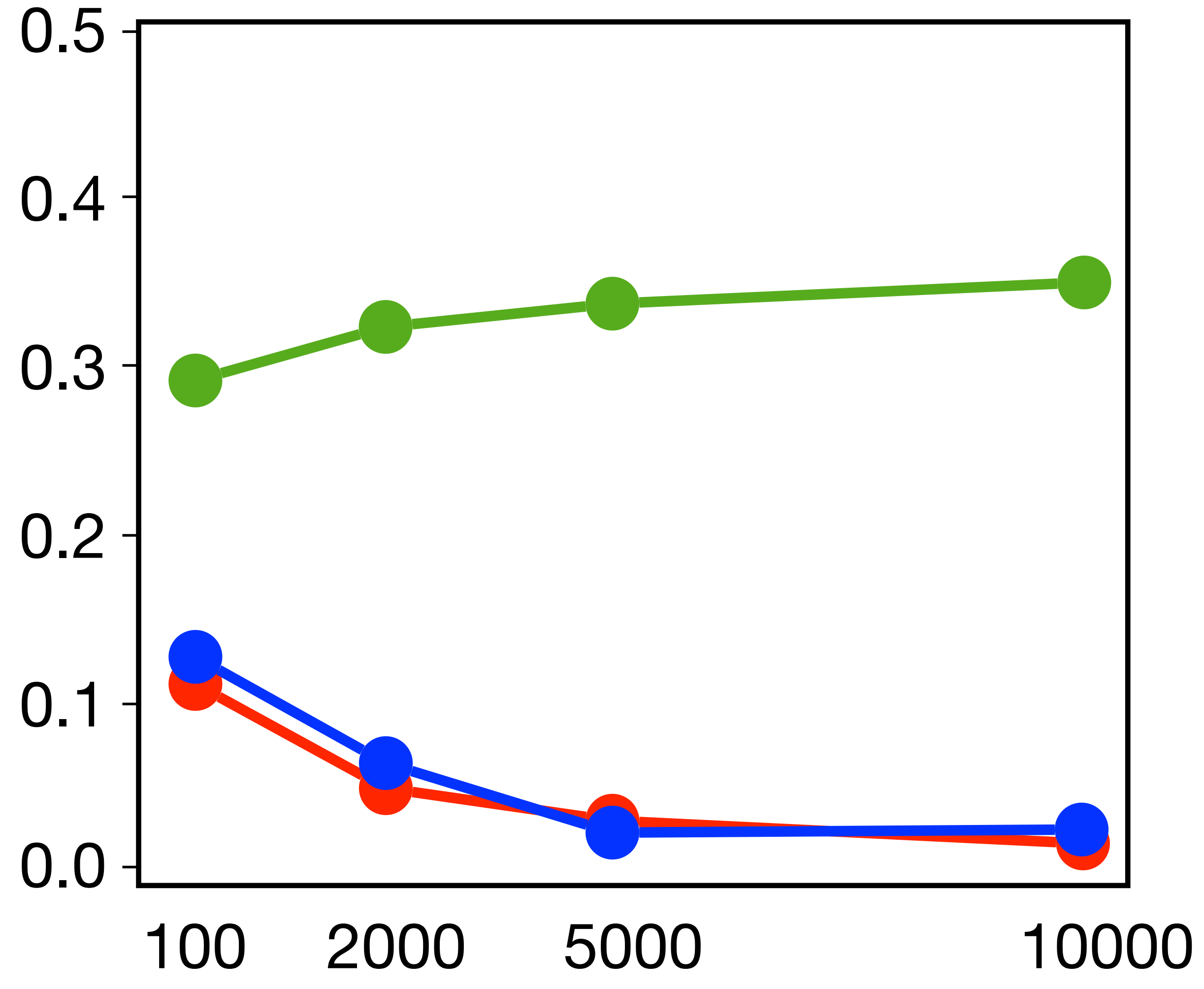
DML-BD+ converges fast, even when $(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\pi}})$ converge slowly

Double Robustness

$\hat{\boldsymbol{\mu}}$ misspecified ($\hat{\boldsymbol{\mu}} \neq \boldsymbol{\mu}$)



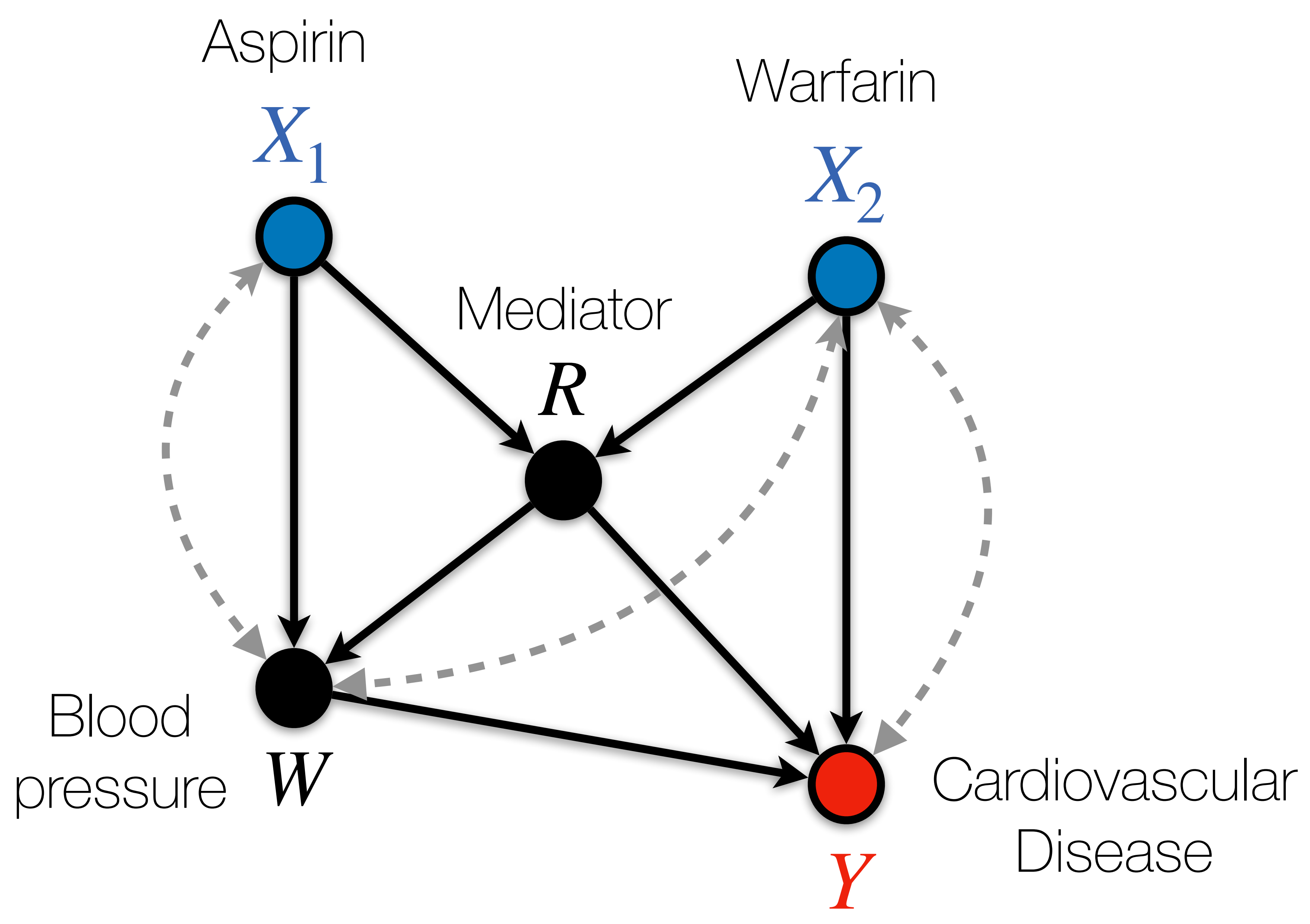
$\hat{\boldsymbol{\pi}}$ misspecified ($\hat{\boldsymbol{\pi}} \neq \boldsymbol{\pi}$)



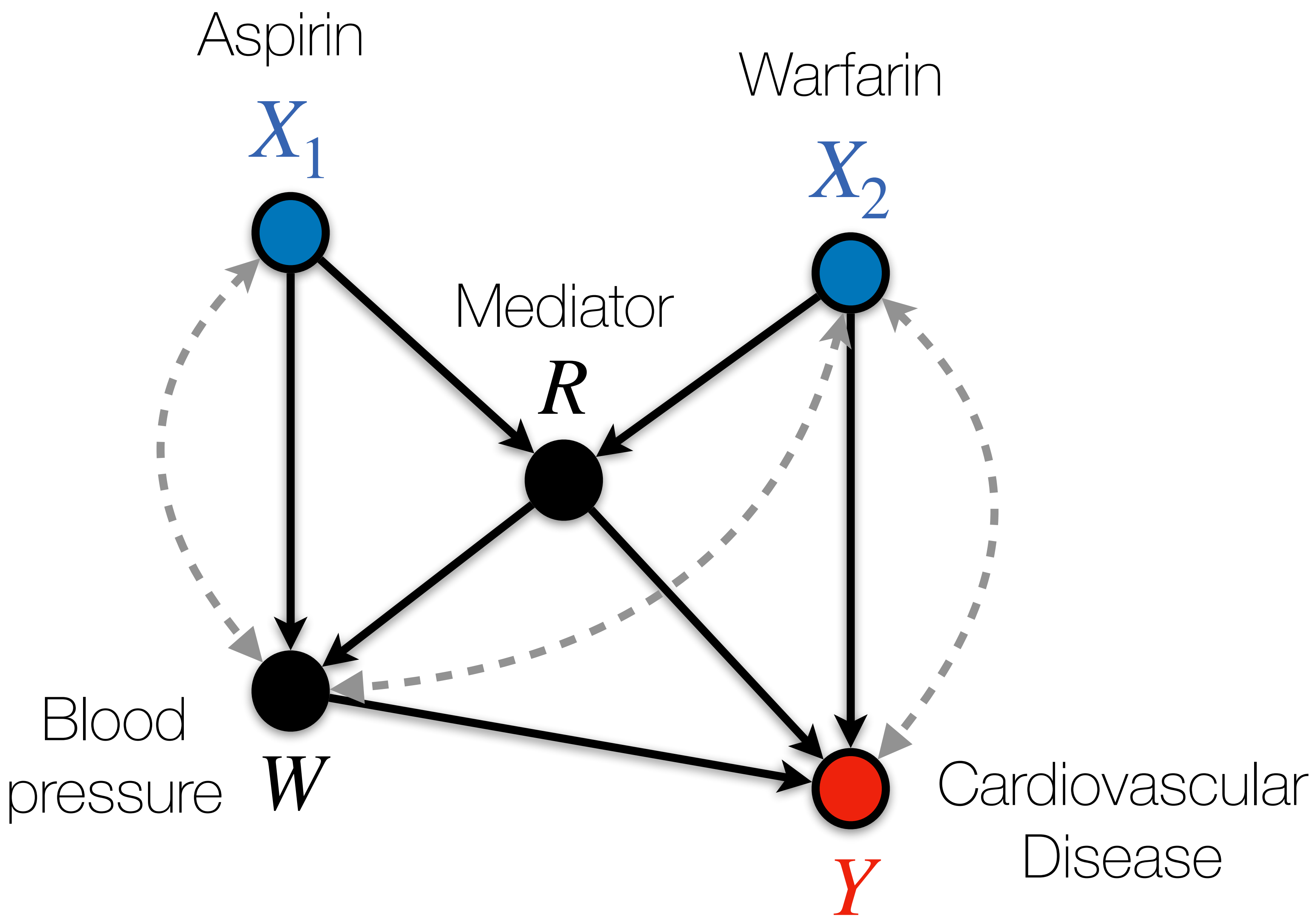
DML-BD+ converges to the true causal effect even when $\hat{\boldsymbol{\mu}}$ or $\hat{\boldsymbol{\pi}}$ are misspecified.

Example where BD^+ Fails

Example where BD+ Fails

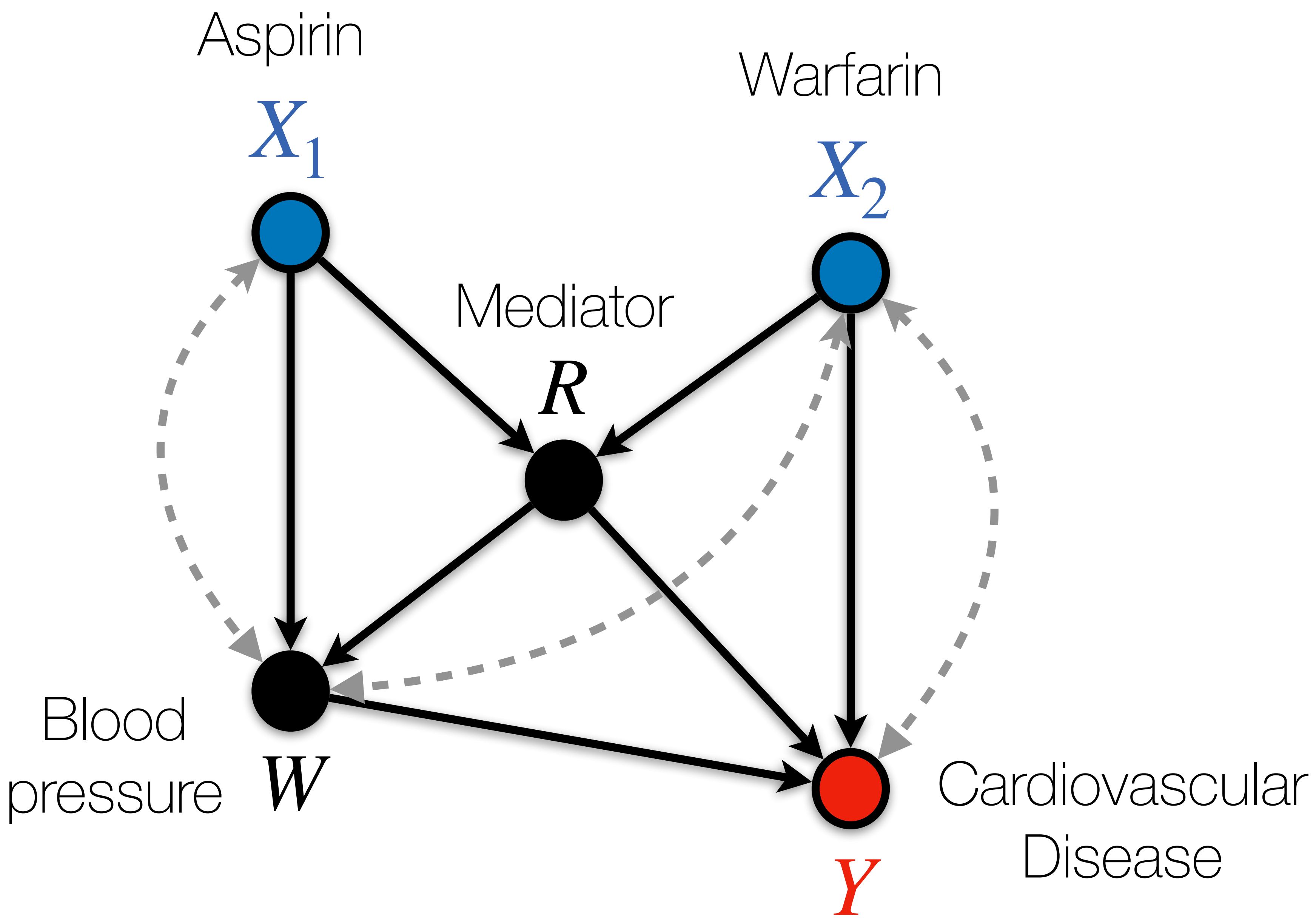


Example where BD+ Fails



$$\sum_{rw} P_{\mathrm{do}(x_1)}(r \mid x_2) P_{\mathrm{do}(x_2)}(y \mid rwx_1) \sum_{x_2'} P_{\mathrm{do}(x_1)}(w \mid r, x_2') P_{\mathrm{do}(x_1)}(x_2')$$

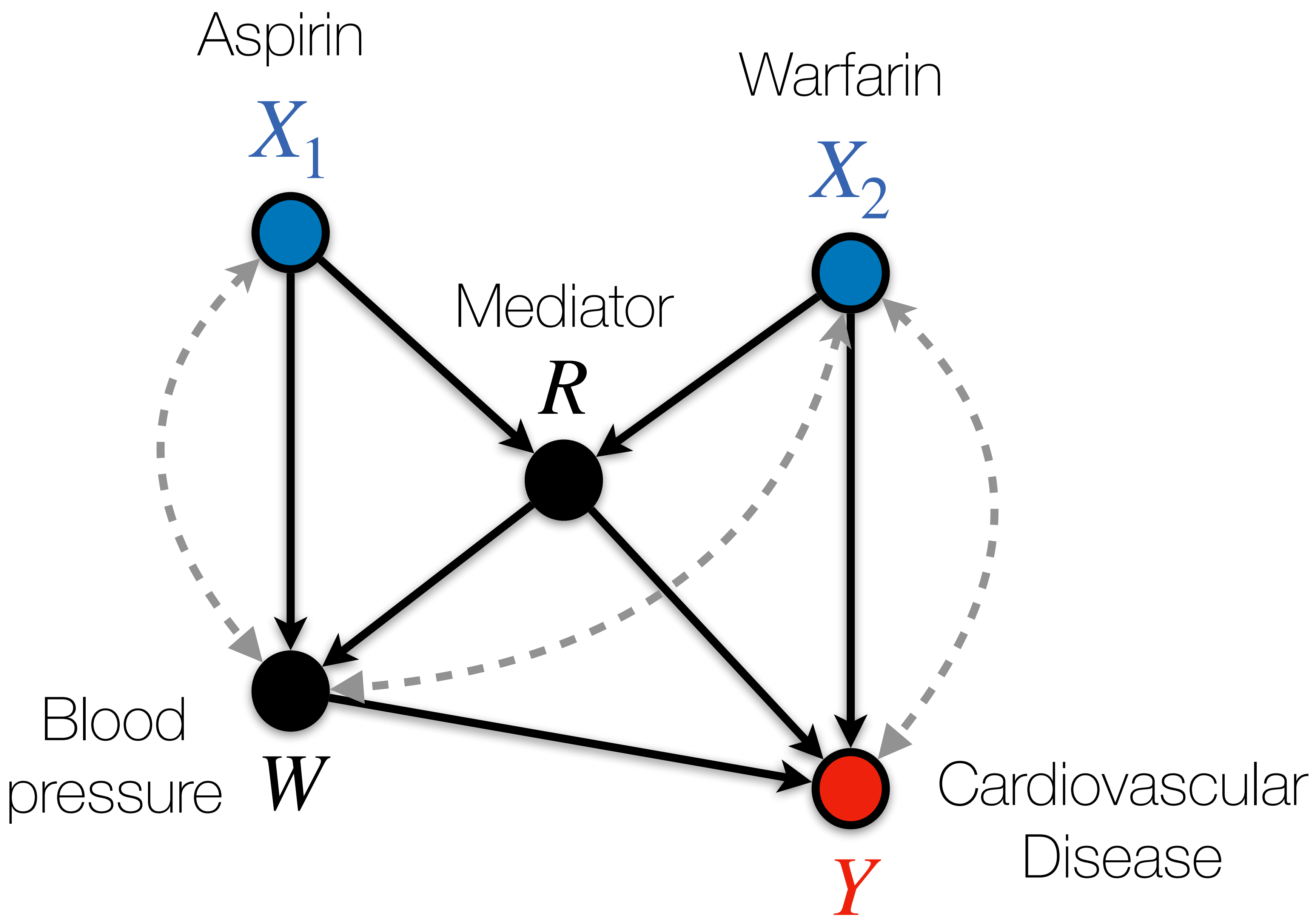
Example where BD+ Fails



✗ BD+ fails

$$\sum_{rw} P_{\mathrm{do}(x_1)}(r \mid x_2) P_{\mathrm{do}(x_2)}(y \mid rwx_1) \sum_{x_2'} P_{\mathrm{do}(x_1)}(w \mid r, x_2') P_{\mathrm{do}(x_1)}(x_2')$$

Example where BD+ Fails



X BD+ fails

$$\sum_{rw} P_{\mathrm{do}(x_1)}(r \mid x_2) P_{\mathrm{do}(x_2)}(y \mid rwx_1) \sum_{x_2'} P_{\mathrm{do}(x_1)}(w \mid r, x_2') P_{\mathrm{do}(x_1)}(x_2')$$

Can $\mathbb{E}[Y \mid \mathrm{do}(x_1, x_2)]$ be sample-efficiently estimated?

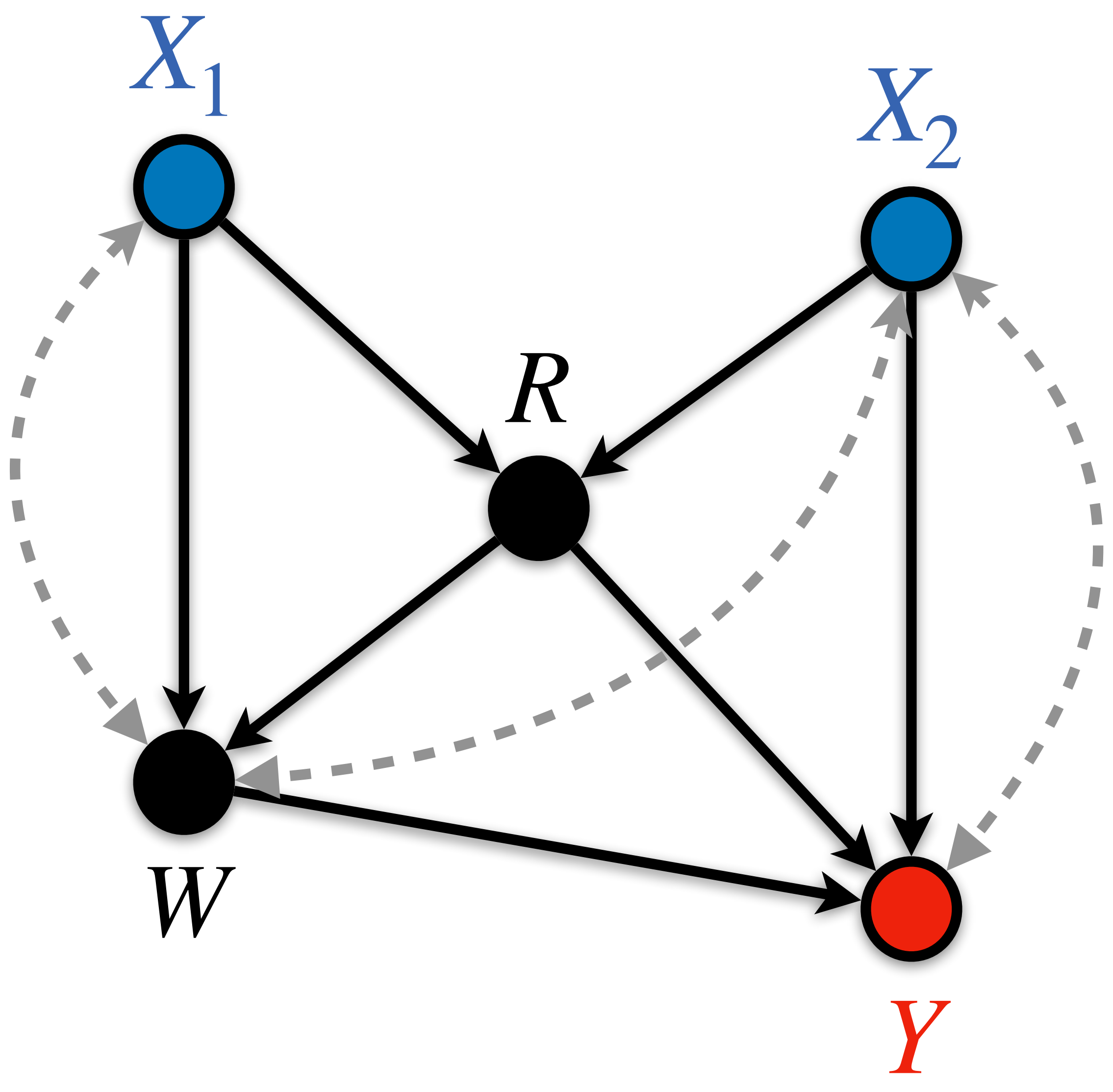
Background: General Identification from Data Fusion

General Identification (gID, Algo 5)

Bareinboim and Pearl, 2012; Lee et al. 2019

- spanning a *tree* from available distributions $\{P_{\mathrm{do}(\mathbf{r}_i)}(\mathbf{V})\}_{\mathbf{R}_i \subseteq \mathbf{V}}$
- to reach to causal distribution $P(\mathbf{Y} \mid \mathrm{do}(\mathbf{X}))$
- through factorization & marginalization of distributions

Background: General Identification from Data Fusion



General Identification (gID, Algo 5)

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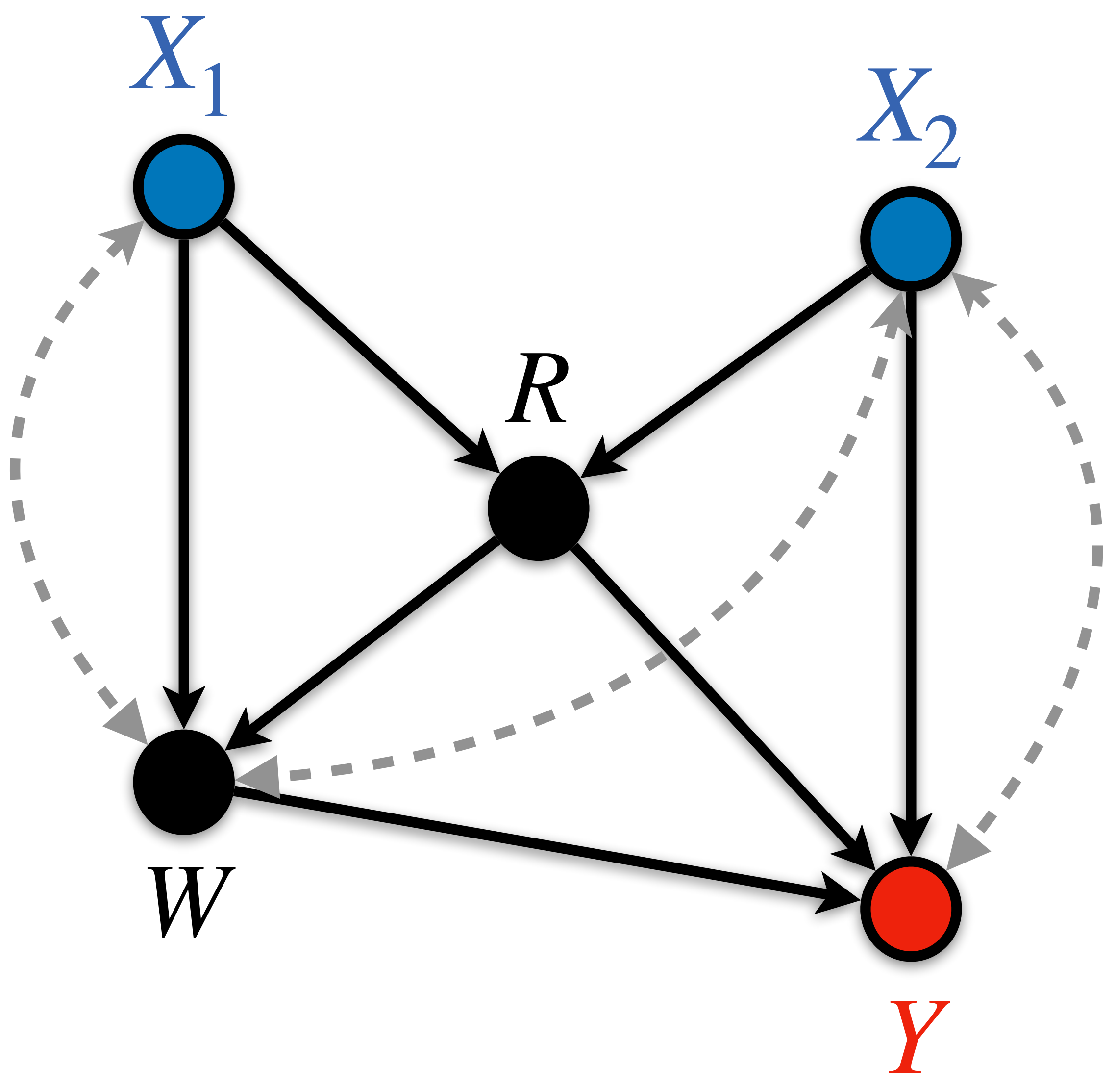
- to reach to causal distribution $P(\mathbf{Y} \mid \text{do}(\mathbf{X}))$
- through factorization & marginalization of distributions

Available distributions

$P_{\text{do}(x_1)}(RWX_2Y)$

$P_{\text{do}(x_2)}(RWX_1Y)$

Background: General Identification from Data Fusion



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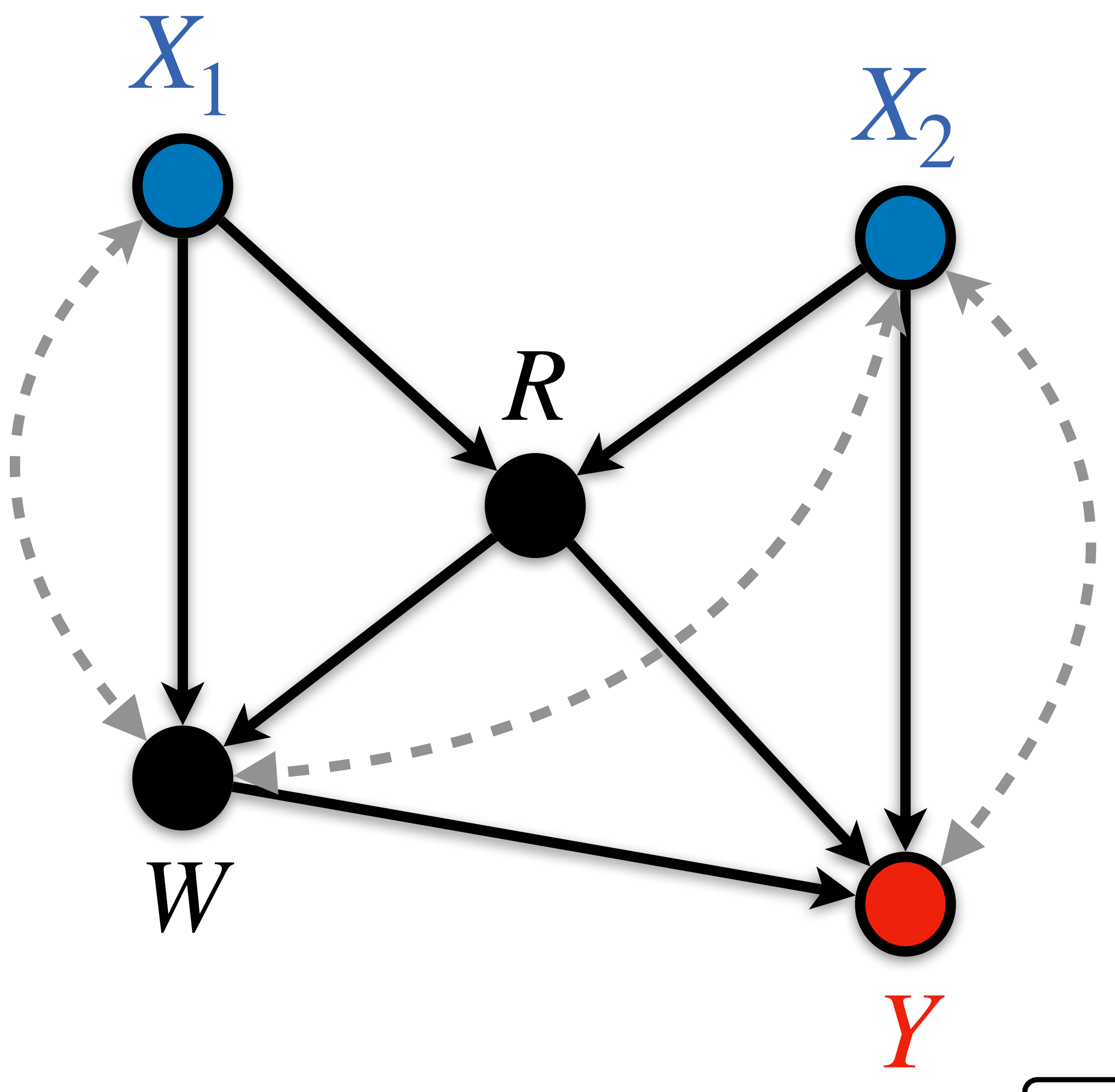
- to reach to causal distribution $P(\mathbf{Y} \mid \mathrm{do}(\mathbf{X}))$
- through factorization & marginalization of distributions

$P_{\mathrm{do}(x_1)}(RWX_2Y)$ —Factorization→ $P_{\mathrm{do}(x_1,r)}(WX_2Y)$ —Marginalization→ $P_{\mathrm{do}(x_1,r)}(W)$

$P_{\mathrm{do}(x_1)}(RWX_2Y)$ —Factorization→ $P_{\mathrm{do}(x_1,x_2)}(R)$

$P_{\mathrm{do}(x_2)}(RWX_1Y)$ —Factorization→ $P_{\mathrm{do}(w,r,x_2)}(Y)$

Background: General Identification from Data Fusion



General Identification (gID, Algo 5)

Bareinboim and Pearl, 2012; Lee et al. 2019

- spanning a *tree* from available distributions $\{P_{do(\mathbf{r}_i)}(\mathbf{V})\}_{\mathbf{R}_i \subseteq \mathbf{V}}$
- to reach to causal distribution $P(\mathbf{Y} \mid do(\mathbf{X}))$
- through factorization & marginalization of distributions

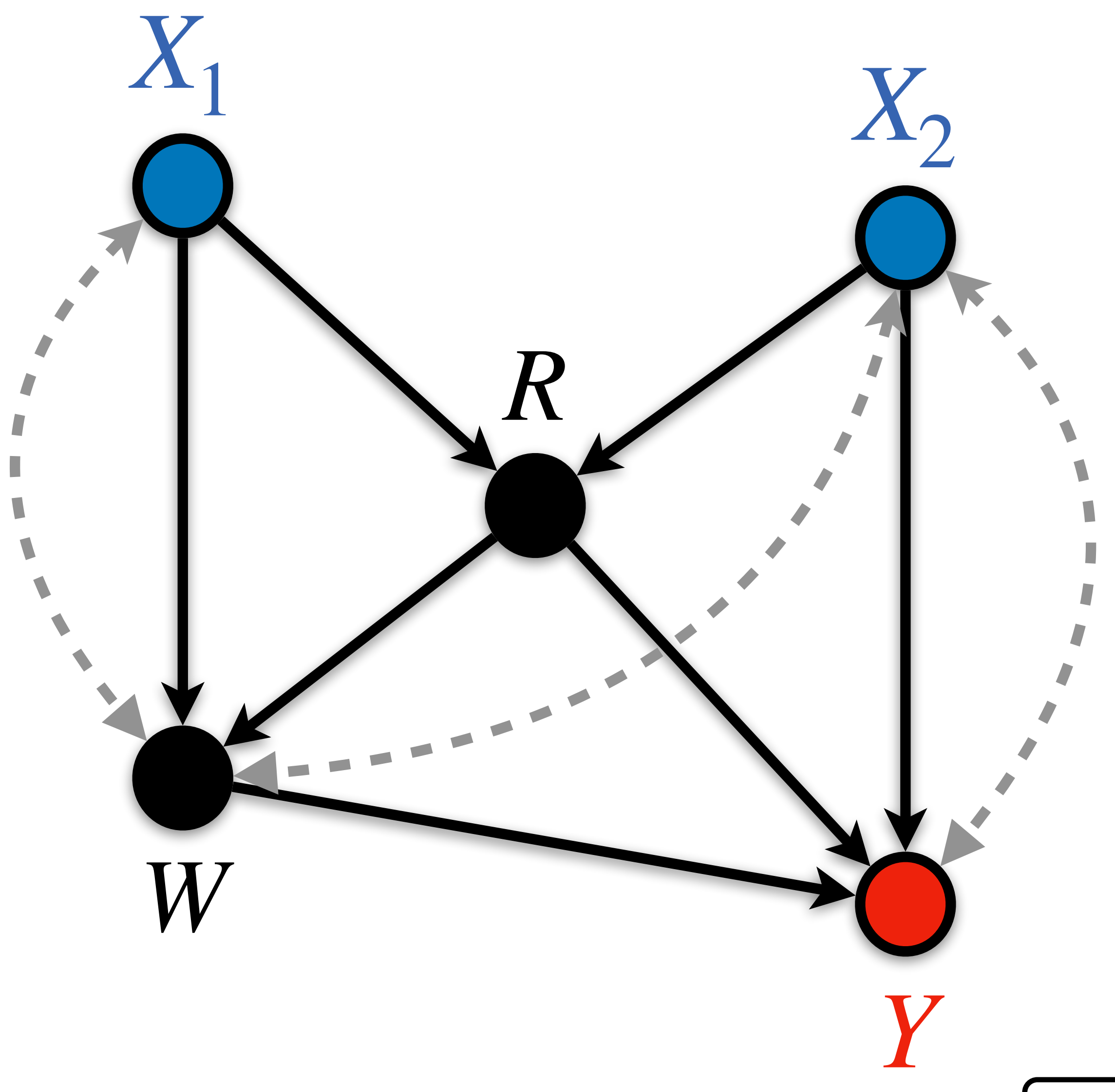
$P_{do(x_1)}(RWX_2Y)$ —Factorization→ $P_{do(x_1,r)}(WX_2Y)$ [$P_{do(x_1)}(y \mid wrx_2)P_{do(x_1)}(w \mid rx_2)P_{do(x_1)}(x_2)$]

$P_{do(x_1,r)}(WX_2Y)$ —Marginalization→ $P_{do(x_1,r)}(W)$ [$\sum_{x_2'} P_{do(x_1)}(w \mid r, x_2')P_{do(x_1)}(x_2')$]

$P_{do(x_1)}(RWX_2Y)$ —Factorization→ $P_{do(x_1,x_2)}(R)$ [$P_{do(x_1)}(r \mid x_2)$]

$P_{do(x_2)}(RWX_1Y)$ —Factorization→ $P_{do(w,r,x_2)}(Y)$ [$P_{do(x_2)}(y \mid rwx_1)$]

Background: General Identification from Data Fusion



General Identification (gID, Algo 5)

Bareinboim and Pearl, 2012; Lee et al. 2019

- spanning a *tree* from available distributions $\{P_{\mathrm{do}(\mathbf{r}_i)}(\mathbf{V})\}_{\mathbf{R}_i \subseteq \mathbf{V}}$
- to reach to causal distribution $P(\mathbf{Y} \mid \mathrm{do}(\mathbf{X}))$
- through factorization & marginalization of distributions

$P_{\mathrm{do}(x_1)}(RWX_2Y)$ —Factorization→ $P_{\mathrm{do}(x_1,r)}(WX_2Y)$

$P_{\mathrm{do}(x_1)}(y \mid wrx_2)P_{\mathrm{do}(x_1)}(w \mid rx_2)P_{\mathrm{do}(x_1)}(x_2)$

$P_{\mathrm{do}(x_1,r)}(WX_2Y)$ —Marginalization→ $P_{\mathrm{do}(x_1,r)}(W)$

$\sum_{x_2'} P_{\mathrm{do}(x_1)}(w \mid r, x_2')P_{\mathrm{do}(x_1)}(x_2')$

$P_{\mathrm{do}(x_1)}(RWX_2Y)$ —Factorization→ $P_{\mathrm{do}(x_1,x_2)}(R)$

$P_{\mathrm{do}(x_1)}(r \mid x_2)$

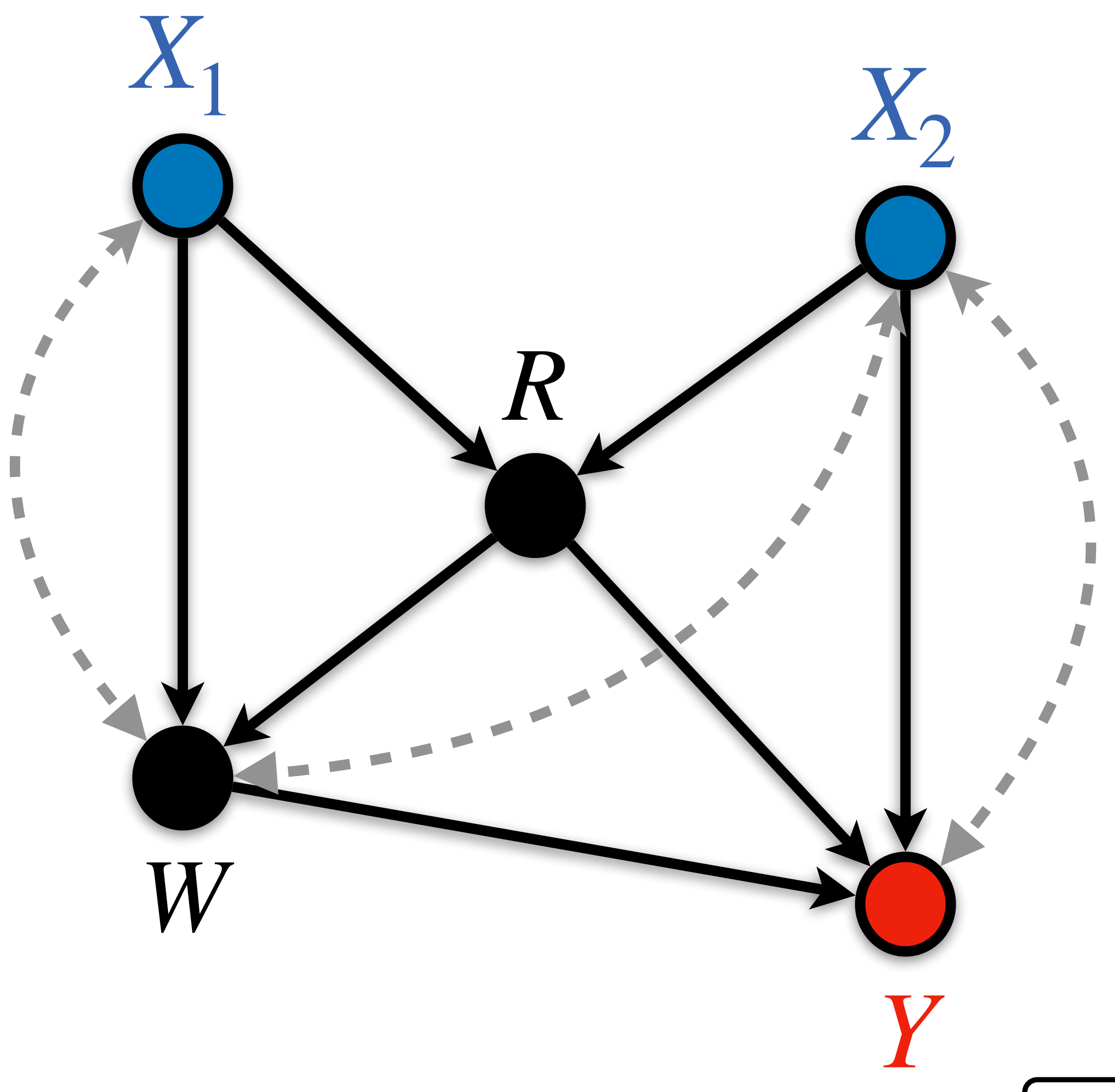
$P_{\mathrm{do}(x_2)}(RWX_1Y)$ —Factorization→ $P_{\mathrm{do}(w,r,x_2)}(Y)$

$P_{\mathrm{do}(x_2)}(y \mid rwx_1)$

$P_{\mathrm{do}(x_1,r)}(W)$ —Factorization→ $P_{\mathrm{do}(x_1,x_2)}(RWY)$; $P_{\mathrm{do}(x_1,x_2)}(R)$ → $P_{\mathrm{do}(x_1,x_2)}(RWY)$; $P_{\mathrm{do}(w,r,x_2)}(Y)$ → $P_{\mathrm{do}(x_1,x_2)}(RWY)$

$P_{\mathrm{do}(x_1)}(r \mid x_2)P_{\mathrm{do}(x_2)}(y \mid rwx_1)\sum_{x_2'} P_{\mathrm{do}(x_1)}(w \mid r, x_2')P_{\mathrm{do}(x_1)}(x_2')$

Background: General Identification from Data Fusion

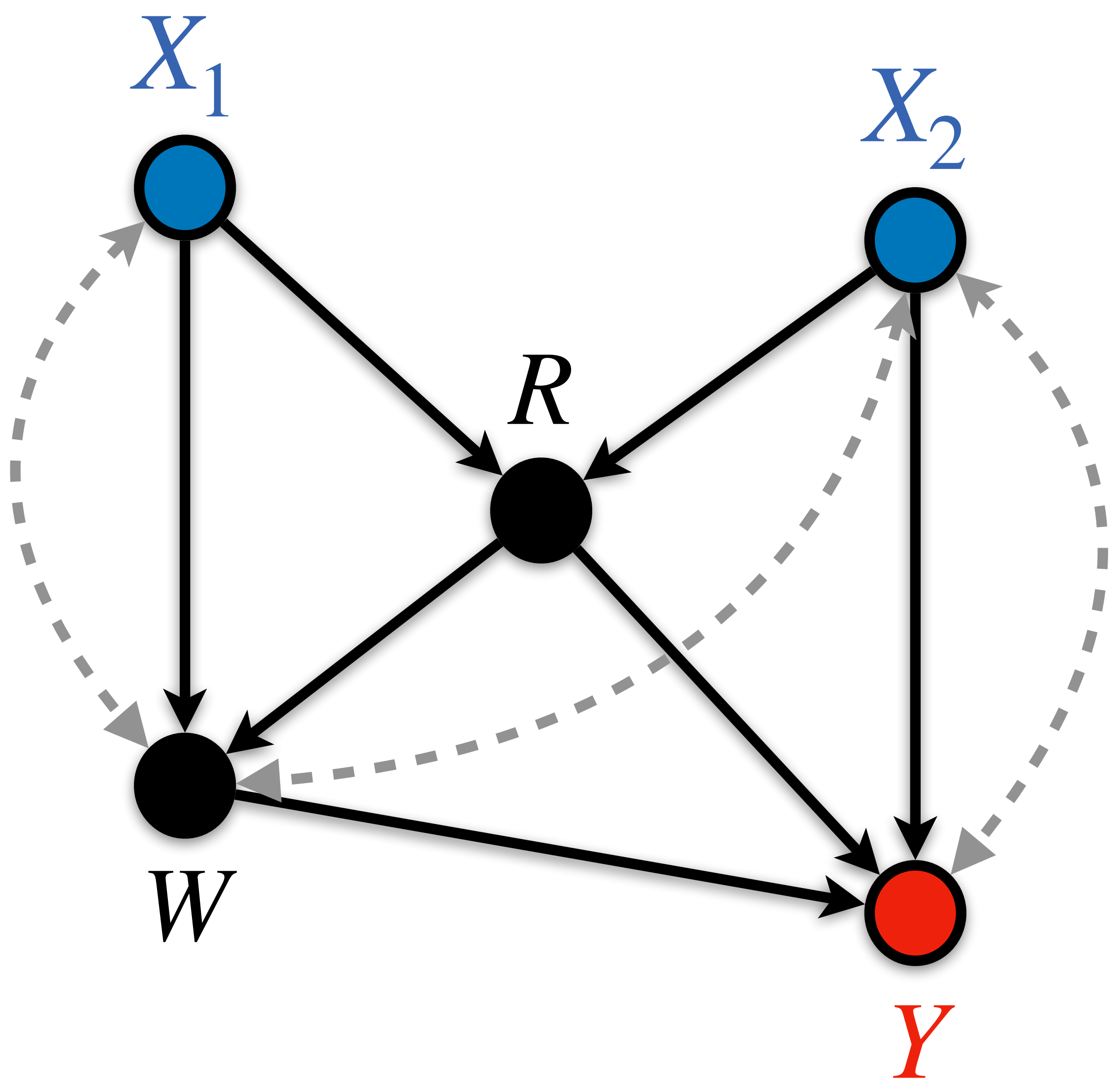


General Identification (gID, Algo 5)

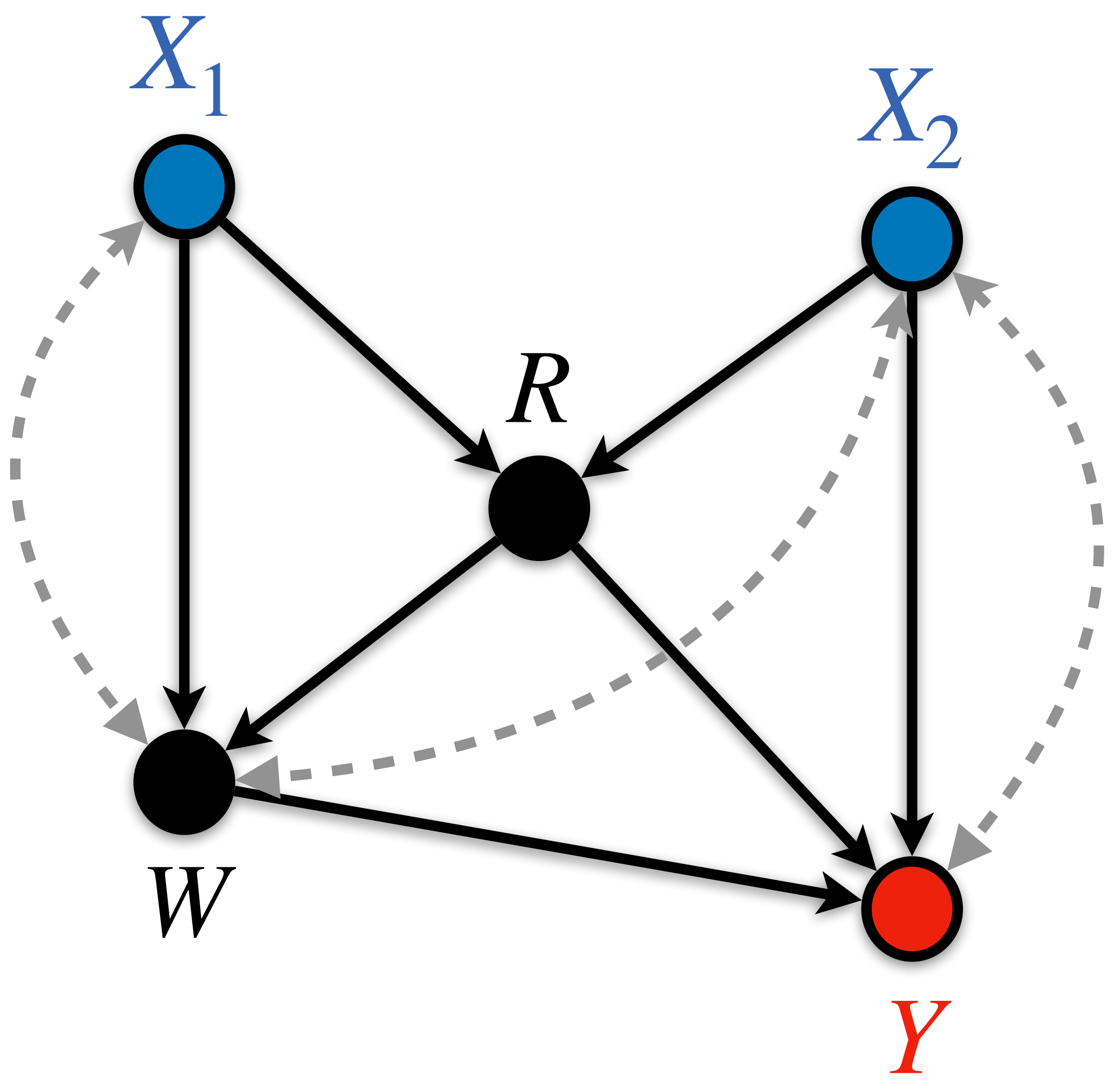
Bareinboim and Pearl, 2012; Lee et al. 2019

- spanning a *tree* from available distributions $\{P_{\mathrm{do}(\mathbf{r}_i)}(\mathbf{V})\}_{\mathbf{R}_i \subseteq \mathbf{V}}$
- to reach to causal distribution $P(\mathbf{Y} \mid \mathrm{do}(\mathbf{X}))$
- through factorization & marginalization of distributions

Causal effects as a function of BD⁺



Causal effects as a function of BD+



$P_{\text{do}(x_1,r)}(WX_2Y) \rightarrow P_{\text{do}(x_1,r)}(W)$

$P_{\text{do}(x_1)}(RWX_2Y) \rightarrow P_{\text{do}(x_1,x_2)}(R)$

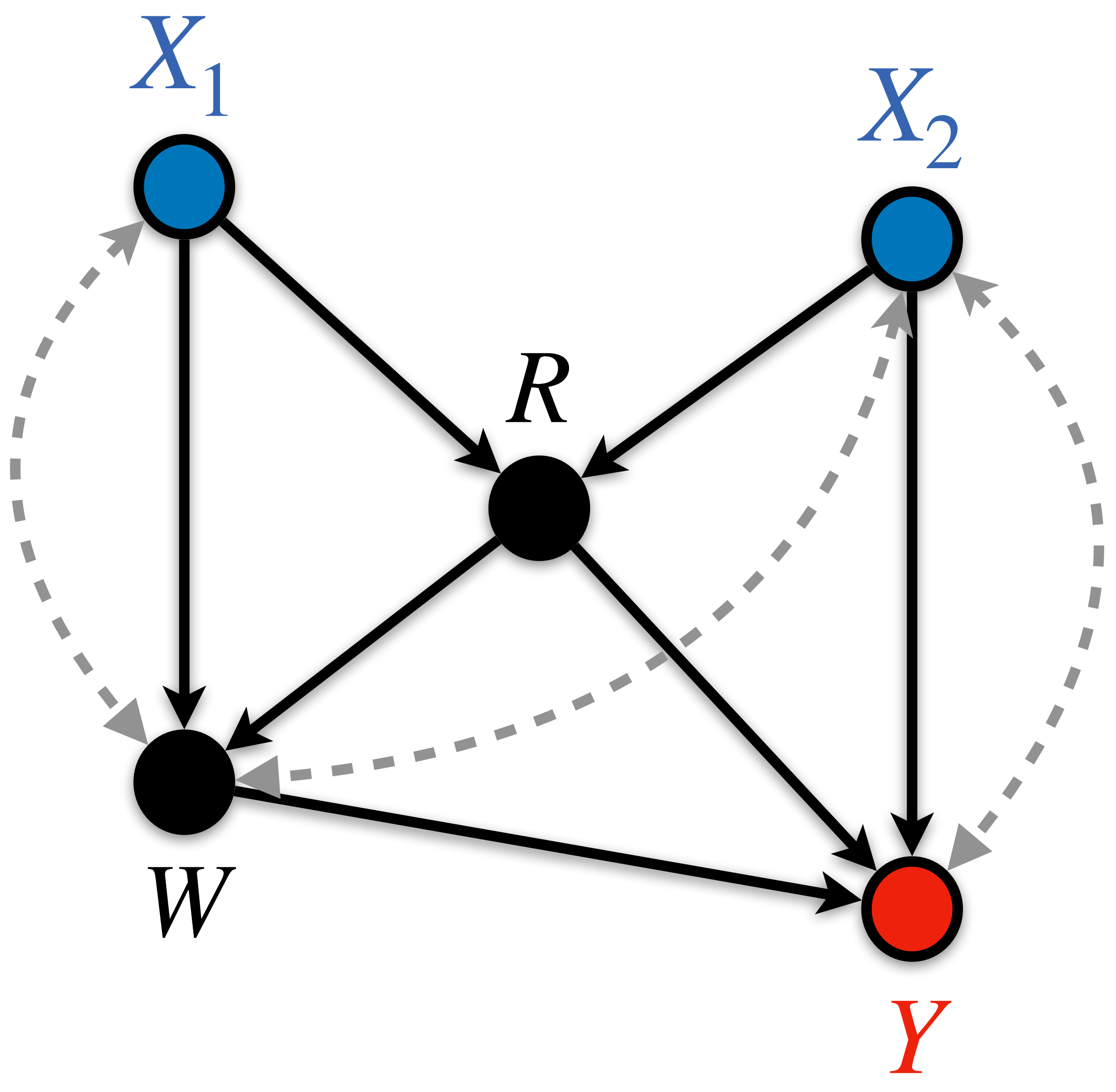
$P_{\text{do}(x_2)}(RWX_1Y) \rightarrow P_{\text{do}(w,r,x_2)}(Y)$

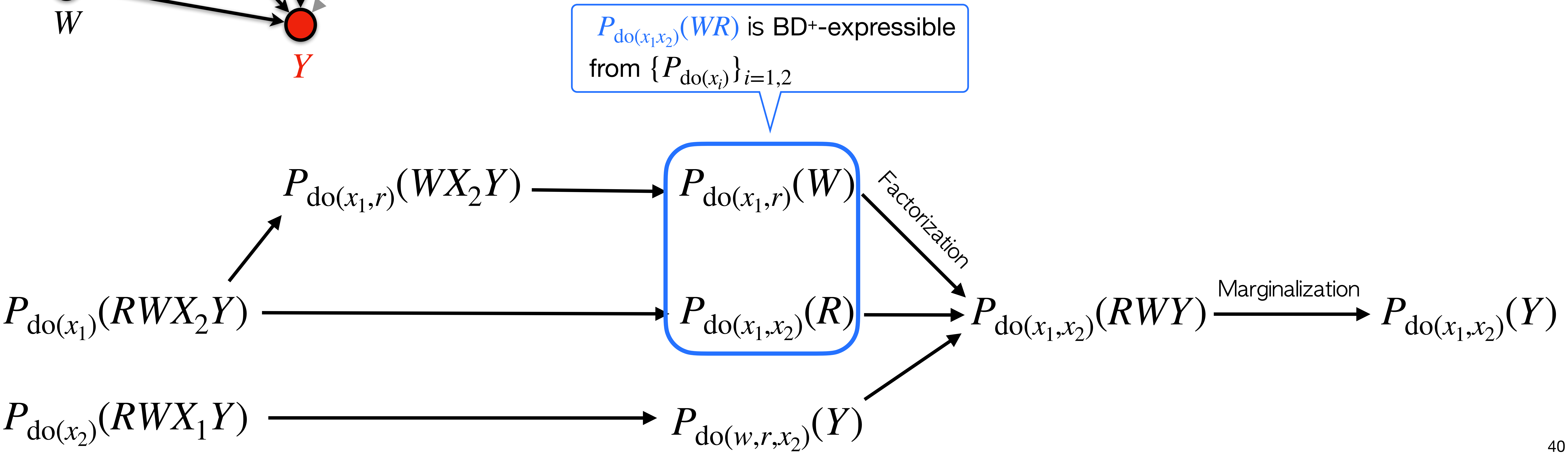
$P_{\text{do}(x_1)}(RWX_2Y) \rightarrow P_{\text{do}(x_1,r)}(WX_2Y)$

[Factorization] $P_{\text{do}(x_1,r)}(W),\ P_{\text{do}(x_1,x_2)}(R),\ P_{\text{do}(w,r,x_2)}(Y) \rightarrow P_{\text{do}(x_1,x_2)}(RWY)$

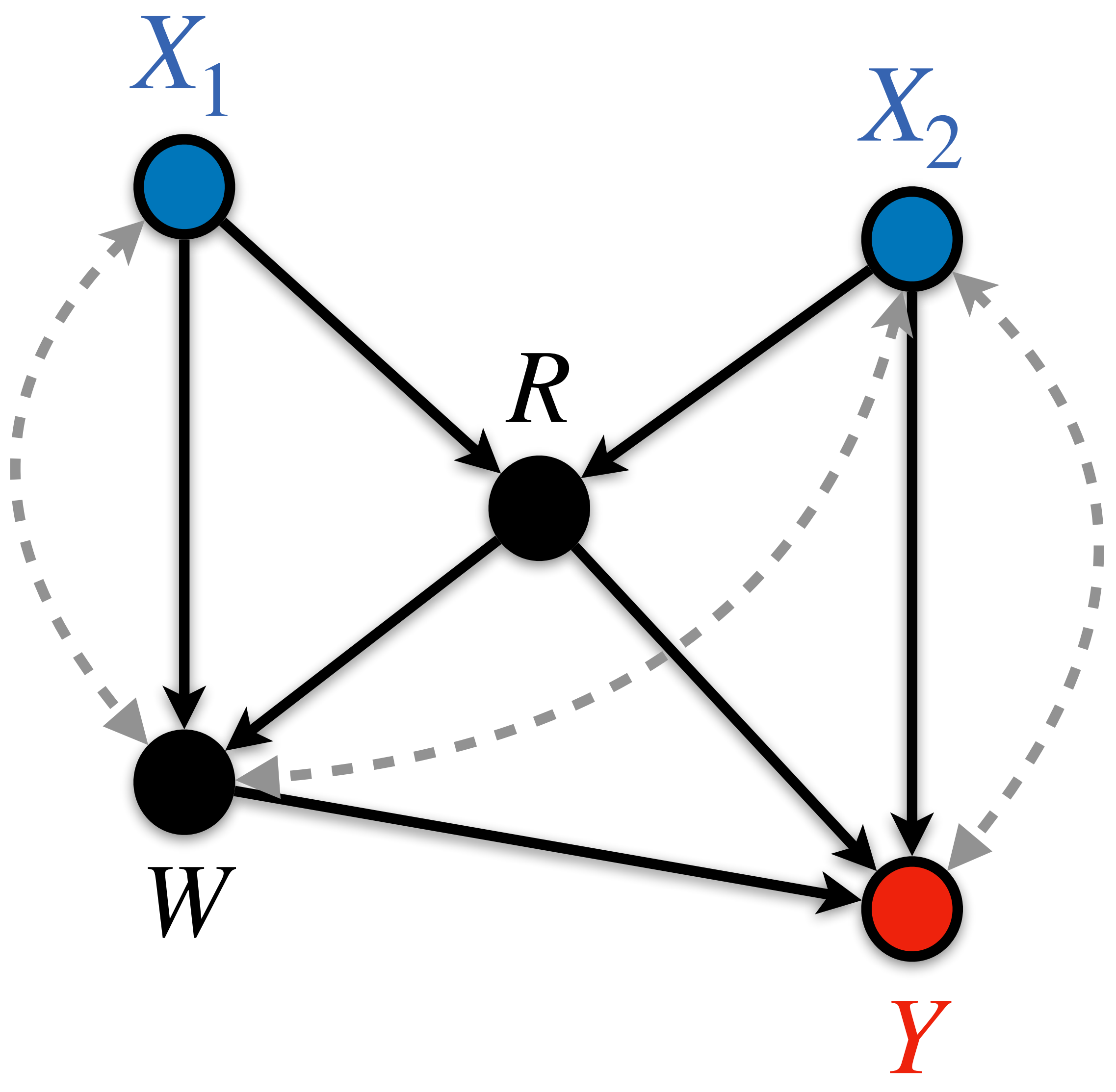
[Marginalization] $P_{\text{do}(x_1,x_2)}(RWY) \rightarrow P_{\text{do}(x_1,x_2)}(Y)$

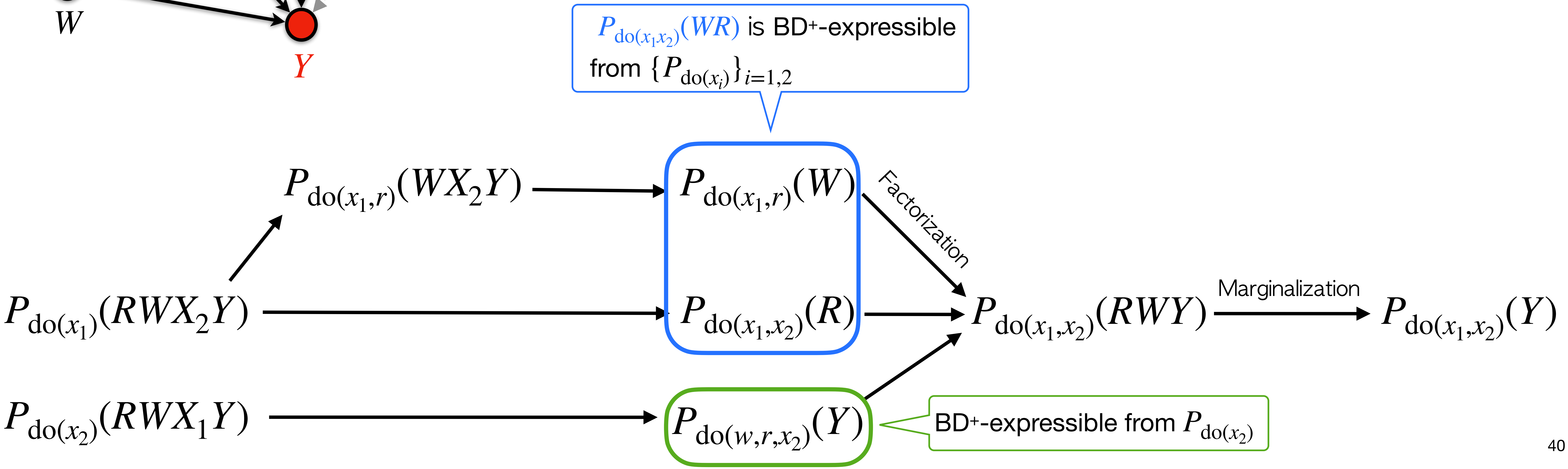
Causal effects as a function of BD+



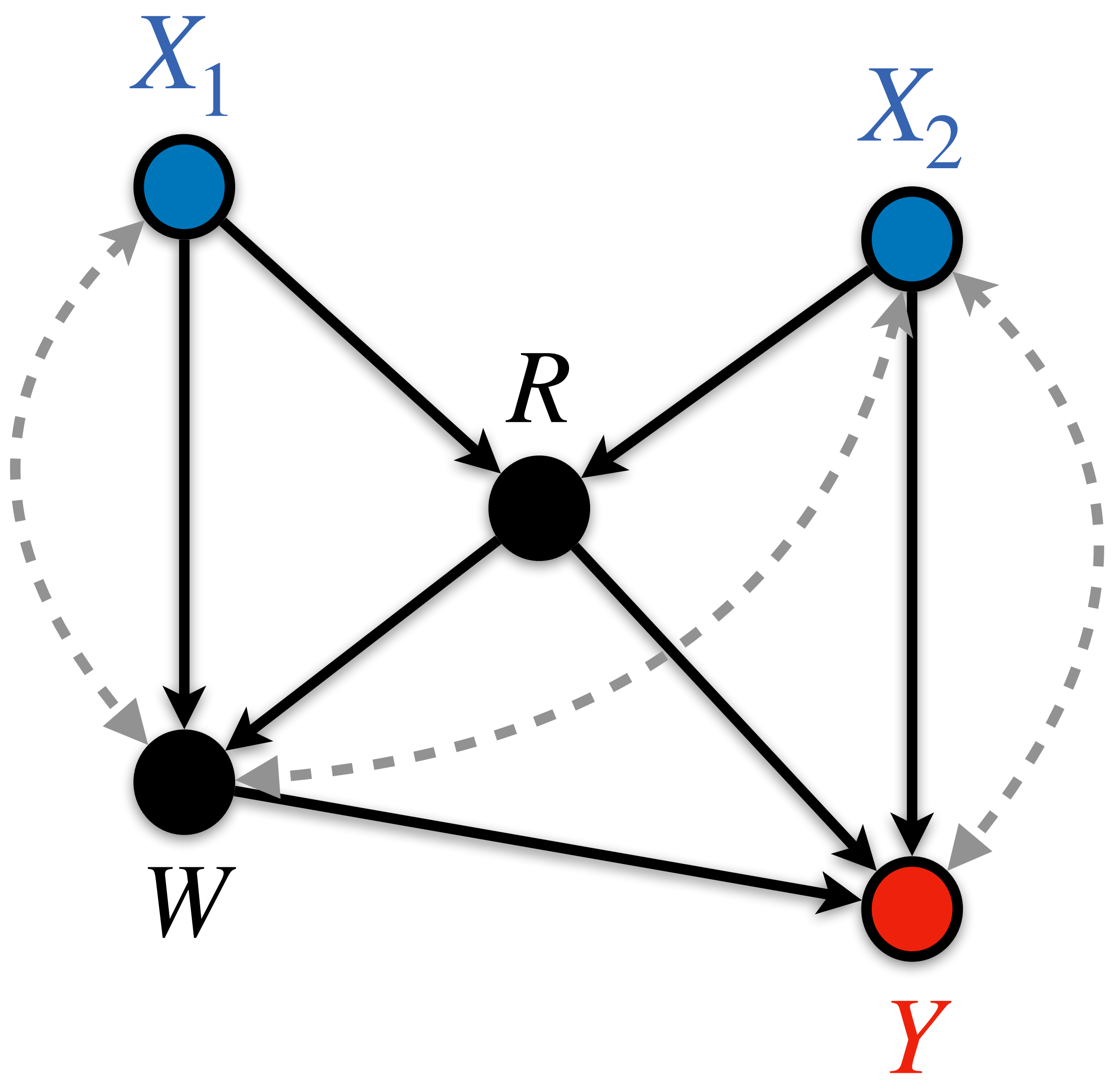


Causal effects as a function of BD+





Causal effects as a function of BD+



$P_{do(x_1 x_2)}(WR)$ is BD+-expressible from $\{P_{do(x_i)}\}_{i=1,2}$

$P_{do(x_1 x_2)}(Y)$ is a function of BD+

$P_{do(x_1)}(RWX_2Y) \rightarrow P_{do(x_1,r)}(WX_2Y) \rightarrow P_{do(x_1,r)}(W)$

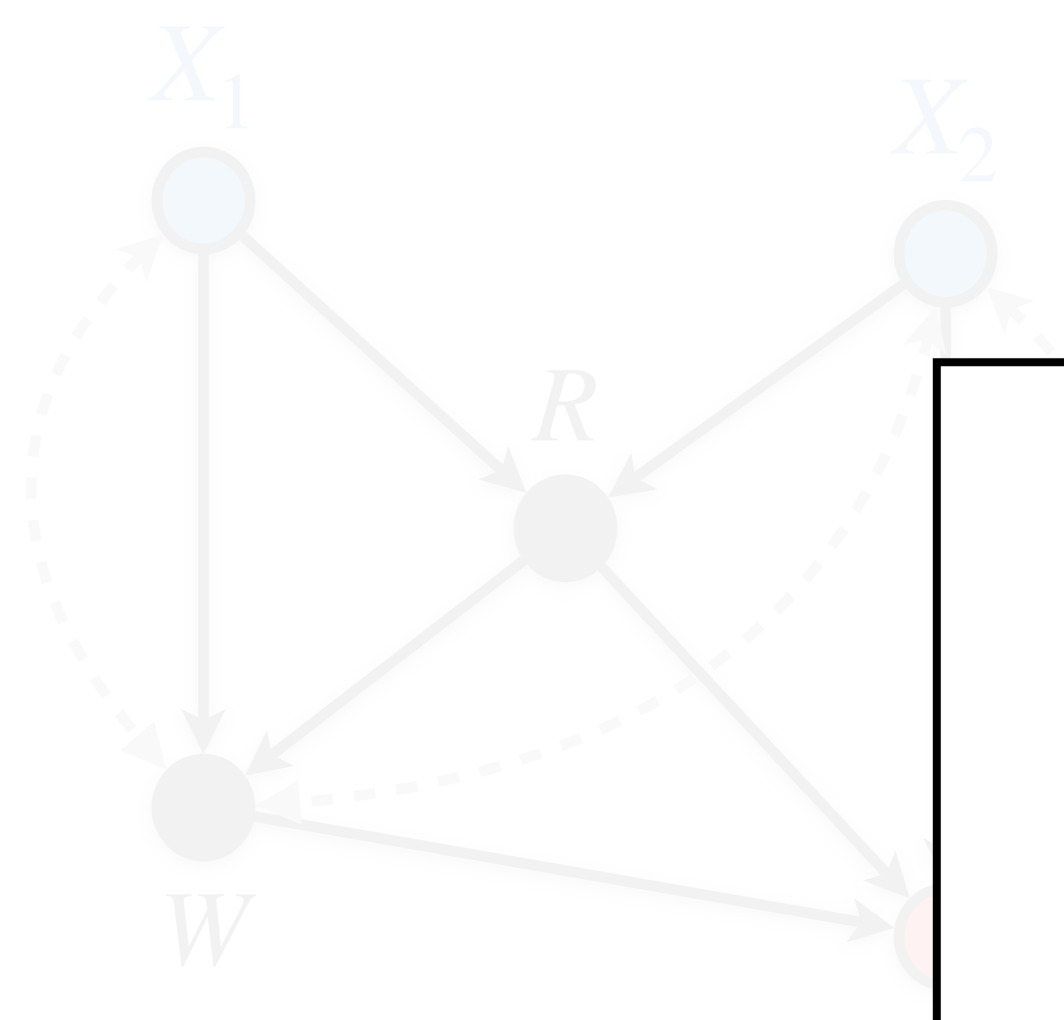
$P_{do(x_1)}(RWX_2Y) \rightarrow P_{do(x_1,x_2)}(R)$

$P_{do(x_2)}(RWX_1Y) \rightarrow P_{do(w,r,x_2)}(Y)$ — BD+-expressible from $P_{do(x_2)}$

Factorization: $P_{do(x_1,r)}(W),\ P_{do(x_1,x_2)}(R),\ P_{do(w,r,x_2)}(Y) \rightarrow P_{do(x_1,x_2)}(RWY)$

Marginalization: $P_{do(x_1,x_2)}(RWY) \rightarrow P_{do(x_1,x_2)}(Y)$

Causal effects as a function of BD+



Theorem 26

The followings are equivalent:

1. $P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}))$ is identifiable from $(\mathcal{G}, \{P_{\mathrm{do}(\mathbf{r}_i)}\})$
2. $P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}))$ is expressible as a ***function of BD+s*** through AdmissibleGID (Algo 6)

DML-gID: Estimator for Causal Effects from Fusion

$$\mathbb{E}[Y \mid \mathrm{do}(\mathbf{x})] = f(\{\text{BD}^{+}(\mu_1, \pi_1),\ \text{BD}^{+}(\mu_2, \pi_2),\ \cdots,\ \text{BD}^{+}(\mu_m, \pi_m)\})$$

DML-gID: Estimator for Causal Effects from Fusion

$$\mathbb{E}[Y \mid \mathrm{do}(\mathbf{x})] = f(\{\mathrm{BD}^{+}(\mu_1, \pi_1), \mathrm{BD}^{+}(\mu_2, \pi_2), \cdots, \mathrm{BD}^{+}(\mu_m, \pi_m)\})$$

$$\mathbb{E}[\widehat{Y \mid \mathrm{do}}(\mathbf{x})] \triangleq f(\{ \qquad \})$$

"DML-gID" (Def 49)

DML-gID: Estimator for Causal Effects from Fusion

$$\mathbb{E}[Y \mid do(\mathbf{x})] = f(\{\text{BD}^+(\mu_1,\pi_1), \text{BD}^+(\mu_2,\pi_2), \cdots, \text{BD}^+(\mu_m,\pi_m)\})$$

DML-BD+ DML-BD+ ⋯ DML-BD+

$$\widehat{\mathbb{E}[Y \mid do(\mathbf{x})]} \triangleq f(\{\widehat{\text{BD}}(\mu_1,\pi_1), \widehat{\text{BD}}(\mu_2,\pi_2), \ldots, \widehat{\text{BD}}(\mu_m,\pi_m)\})$$

"DML-gID" (Def 49)

Robustness of DML-gID

Theorem 27

$$\text{Error}(\text{DML-gID}, \mathbb{E}[Y \mid do(\mathbf{x})]) = \sum_{i=1}^{m} \text{Error}(\hat{\mu}_i, \mu_i) \times \text{Error}(\hat{\pi}_i, \pi_i)$$

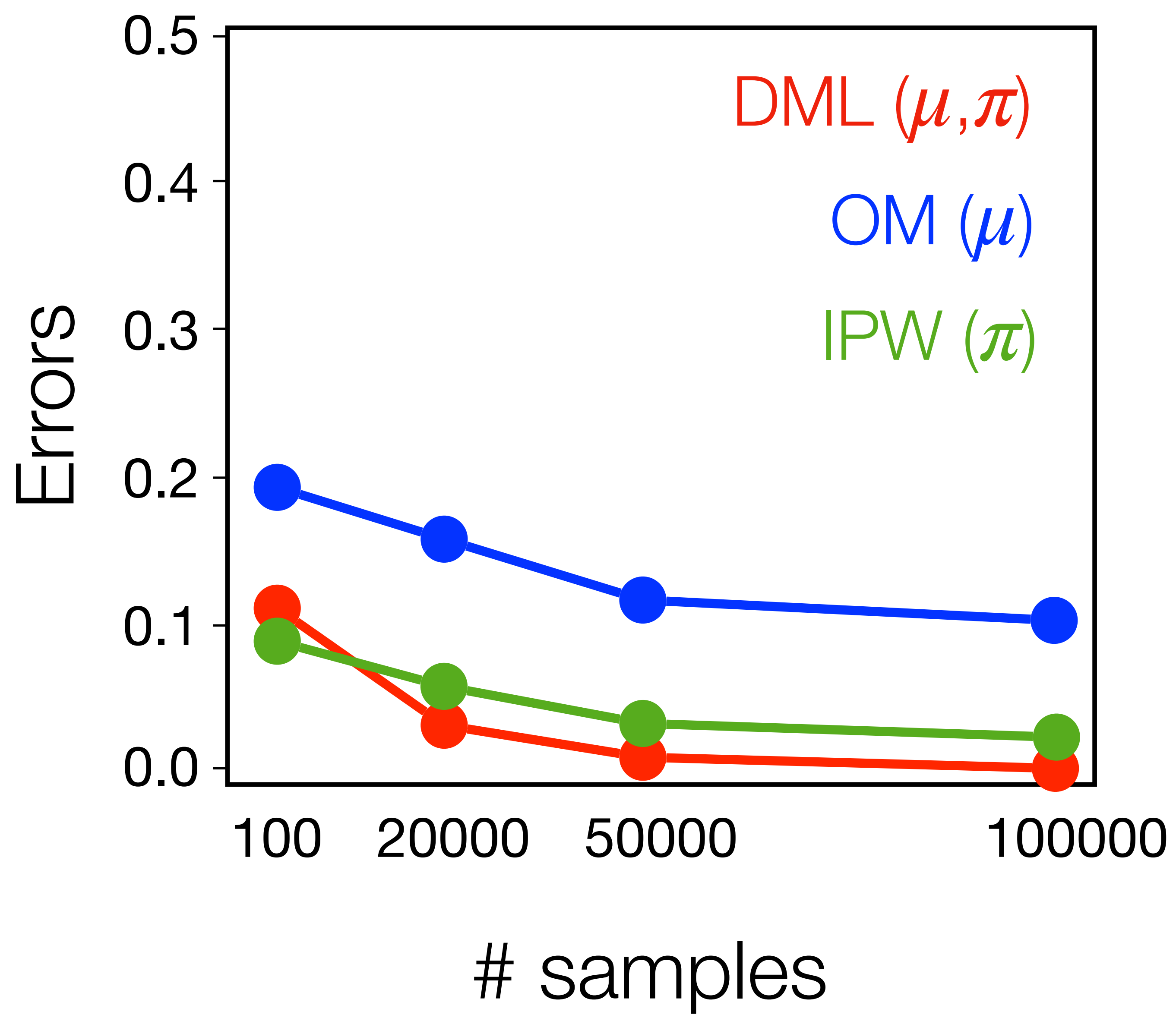
- **Double Robustness:** Error $= 0$ if either $\hat{\mu}_i = \mu_i$ or $\hat{\pi}_i = \pi_i$ for all $i = 1, \cdots, m$.
- **Fast Convergence:** Error $\rightarrow 0$ *fast* even when $\hat{\mu}_i \rightarrow \mu_i$ and $\hat{\pi}_i \rightarrow \pi_i$ *slow*.

DML-gID - Simulation (Sec. 4.6)

DML-gID - Simulation (Sec. 4.6)

Fast Convergence

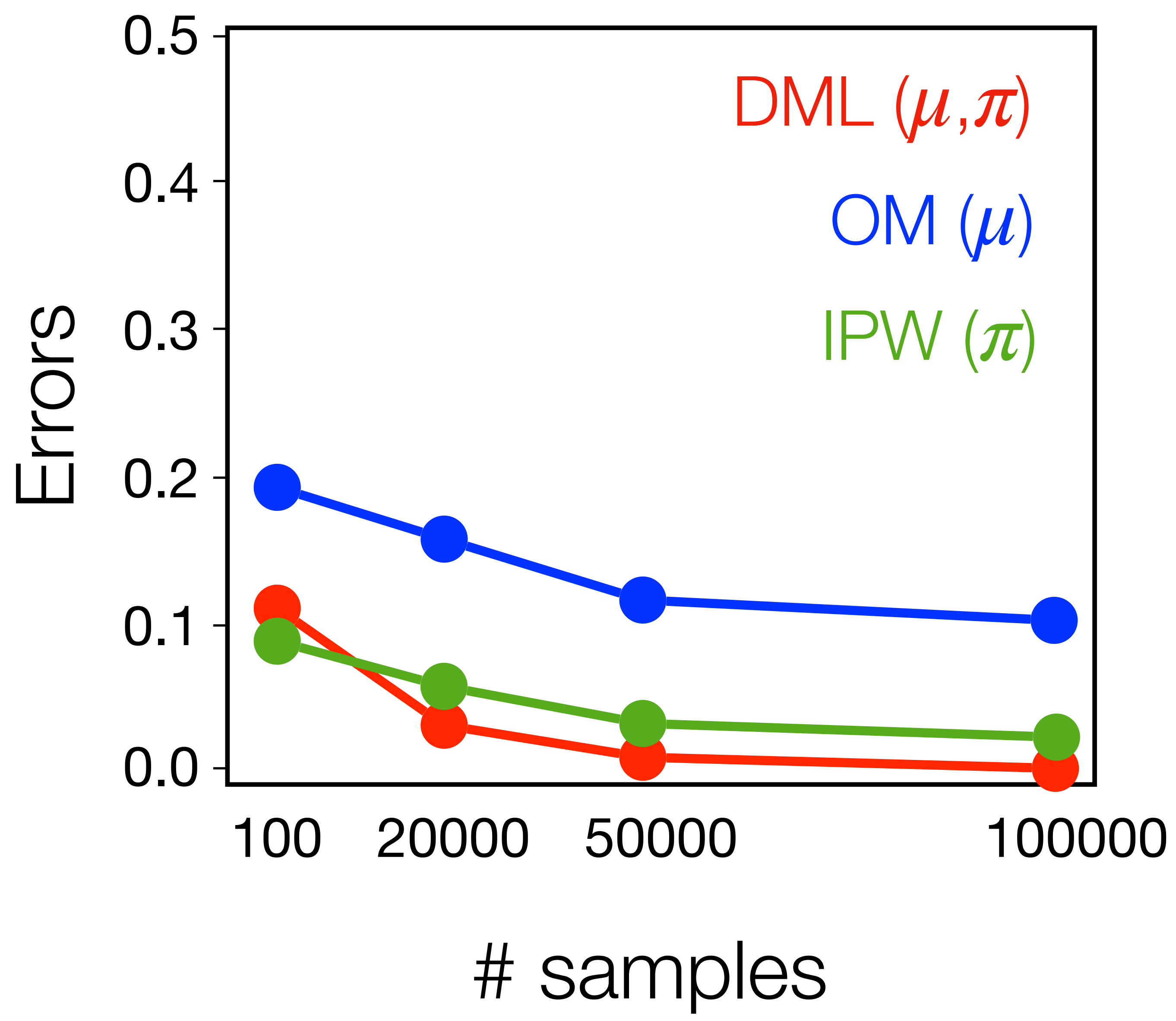
$(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\pi}}) \rightarrow (\boldsymbol{\mu}_0, \boldsymbol{\pi}_0)$ slowly



DML-gID - Simulation (Sec. 4.6)

Fast Convergence

$(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\pi}}) \rightarrow (\boldsymbol{\mu}_0, \boldsymbol{\pi}_0)$ slowly

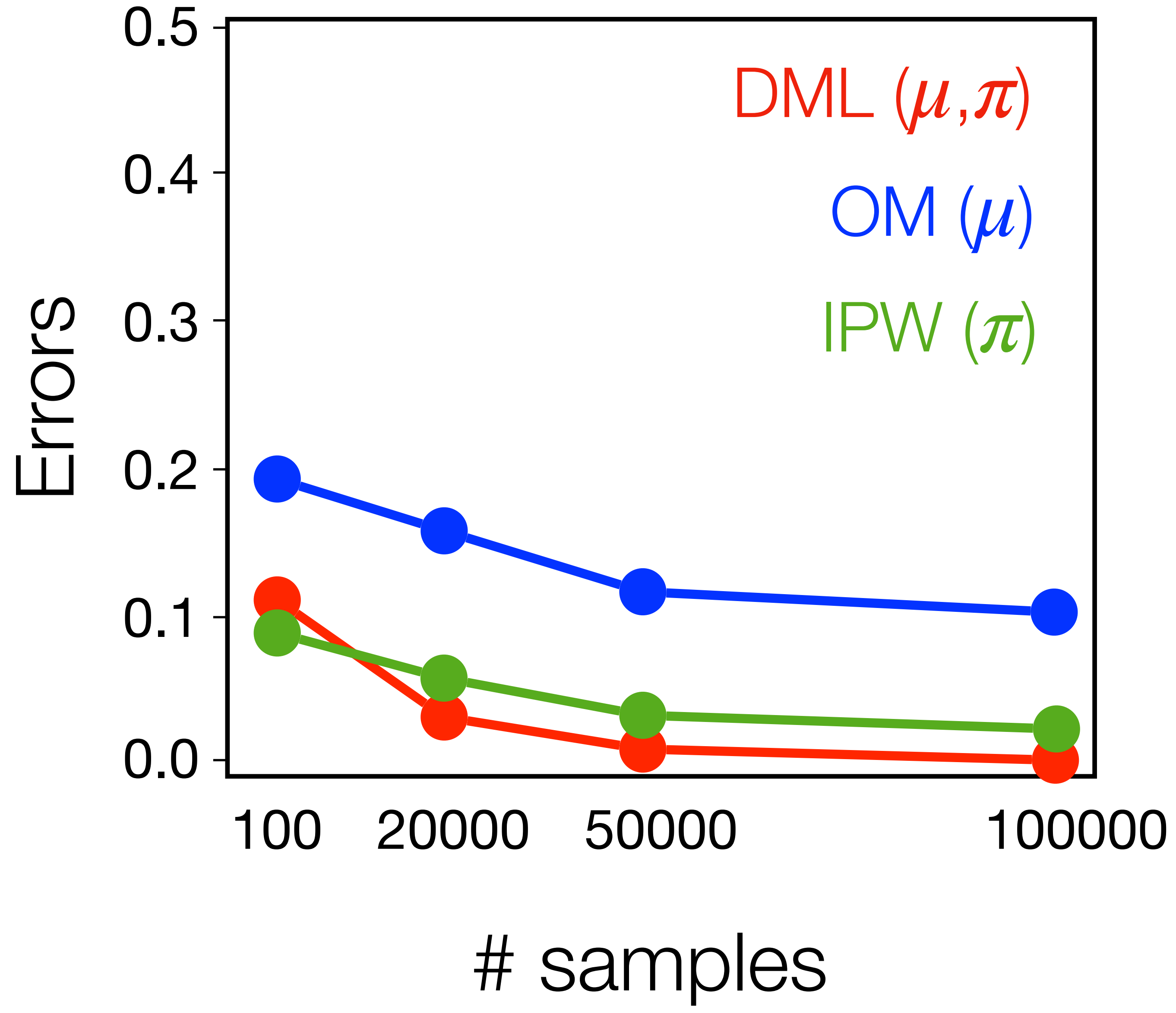


DML-gID converges fast, even when $(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\pi}})$ converge slowly

DML-gID - Simulation (Sec. 4.6)

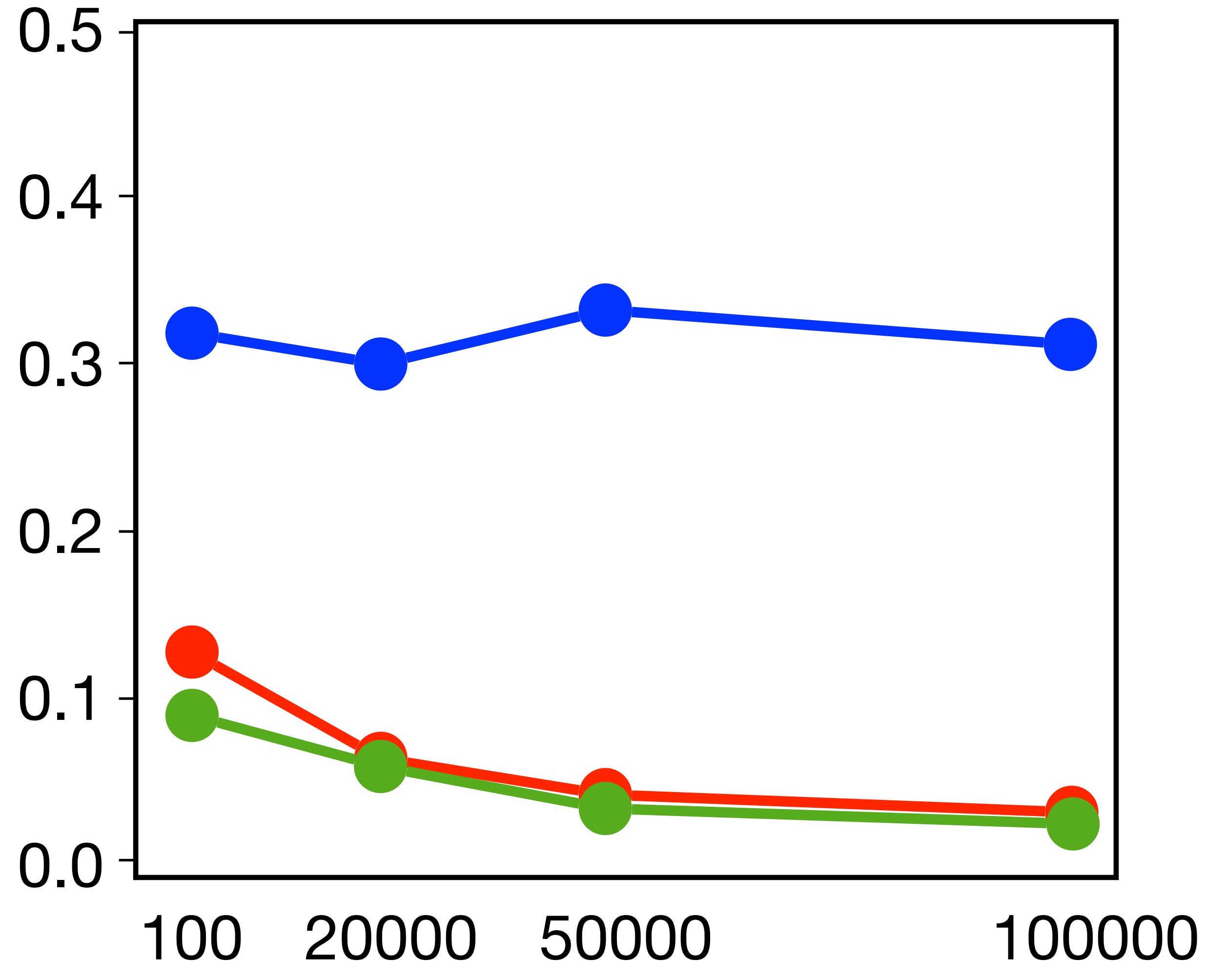
Fast Convergence

$(\hat{\mu},\hat{\pi}) \rightarrow (\mu_0,\pi_0)$ slowly

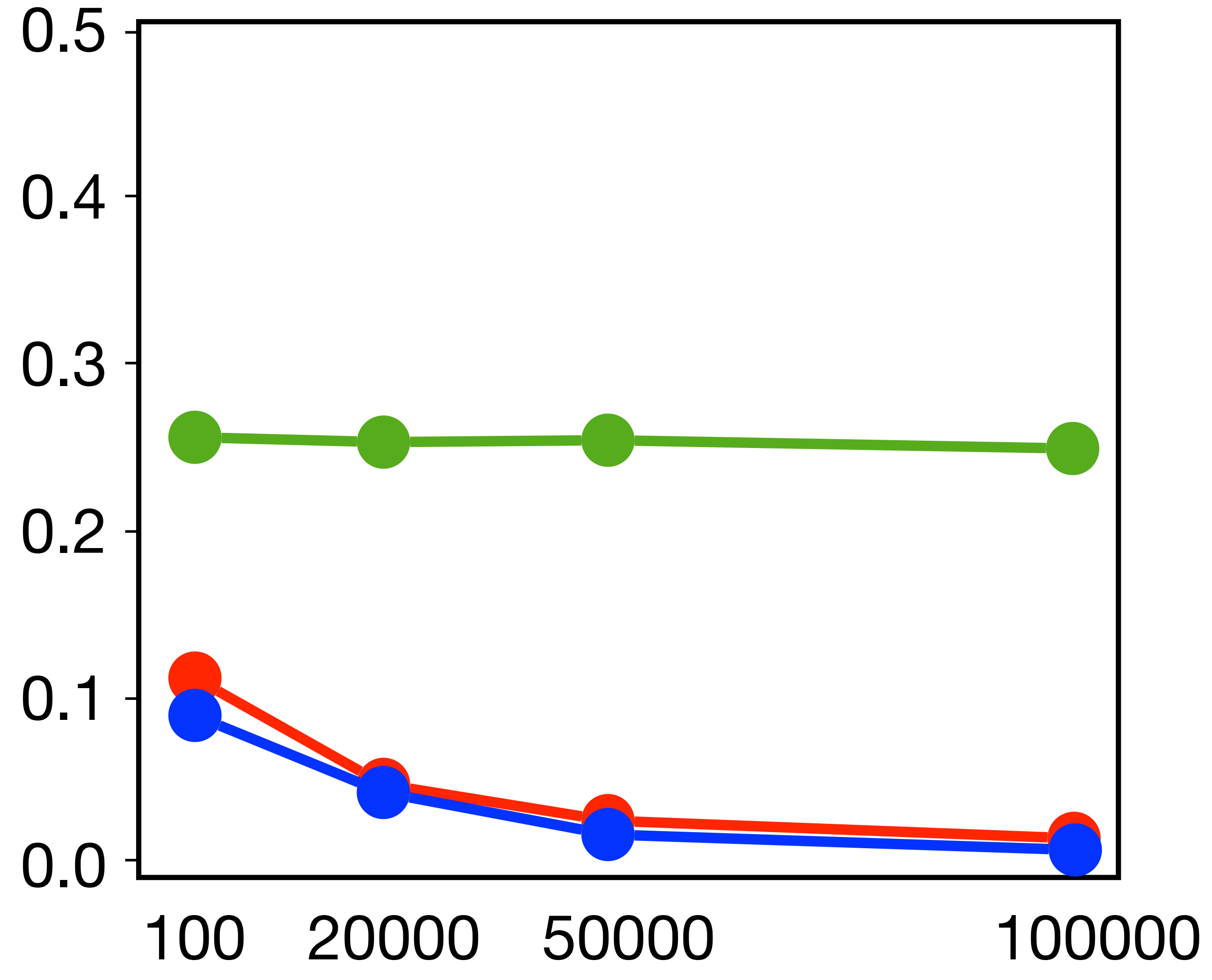


Double Robustness

$\hat{\mu}$ misspecified ($\hat{\mu}\neq\mu$)



$\hat{\pi}$ misspecified ($\hat{\pi}\neq\pi$)

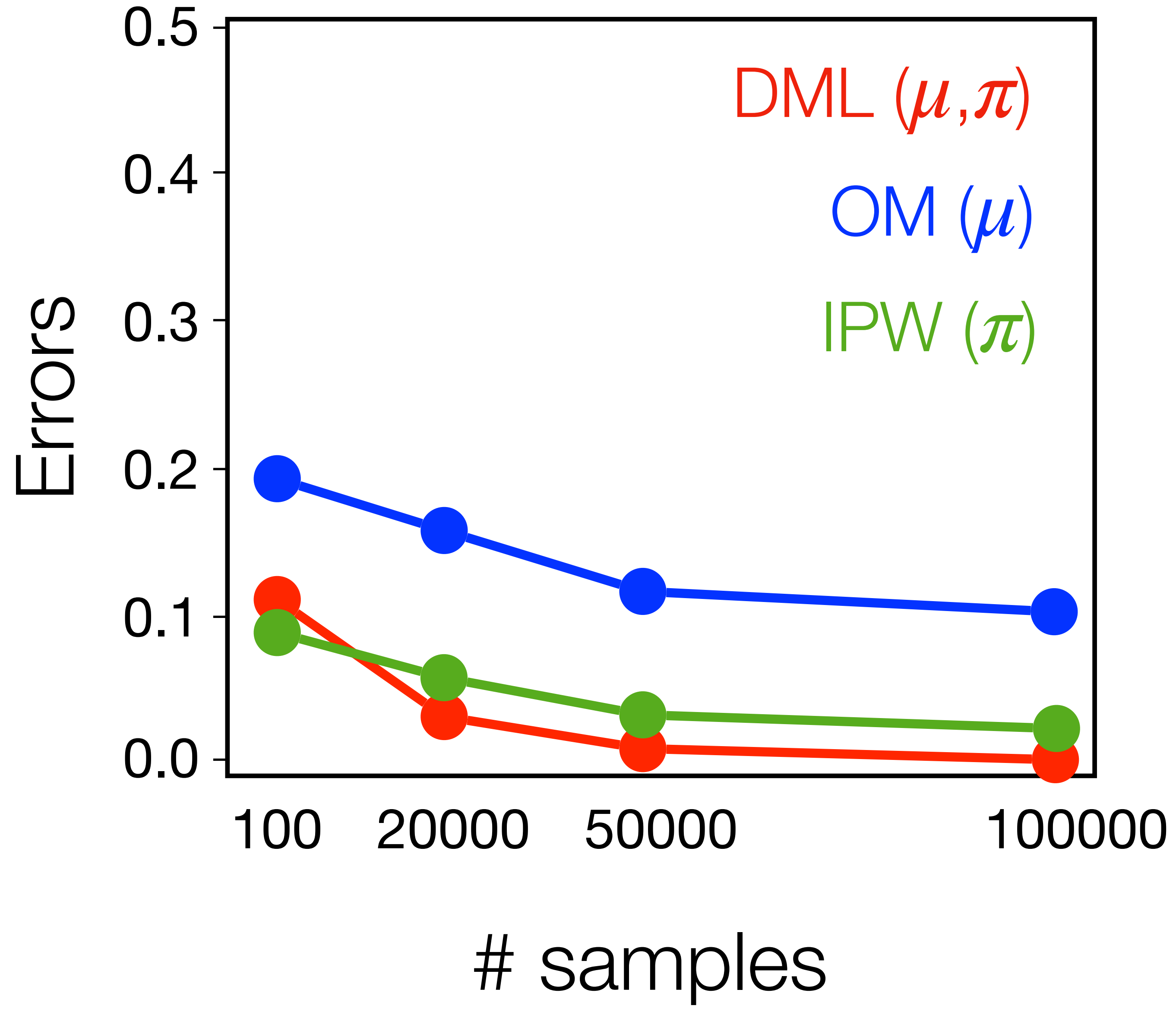


DML-gID converges fast, even when $(\hat{\mu},\hat{\pi})$ converge slowly

DML-gID - Simulation (Sec. 4.6)

Fast Convergence

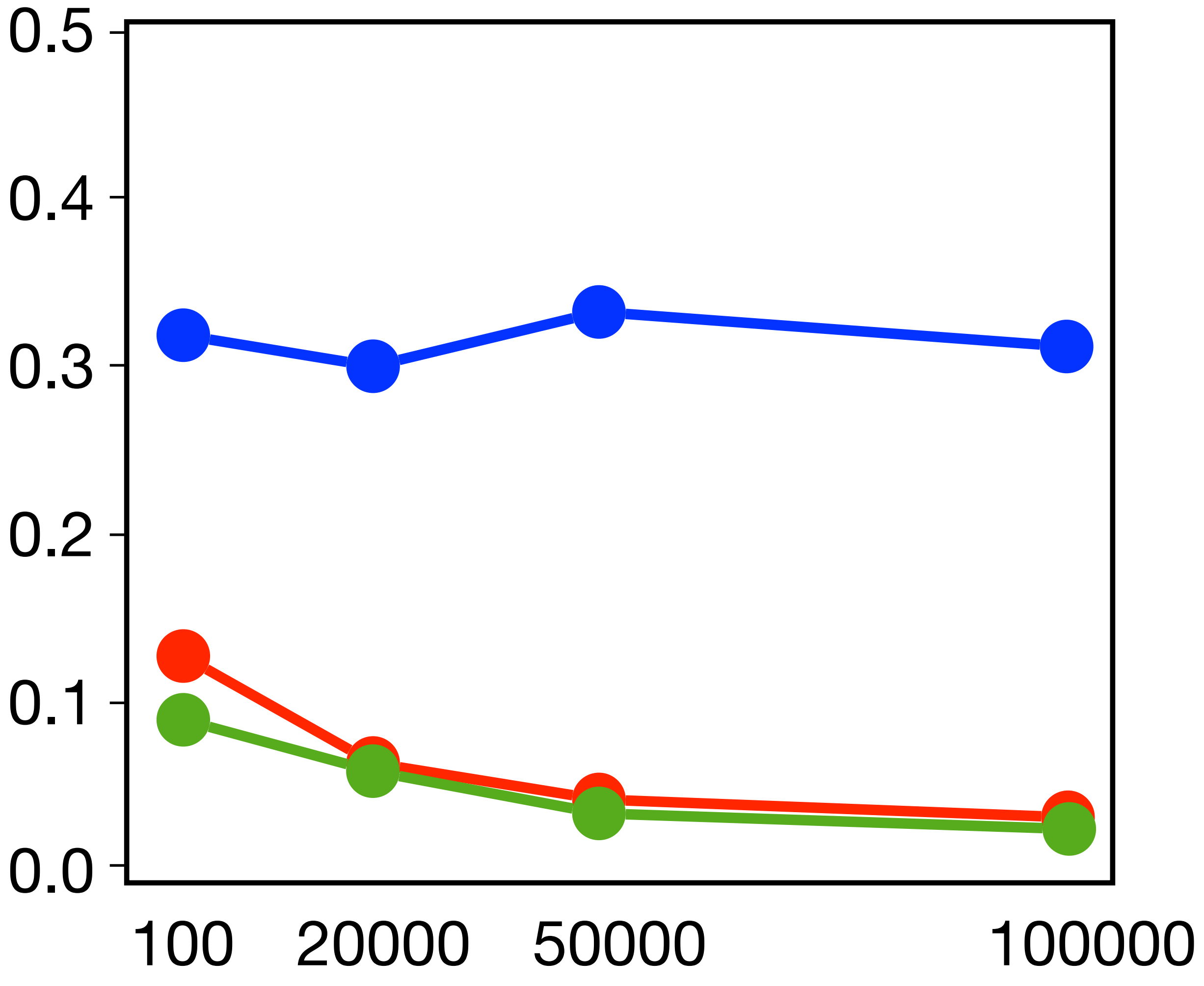
$(\hat{\mu},\hat{\pi}) \rightarrow (\mu_0, \pi_0)$ slowly



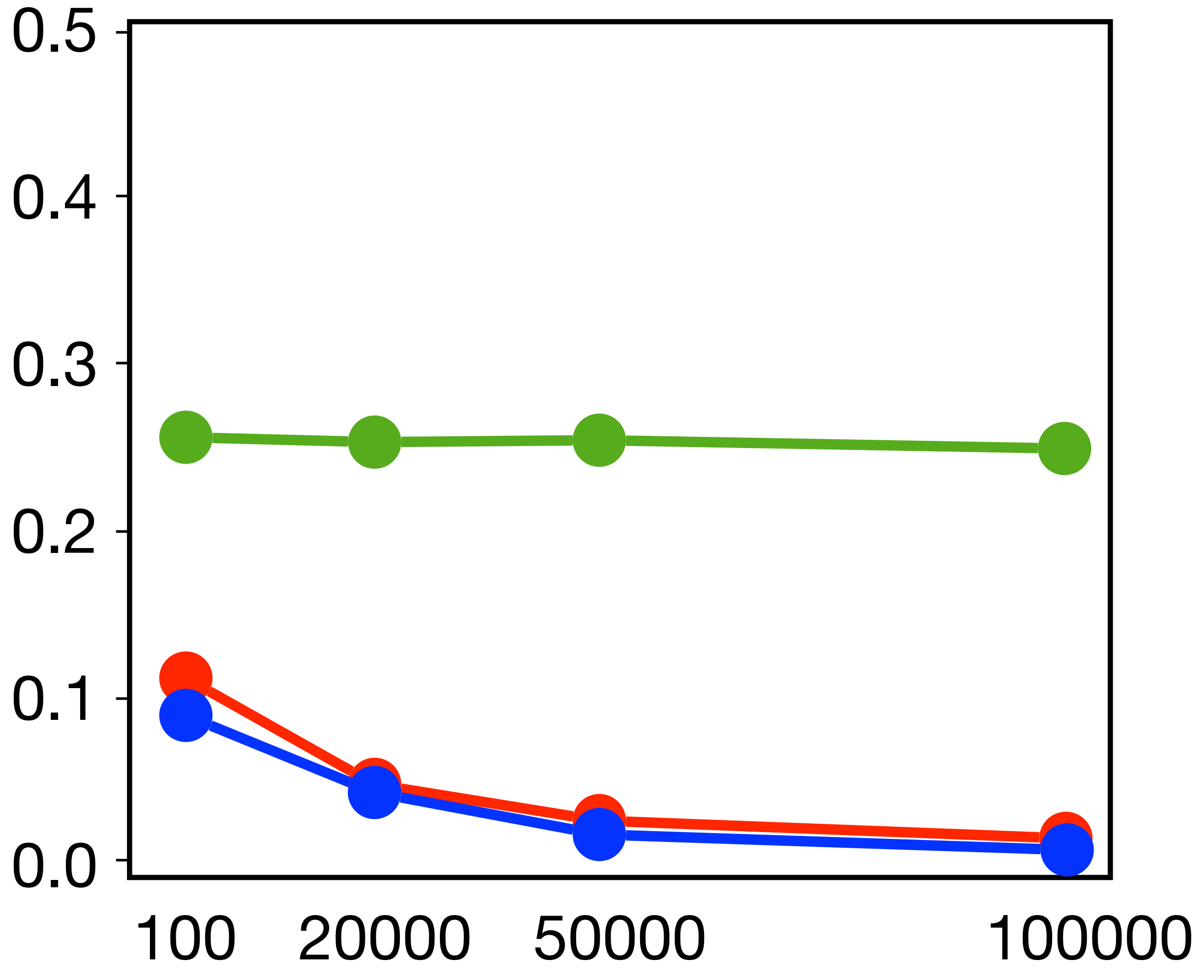
DML-gID converges fast, even when $(\hat{\mu},\hat{\pi})$ converge slowly

Double Robustness

$\hat{\mu}$ misspecified ($\hat{\mu}\neq\mu$)



$\hat{\pi}$ misspecified ($\hat{\pi}\neq\pi$)



DML-gID converges to the true causal effect even when $\hat{\mu}$ or $\hat{\pi}$ are misspecified.

Talk Outline

1. **Ch.3** Estimating causal effects from observations
2. **Ch.4** Estimating causal effects from data fusion
3. **Ch.5** Unified causal effect estimation method
4. Conclusion

Talk Outline

✔

➡ ❸ **Ch.5** Unified causal effect estimation method

Towards More General Causal Inference Queries

Estimating the
interventional effects
$\mathbb{E}[Y \mid \mathrm{do}(x)]$

Towards More General Causal Inference Queries

Fairness Analysis

$$\mathbb{E}[Y_{x, M_{\neg x}}]$$

Salary a man would earn if he had the opportunities that other genders would receive

Towards More General Causal Inference Queries

Offline Policy Evaluation

$$\mathbb{E}[Y_{\tau(X|C)}]$$

Recovery rate of a drug dosage policy given baseline characteristics

Towards More General Causal Inference Queries

Joint Treatment Effect

$$\mathbb{E}[Y \mid \mathrm{do}(x_1, x_2)]$$

Effect of drugs x_1 and x_2 from two trials $\mathrm{do}(x_1)$ and $\mathrm{do}(x_2)$, respectively

Towards More General Causal Inference Queries

Counterfactual

$$\mathbb{E}[Y_x | \neg x]$$

The headache intensity for patients who took aspirin, had they not taken it

Towards More General Causal Inference Queries

Missing Data

$$\mathbb{E}[Y \mid \text{do}(x), \text{mis}{=}0]$$

The effect of a treatment identifiable from missing data

Towards More General Causal Inference Queries

Domain Transfer

$$\mathbb{E}[Y \mid \mathrm{do}(x), S{=}\mathrm{NY}]$$

The effect of a treatment in NY identifiable from trials in Chicago

Towards More General Causal Inference Queries

Towards More General Causal Inference Queries

Fairness Analysis

$$\sum_m \mathbb{E}[Y \mid m, x) P(m \mid \neg x)$$

Domain Transfer

$$\sum_c \mathbb{E}_{\text{do}(x)}[Y \mid c, S{=}\text{Chi}] P(c \mid S{=}\text{NY})$$

Offline Policy Evaluation

$$\sum_c \mathbb{E}[Y \mid c, x) \pi(x \mid c) P(c)$$

Missing Data

$$\sum_c \mathbb{E}[Y \mid x, c, \text{mis}{=}1] P(c \mid \text{mis}{=}1)$$

Joint Treatment Effect

$$\sum_w \mathbb{E}_{\text{do}(x_2)}[Y \mid x_1, c] P_{\text{do}(x_2)}(w)$$

Counterfactual

$$\sum_c \mathbb{E}[Y \mid c, x) P(c \mid \neg x)$$

Towards More General Causal Inference Queries

Fairness Analysis

Estimator 1

Domain Transfer

Estimator 6

Offline Policy Evaluation

Estimator 2

Missing Data

Estimator 5

Joint Treatment Effect

Estimator 3

Counterfactual

Estimator 4

Towards More General Causal Inference Queries

Jung et al., NeurIPS 2024

Chapter 5

Unified Covariate Adjustment (UCA)

Unified causal estimation for summation of the product of arbitrary conditional distributions

Kernel Policy Product & Unified Covariate Adjustment

Kernel Policy Product & Unified Covariate Adjustment

Kernel Policy Product (Def. 50)

$$P_{m+1}(Y \mid \mathbf{S}^Z_m)\prod_{i=1}^{m}\sigma_i(\mathbf{X}_i \mid \mathbf{S}^X_i)P_i(\mathbf{Z}_i \mid \mathbf{S}^Z_{i-1})$$

Kernel Policy Product & Unified Covariate Adjustment

Kernel Policy Product (Def. 50)

Arbitrary prob. kernel
(distributions)

$$P_{m+1}(Y \mid \mathbf{S}^{Z}_{m}) \prod_{i=1}^{m} \sigma_i(\mathbf{X}_i \mid \mathbf{S}^{X}_{i}) P_i(\mathbf{Z}_i \mid \mathbf{S}^{Z}_{i-1})$$

Kernel Policy Product & Unified Covariate Adjustment

Kernel Policy Product (Def. 50)

Arbitrary prob. kernel (distributions)

Arbitrary vectors

$$P_{m+1}(Y \mid \mathbf{S}^{Z}_{m})\prod_{i=1}^{m} \sigma_i(\mathbf{X}_i \mid \mathbf{S}^{X}_{i})P_i(\mathbf{Z}_i \mid \mathbf{S}^{Z}_{i-1})$$

Kernel Policy Product & Unified Covariate Adjustment

Kernel Policy Product (Def. 50)

Arbitrary prob. kernel (distributions)

Arbitrary vectors

$$P_{m+1}(Y \mid \mathbf{S}^{Z}_{m}) \prod_{i=1}^{m} \sigma_i(\mathbf{X}_i \mid \mathbf{S}^{X}_{i}) P_i(\mathbf{Z}_i \mid \mathbf{S}^{Z}_{i-1})$$

outcome

Policies

Covariates

Kernel Policy Product & Unified Covariate Adjustment

Kernel Policy Product (Def. 50)

Arbitrary prob. kernel (distributions)

Arbitrary vectors

$$P_{m+1}(Y \mid \mathbf{S}^{Z}_{m}) \prod_{i=1}^{m} \sigma_i(\mathbf{X}_i \mid \mathbf{S}^{X}_{i}) P_i(\mathbf{Z}_i \mid \mathbf{S}^{Z}_{i-1})$$

outcome

Policies

Covariates

Unified Covariate Adjustment (Def. 51)

Expectation of Y over the KPP

Canonical Example of UCA

arbitrary distributions

outcome

arbitrary policy of treatments

arbitrary distributions

set of variables

$$\sum_{x,c} \mathbb{E}_{P_2}[Y \mid x, z]\sigma(x \mid z)P_1(z)$$

Canonical Example of UCA

arbitrary distributions — outcome — arbitrary policy of treatments — arbitrary distributions — set of variables

$$\sum_{x,c} \mathbb{E}_{P_2}[Y \mid x, z]\sigma(x \mid z)P_1(z)$$

UCA

$$\sum_{x,c} \mathbb{E}_{P_2}[Y \mid x, z]\sigma(x \mid z)P_1(z)$$

→

BD+

$$\sum_{w} \mathbb{E}_{P_{\mathrm{do}(x_2)}}[Y \mid x, w]P_{\mathrm{do}(x_1)}(w)$$

$P_1 \leftarrow P_{\mathrm{do}(x_1)}$

$P_2 \leftarrow P_{\mathrm{do}(x_2)}$

$\sigma \leftarrow \mathbb{1}_x(X)$

$Z \leftarrow \{W\}$

Canonical Example of UCA

arbitrary distributions
outcome
arbitrary policy of treatments
arbitrary distributions
set of variables

$$\sum_{x,c}\mathbb{E}_{P_2}[Y \mid x, z]\sigma(x \mid z)P_1(z)$$

UCA

BD+

Theorem 28

UCA can represent ***any*** causal effects expressible as a sum of products of arbitrary conditional distributions (*Kernel-Policy Product*), by choosing $Z, P_1, P_2, \sigma(\cdot \mid \cdot)$ properly.

Parameterization for UCA (Def. 54)

$$\psi_0 \triangleq \sum_{x,c} \mathbb{E}_{P_2}[Y \mid x, z]\sigma(x \mid z)P_1(z)$$

Parameterization for UCA (Def. 54)

$$\psi_0 \triangleq \sum_{x,c} \mathbb{E}_{P_2}[Y \mid x, z]\sigma(x \mid z)P_1(z)$$

$$\mu(\mathbf{X},\mathbf{Z}) \triangleq \mathbb{E}_{P_2}[Y \mid \mathbf{X}, \mathbf{Z}]$$

Parameterization for UCA (Def. 54)

$$\psi_0 \triangleq \sum_{x,c} \mathbb{E}_{P_2}[Y \mid x, z]\sigma(x \mid z)P_1(z)$$

$$\mu(\mathbf{X},\mathbf{Z}) \triangleq \mathbb{E}_{P_2}[Y \mid \mathbf{X}, \mathbf{Z}]$$

$$\mathbb{E}_{P_1}[\mathbb{E}_{\sigma(\mathbf{X}|\mathbf{Z})}[\mu(\mathbf{X},\mathbf{Z})]]$$

$$= \sum_{\mathbf{z},\mathbf{x}} \mu(\mathbf{x},\mathbf{z})\sigma_X(\mathbf{x}|\mathbf{z})P_1(\mathbf{z})$$

$$= \psi_0$$

Parameterization for UCA (Def. 54)

$$\psi_0 \triangleq \sum_{x,c} \mathbb{E}_{P_2}[Y \mid x, z]\sigma(x \mid z)P_1(z)$$

$$\mu(\mathbf{X},\mathbf{Z}) \triangleq \mathbb{E}_{P_2}[Y \mid \mathbf{X}, \mathbf{Z}]$$

$$\mathbb{E}_{P_1}[\mathbb{E}_{\sigma(\mathbf{X}|\mathbf{Z})}[\mu(\mathbf{X},\mathbf{Z})]]$$

$$= \sum_{\mathbf{z},\mathbf{x}} \mu(\mathbf{x},\mathbf{z})\sigma_X(\mathbf{x}|\mathbf{z})P_1(\mathbf{z})$$

$$= \psi_0$$

$\pi(\mathbf{X},\mathbf{Z})$: Solution of

$$\mathbb{E}_{P_2}[\pi(\mathbf{X},\mathbf{Z}) \times \mu(\mathbf{X},\mathbf{Z})] = \mathbb{E}_{P_1}[\mathbb{E}_{\sigma_X}[\mu(\mathbf{X},\mathbf{Z})]]$$

Parameterization for UCA (Def. 54)

$$\psi_0 \triangleq \sum_{x,c} \mathbb{E}_{P_2}[Y \mid x, z]\sigma(x \mid z)P_1(z)$$

$$\mu(\mathbf{X},\mathbf{Z}) \triangleq \mathbb{E}_{P_2}[Y \mid \mathbf{X}, \mathbf{Z}]$$

$$\mathbb{E}_{P_1}[\mathbb{E}_{\sigma(\mathbf{X}|\mathbf{Z})}[\mu(\mathbf{X},\mathbf{Z})]]$$

$$= \sum_{\mathbf{z},\mathbf{x}} \mu(\mathbf{x},\mathbf{z})\sigma_X(\mathbf{x}|\mathbf{z})P_1(\mathbf{z})$$

$$= \psi_0$$

$\pi(\mathbf{X},\mathbf{Z})$: Solution of

$$\mathbb{E}_{P_2}[\pi(\mathbf{X},\mathbf{Z}) \times \mu(\mathbf{X},\mathbf{Z})] = \mathbb{E}_{P_1}[\mathbb{E}_{\sigma_X}[\mu(\mathbf{X},\mathbf{Z})]]$$

$$\mathbb{E}_{P_2}[\pi(\mathbf{X},\mathbf{Z}) \times Y]$$

$$= \mathbb{E}_{P_2}[\pi(\mathbf{X},\mathbf{Z}) \times \mu(\mathbf{X},\mathbf{Z})]$$

$$= \mathbb{E}_{P_1}[\mathbb{E}_{\sigma_X}[\mu(\mathbf{X},\mathbf{Z})]]$$

$$= \psi_0$$

Doubly Robust Estimator for UCA

$$\text{UCA}(\mu, \pi) \triangleq \mathbb{E}_{P_2}[\mu \times \pi]$$

Doubly Robust Estimator for UCA

$$\text{UCA}(\mu, \pi) \triangleq \mathbb{E}_{P_2}[\mu \times \pi]$$

"Double Robustness"

$$?(\hat{\mu}, \hat{\pi}) - \mathbb{E}_{P_2}[\mu \times \pi] = \mathbb{E}_{P_2}[\{\hat{\mu} - \mu\} \times \{\pi - \hat{\pi}\}]$$

Doubly Robust Estimator for UCA

$$\text{UCA}(\mu, \pi) \triangleq \mathbb{E}_{P_2}[\mu \times \pi]$$

$$?(\hat{\mu}, \hat{\pi}) = \mathbb{E}_{P_2}[\{\hat{\mu} - \mu\} \times \{\pi - \hat{\pi}\}] + \mathbb{E}_{P_2}[\mu \times \pi]$$

Doubly Robust Estimator for UCA

$$\text{UCA}(\mu, \pi) \triangleq \mathbb{E}_{P_2}[\mu \times \pi]$$

$$?(\hat{\mu}, \hat{\pi}) = \mathbb{E}_{P_2}[\{\hat{\mu} - \mu\} \times \{\pi - \hat{\pi}\}] + \mathbb{E}_{P_2}[\mu \times \pi]$$

$$= \mathbb{E}_{P_2}[\hat{\pi}\{\mu - \hat{\mu}\} + \pi\hat{\mu}]$$

Doubly Robust Estimator for UCA

$$\text{UCA}(\mu, \pi) \triangleq \mathbb{E}_{P_2}[\mu \times \pi]$$

$$\begin{aligned} \textbf{?}(\hat{\mu}, \hat{\pi}) &= \mathbb{E}_{P_2}[\{\hat{\mu} - \mu\} \times \{\pi - \hat{\pi}\}] + \mathbb{E}_{P_2}[\mu \times \pi] \\ &= \mathbb{E}_{P_2}[\hat{\pi}\{\mu - \hat{\mu}\} + \pi\hat{\mu}] \\ &= \mathbb{E}_{P_2}[\hat{\pi}\{Y - \hat{\mu}\}] + \mathbb{E}_{P_1}[\mathbb{E}_{\sigma_X}[\hat{\mu}]] \end{aligned}$$

Doubly Robust Estimator for UCA

$$\text{UCA}(\mu, \pi) \triangleq \mathbb{E}_{P_2}[\mu \times \pi]$$

$$\textbf{?}(\hat{\mu}, \hat{\pi}) = \mathbb{E}_{P_2}[\{\hat{\mu} - \mu\} \times \{\pi - \hat{\pi}\}] + \mathbb{E}_{P_2}[\mu \times \pi]$$

$$= \mathbb{E}_{P_2}[\hat{\pi}\{\mu - \hat{\mu}\} + \pi\hat{\mu}]$$

$$= \mathbb{E}_{P_2}[\hat{\pi}\{Y - \hat{\mu}\}] + \mathbb{E}_{P_1}[\mathbb{E}_{\sigma_X}[\hat{\mu}]]$$

DML-UCA (Double Machine Learning estimator for UCA)

$$\widehat{\text{UCA}}(\hat{\mu}, \hat{\pi}) \triangleq \mathbb{E}_{P_2}[\hat{\pi}\{Y - \hat{\mu}\}] + \mathbb{E}_{P_1}[\mathbb{E}_{\sigma_X}[\hat{\mu}]]$$

Robustness Property of DML-UCA

Theorem 33

$$\text{Error}(\text{DML-UCA}, \psi_0) = \sum_{i=1}^{m} \text{Error}(\hat{\mu}_i, \mu_i) \times \text{Error}(\hat{\pi}_i, \pi_i)$$

- **Double Robustness:** $\text{Error} = 0$ if either $\hat{\mu}_i = \mu_i$ or $\hat{\pi}_i = \pi_i$ for all $i = 1, \cdots, m$.
- **Fast Convergence:** $\text{Error} \to 0$ *fast* even when $\hat{\mu}_i \to \mu_i$ and $\hat{\pi}_i \to \pi_i$ *slow*.

Simulation: DML-UCA

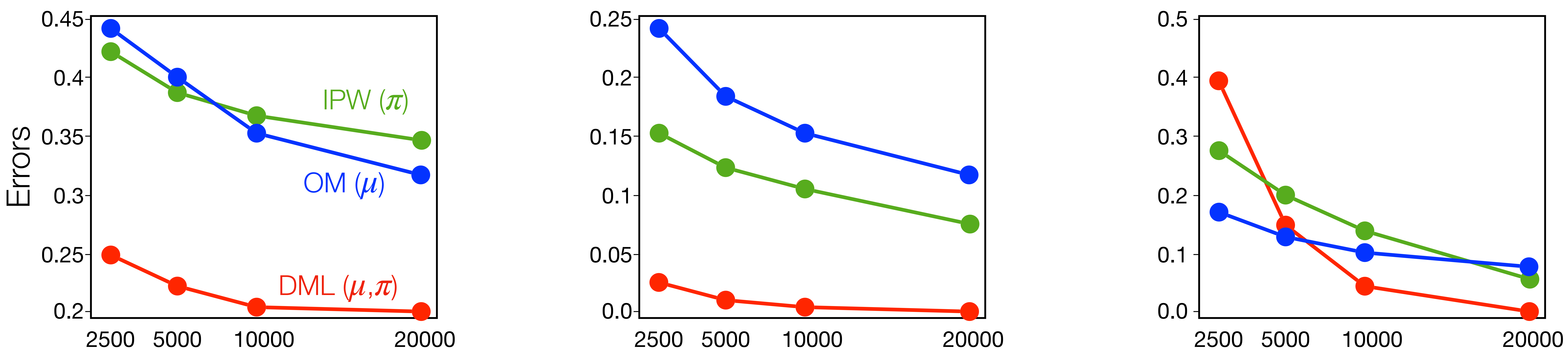
Simulation: DML-UCA

$(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\pi}}) \to (\boldsymbol{\mu}_0, \boldsymbol{\pi}_0)$ slowly

Simulation: DML-UCA

$(\hat{\mu}, \hat{\pi}) \rightarrow (\mu_0, \pi_0)$ slowly

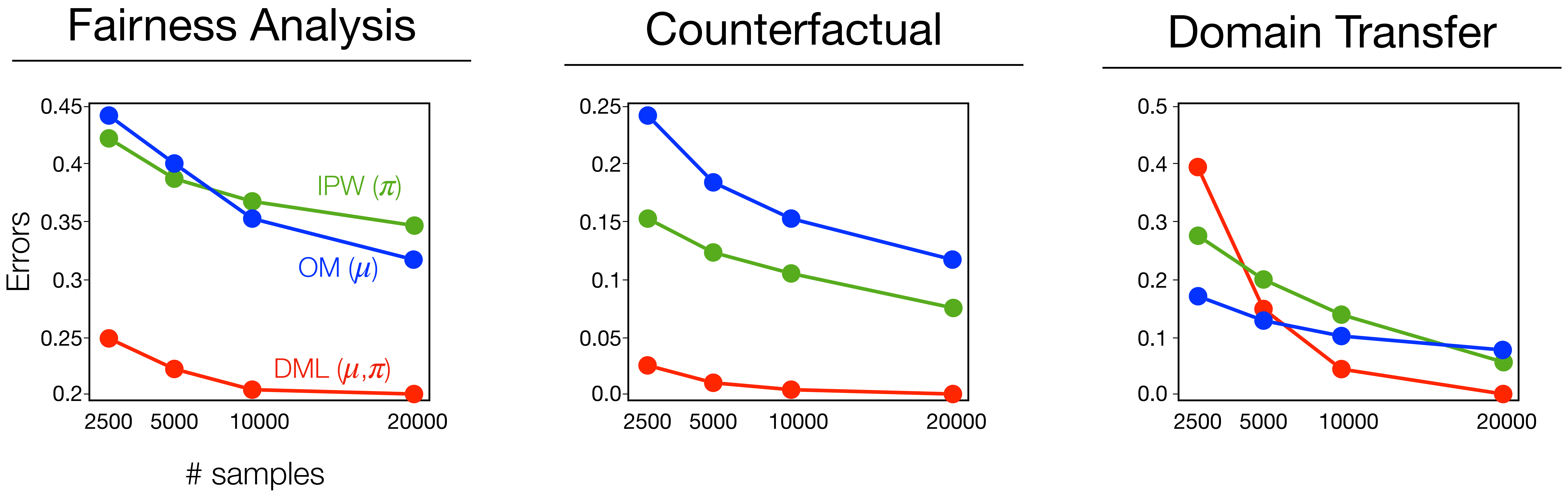
Fairness Analysis | Counterfactual | Domain Transfer



samples

Simulation: DML-UCA

$(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\pi}}) \to (\boldsymbol{\mu}_0, \boldsymbol{\pi}_0)$ slowly



DML-UCA converges fast even when $(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\pi}}) \to (\boldsymbol{\mu}_0, \boldsymbol{\pi}_0)$ slowly

Talk Outline

1. **Ch.3** Estimating causal effects from observations
2. **Ch.4** Estimating causal effects from data fusion
3. **Ch.5** Unified causal effect estimation method
4. Conclusion

Talk Outline

✔

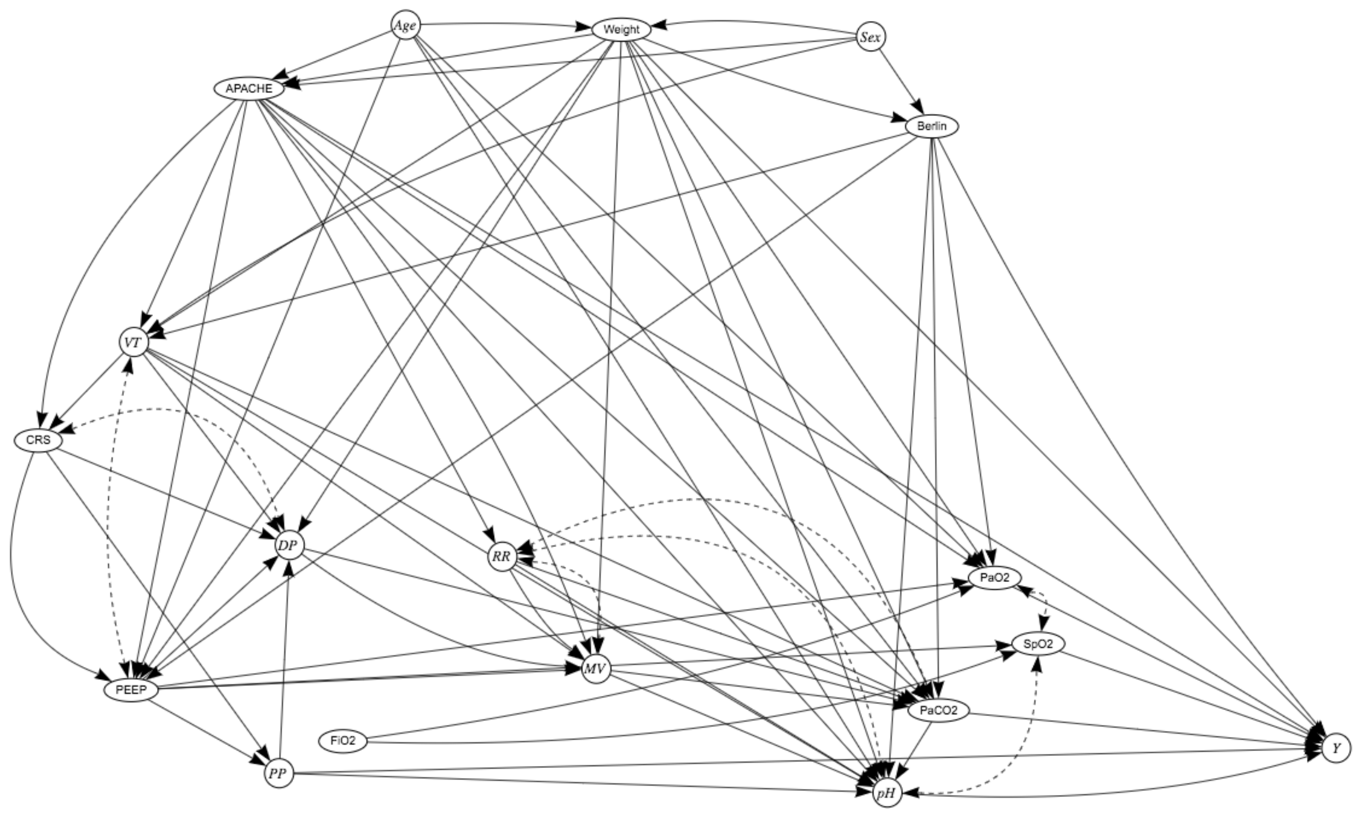
4 Conclusion

This Talk: Estimating Causal Effects

This Talk: Estimating Causal Effects

Tasks

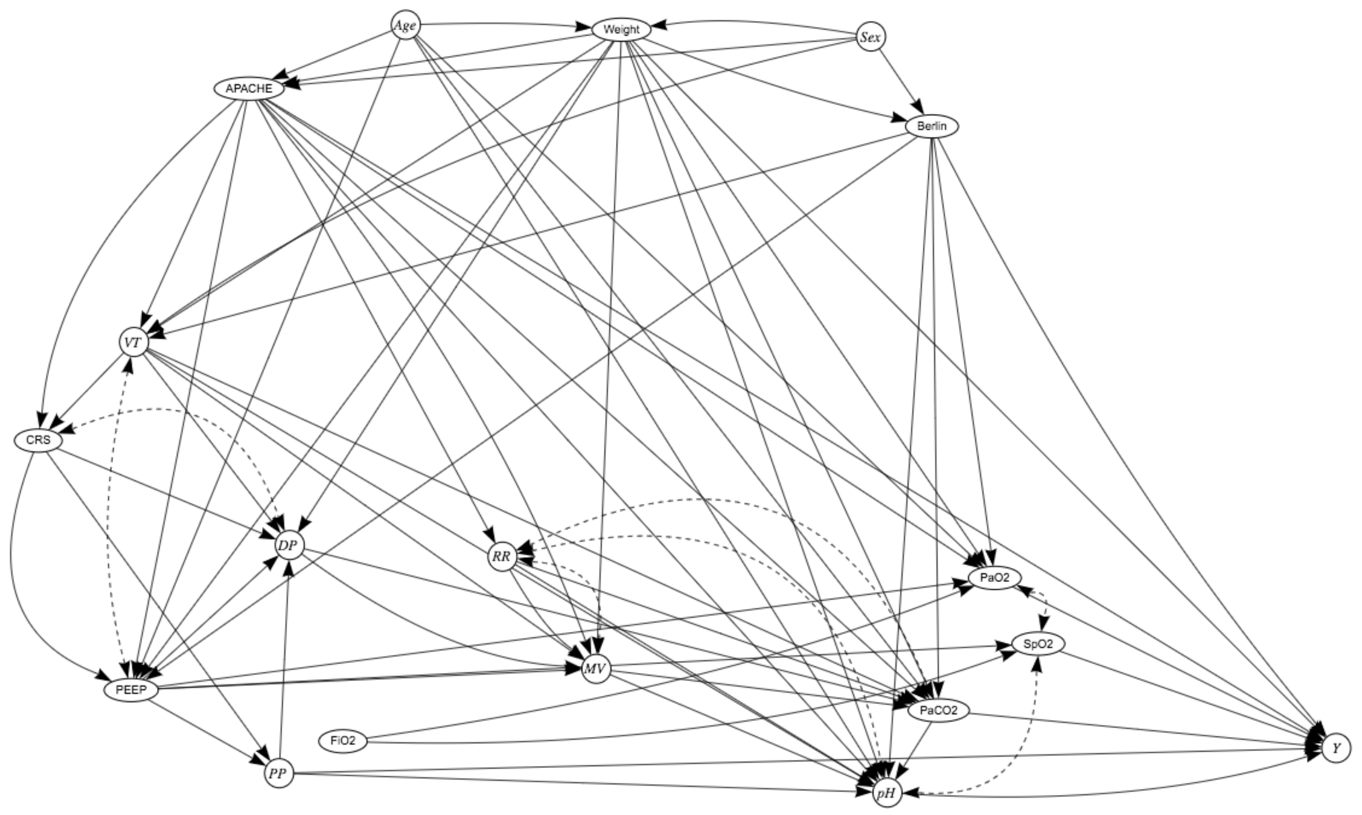
1. From Observation



This Talk: Estimating Causal Effects

Tasks

1. From Observation



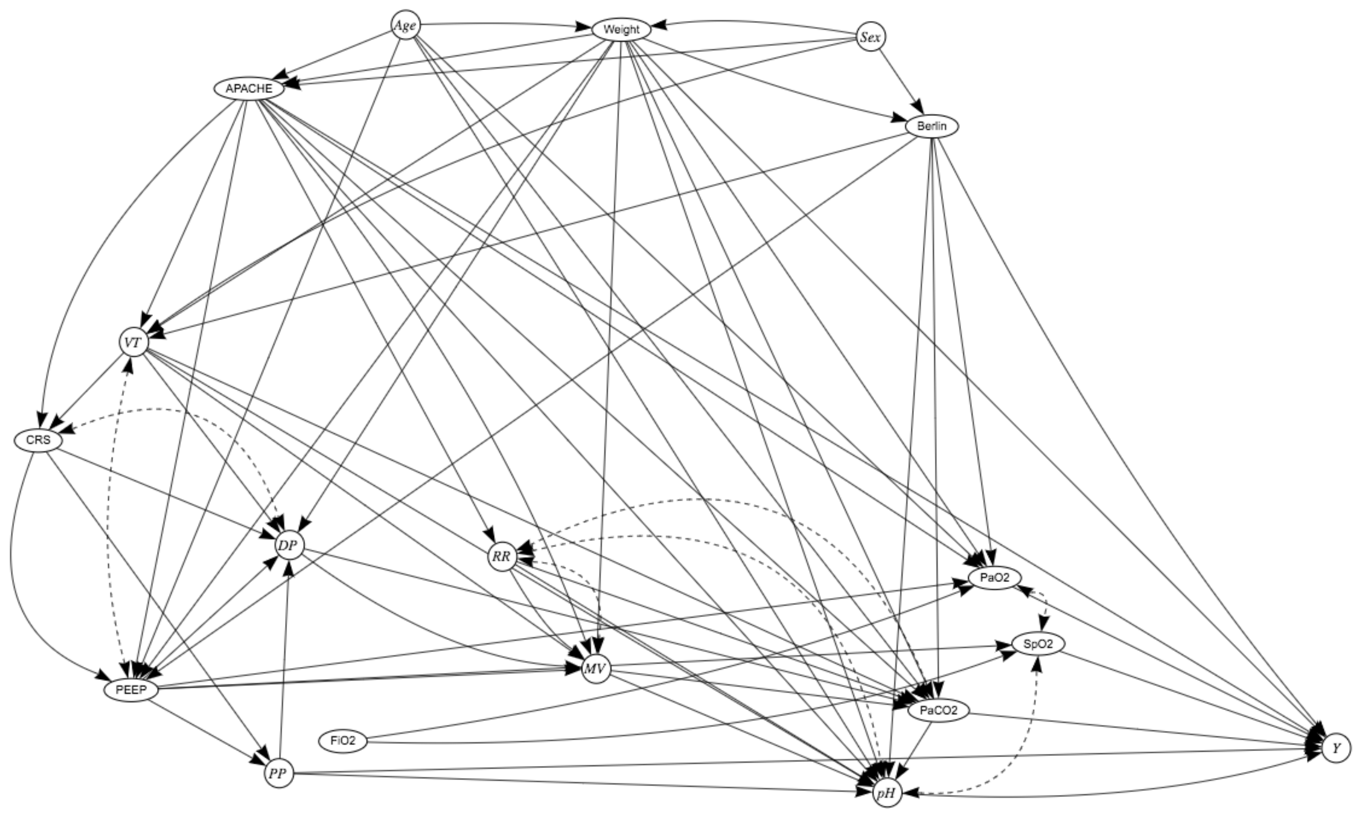
Solution

DML-ID

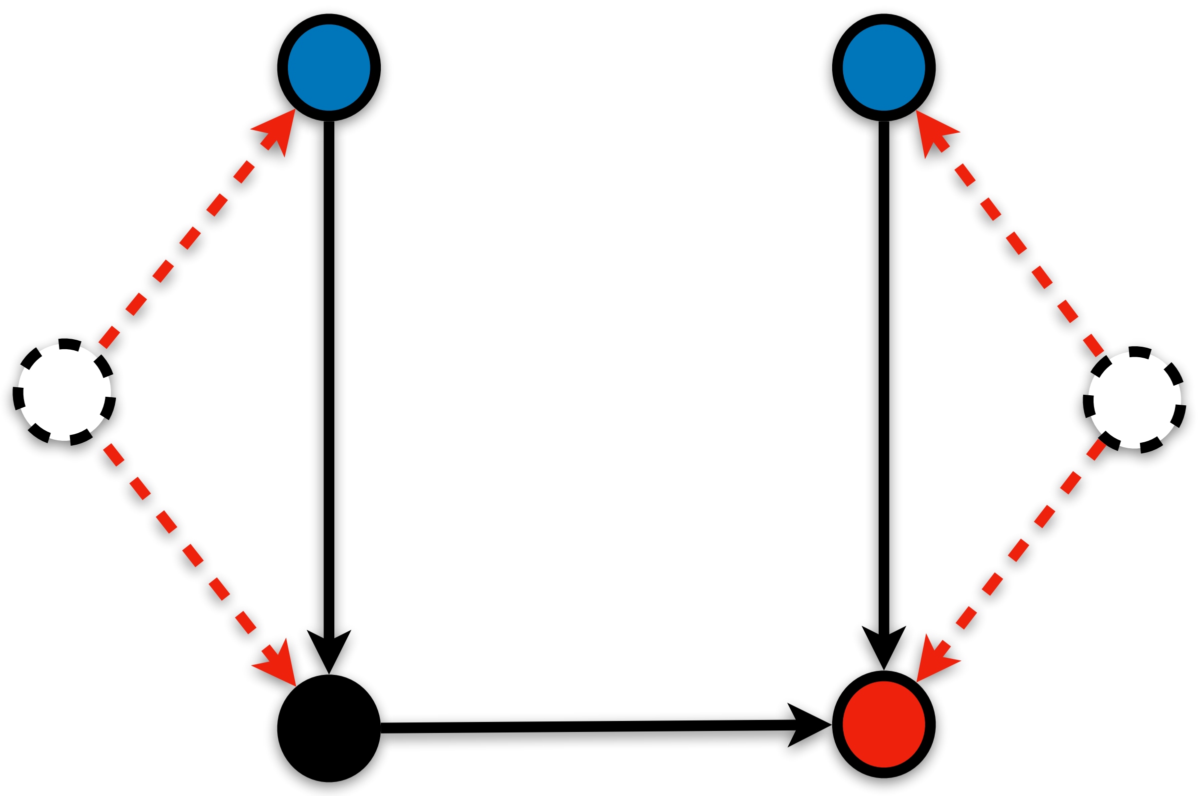
This Talk: Estimating Causal Effects

Tasks

1. From Observation



2. From Data Fusion



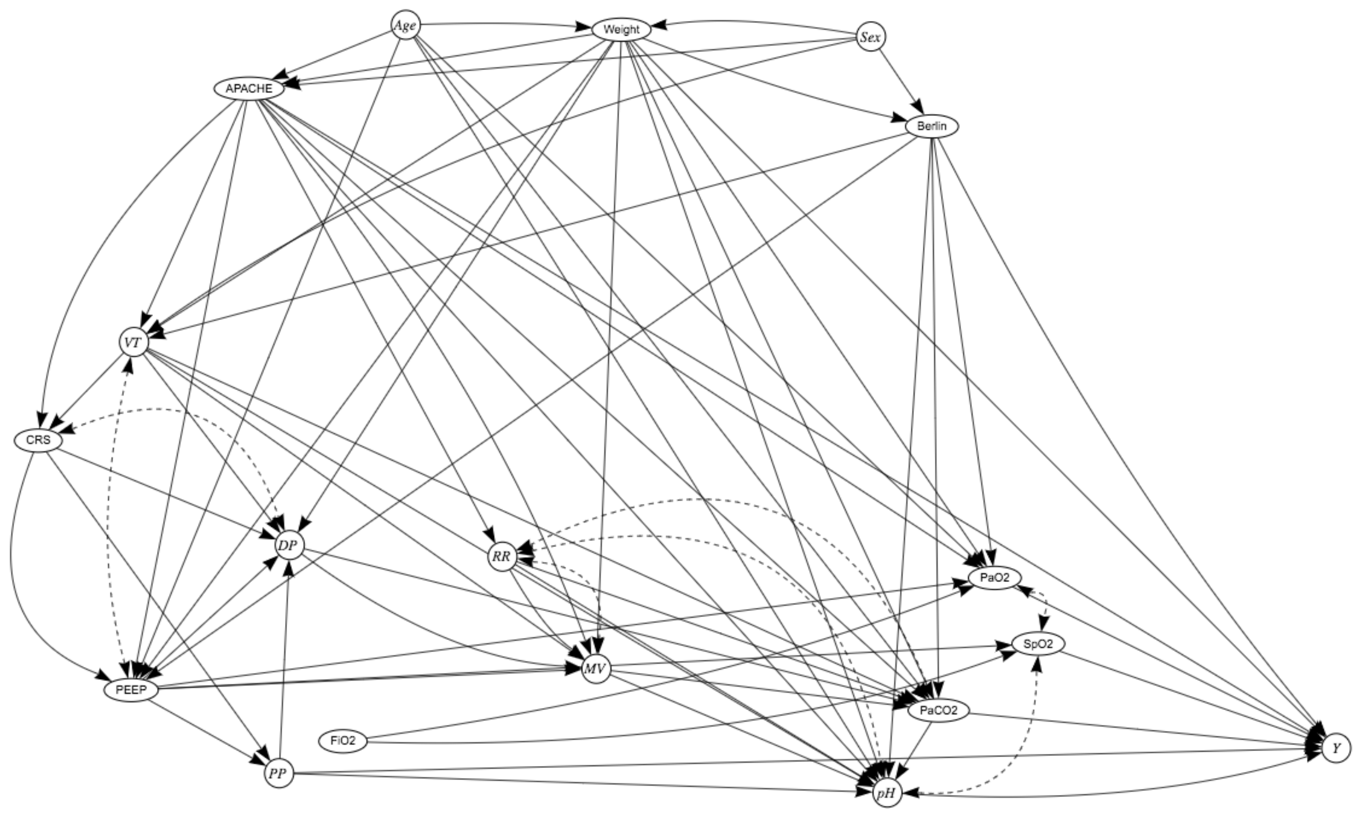
Solution

DML-ID

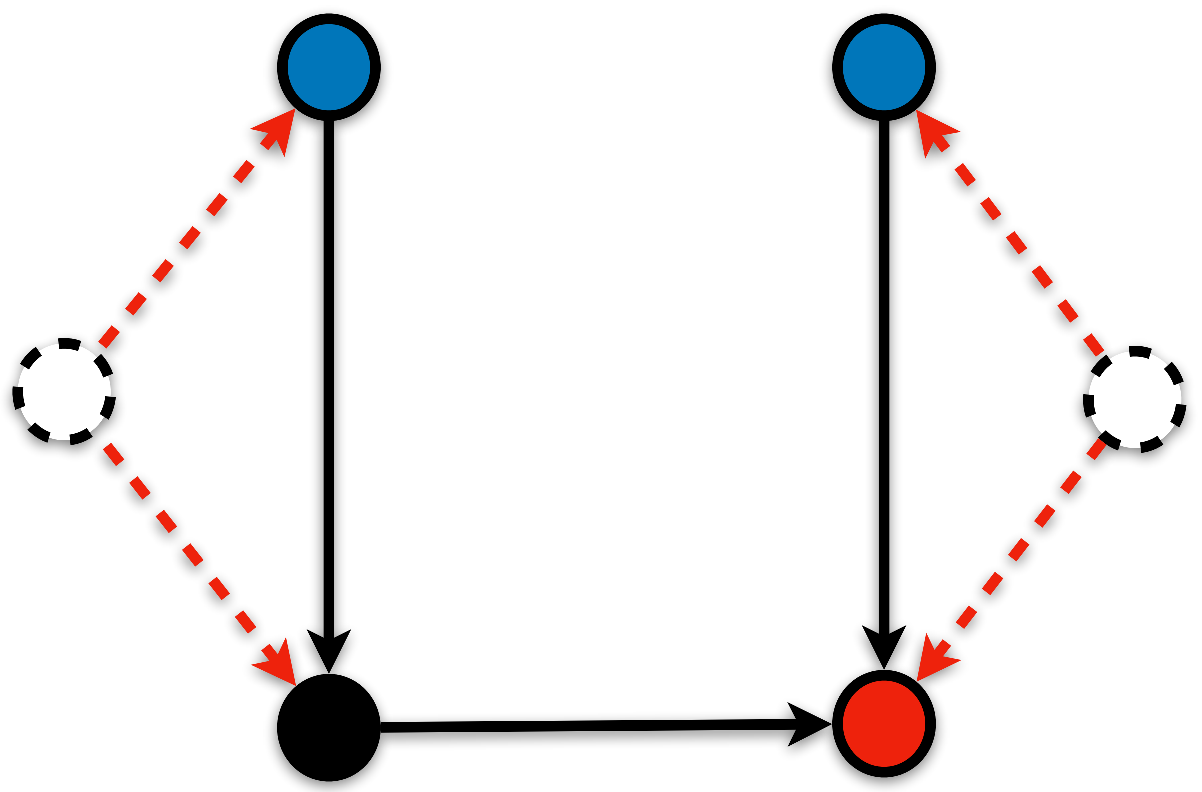
This Talk: Estimating Causal Effects

Tasks

1. From Observation



2. From Data Fusion



Solution

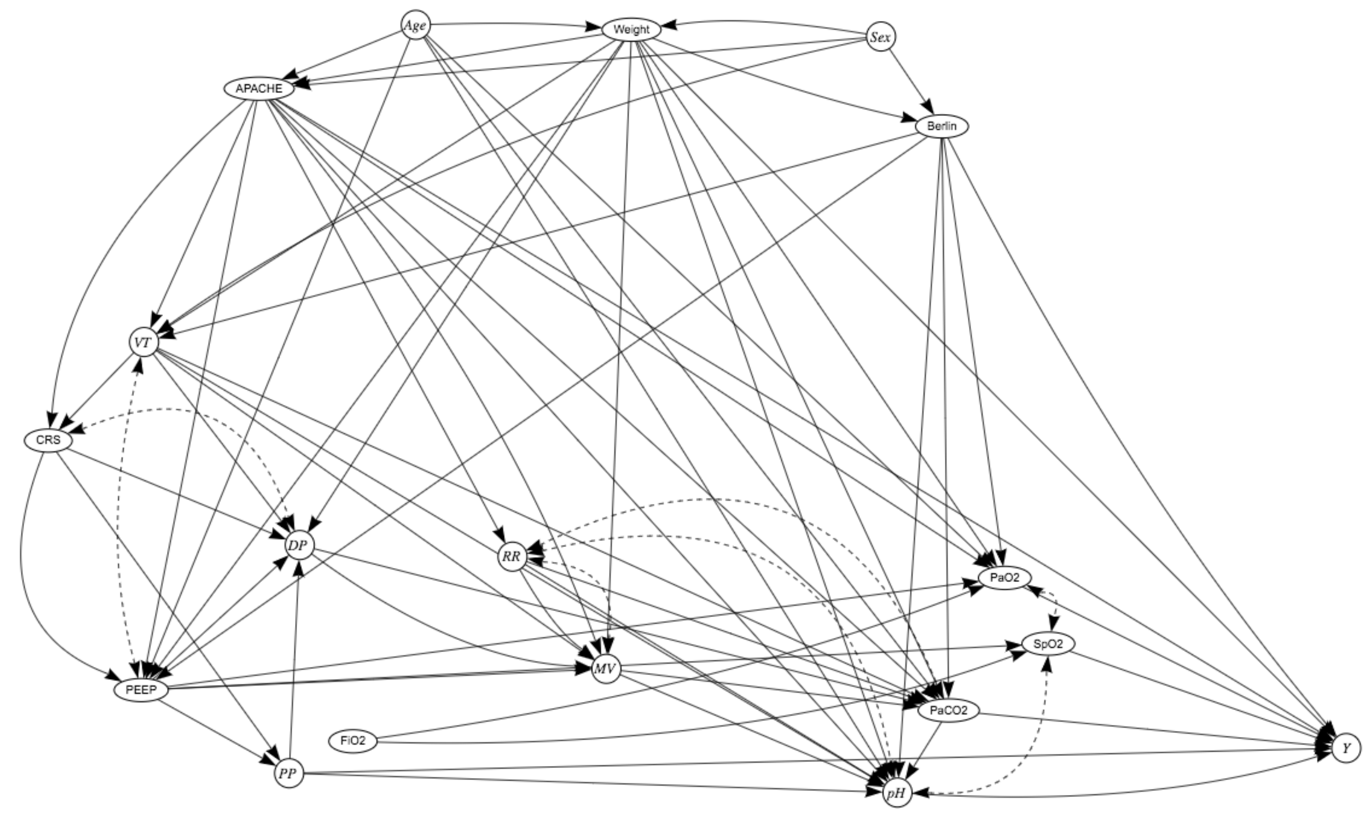
DML-ID

- DML-BD⁺
- DML-gID

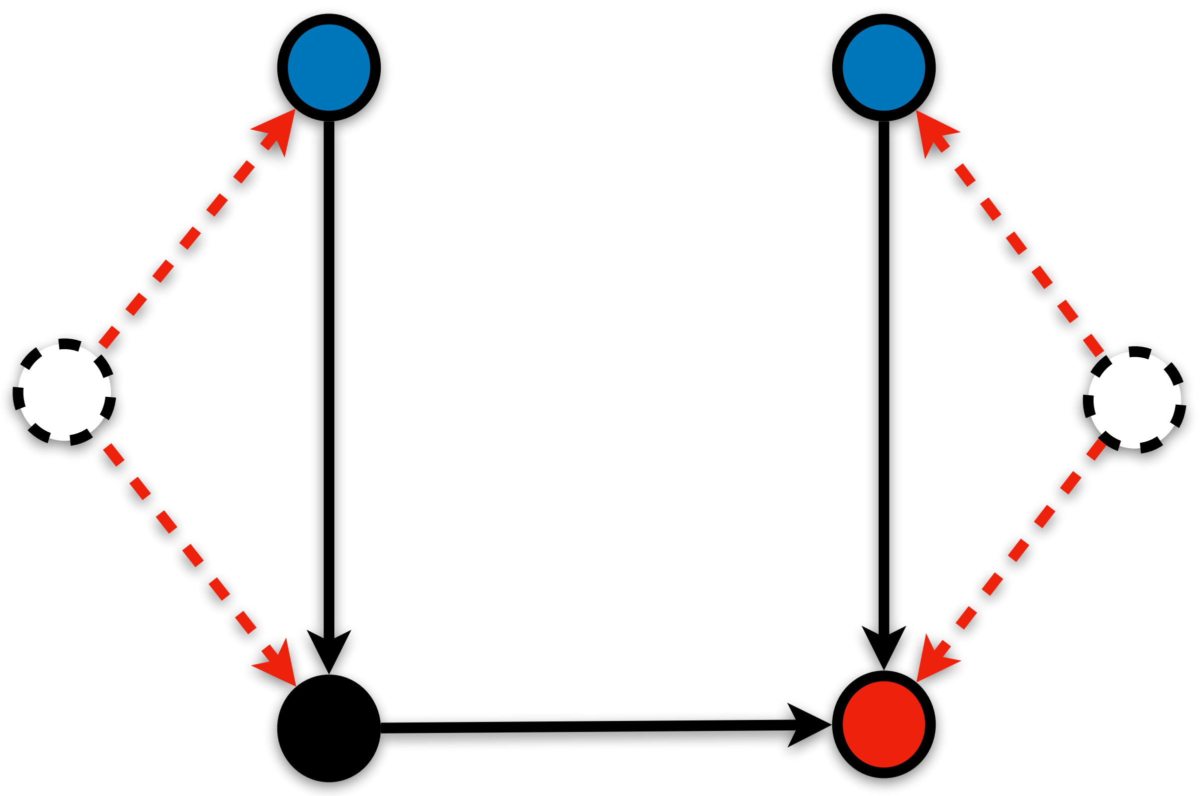
This Talk: Estimating Causal Effects

Tasks

1. From Observation



2. From Data Fusion



3. Unified Estimation

Fairness $\mathbb{E}[Y_{x,M_{\neg x}}]$

Off-policy evaluation $\mathbb{E}[Y_{\tau(X|C)}]$

Counterfactuals $\mathbb{E}[Y_x | \neg x]$

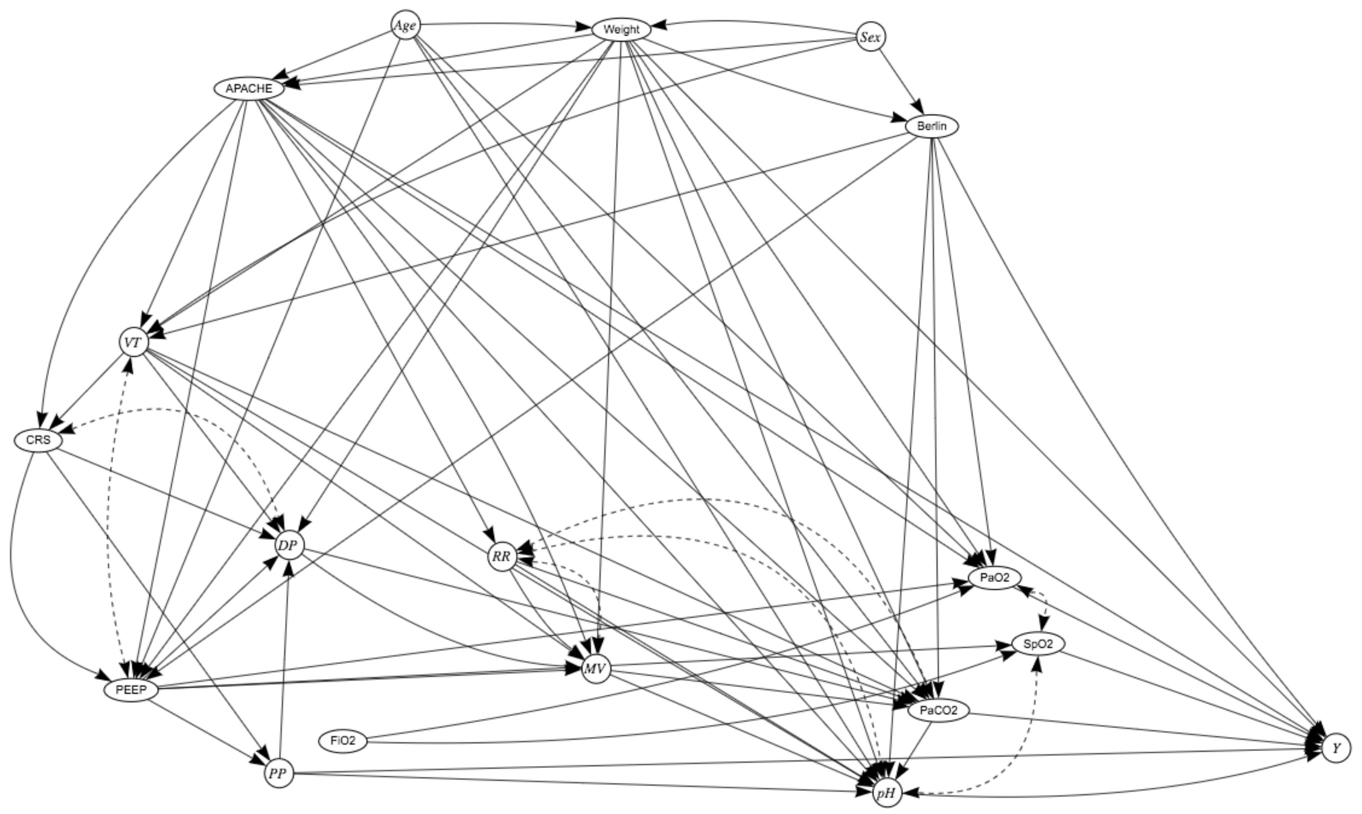
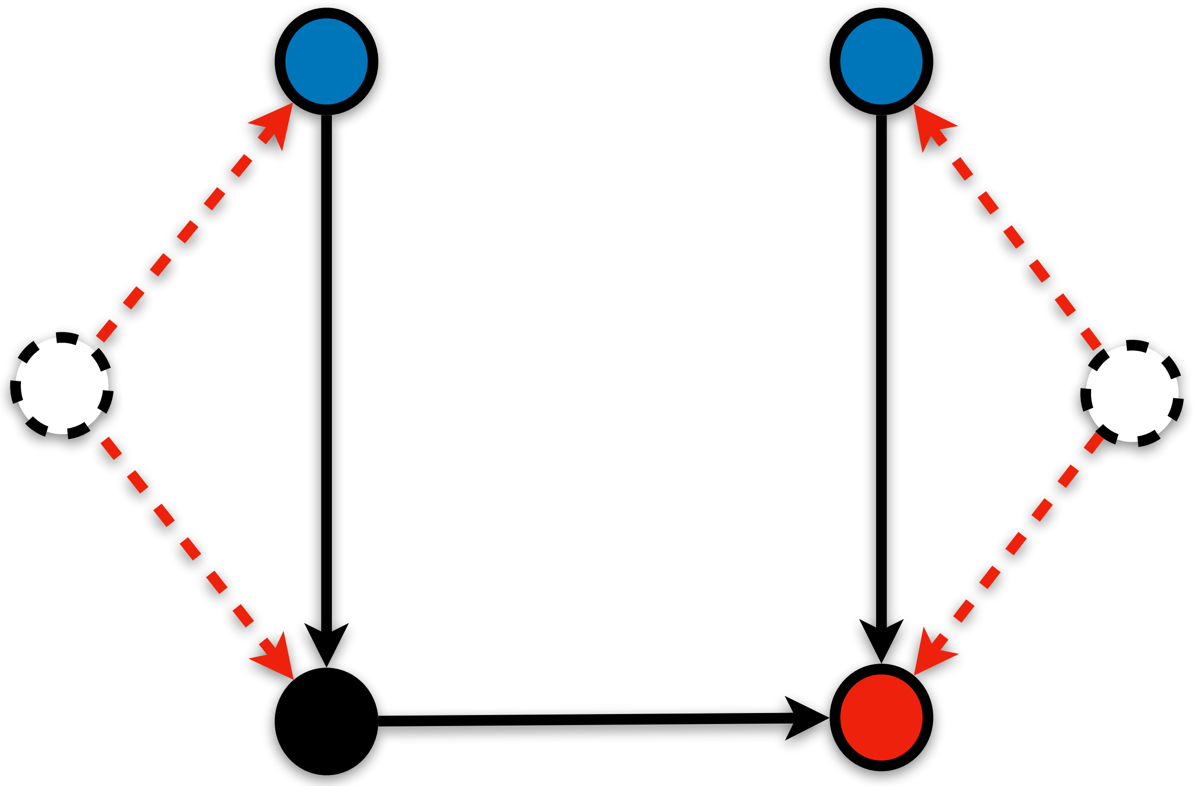
...

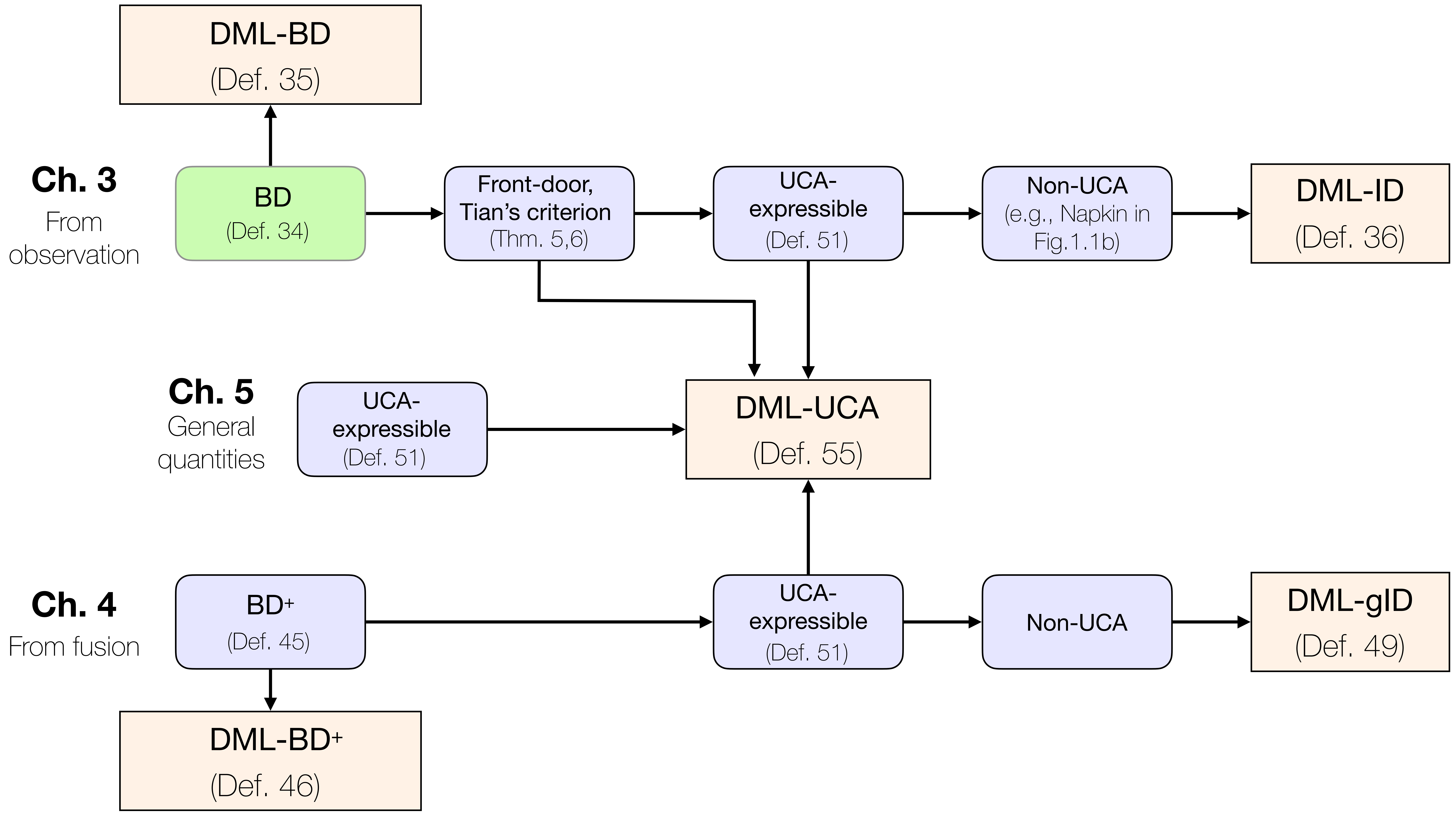
Solution

DML-ID

- DML-BD^{+}
- DML-gID

This Talk: Estimating Causal Effects

	1. From Observation	2. From Data Fusion	3. Unified Estimation
Tasks			Fairness $\mathbb{E}[Y_{x,M_{\neg x}}]$ Off-policy evaluation $\mathbb{E}[Y_{\tau(X\|C)}]$ Counterfactuals $\mathbb{E}[Y_x \| \neg x]$ …
Solution	DML-ID	• DML-BD+ • DML-gID	DML-UCA



DML-BD
(Def. 35)
Ch. 3
From observation
BD
(Def. 34)
Front-door, Tian's criterion
(Thm. 5,6)
UCA-expressible
(Def. 51)
Non-UCA
(e.g., Napkin in Fig.1.1b)
DML-ID
(Def. 36)
Ch. 5
General quantities
UCA-expressible
(Def. 51)
DML-UCA
(Def. 55)
Ch. 4
From fusion
BD+
(Def. 45)
UCA-expressible
(Def. 51)
Non-UCA
DML-gID
(Def. 49)
DML-BD+
(Def. 46)

Thank you

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Appendix

Logistics

Logistics

- **Professor Neville**
  - Please initiate and sign "Form 11: Report of the Final Examination"
  - Please approve the "Form 9: Electronic Thesis Acceptance Form (ETAF)" after reviewing the thesis.

Logistics

- **Professor Neville**
  - Please initiate and sign "Form 11: Report of the Final Examination"
  - Please approve the "Form 9: Electronic Thesis Acceptance Form (ETAF)" after reviewing the thesis.
- **Other Professors**
  - Please sign "Form 11: Report of the Final Examination"
  - Please approve the "Form 9: Electronic Thesis Acceptance Form (ETAF)" after reviewing the thesis.

Logistics

- **Professor Neville**
  - Please initiate and sign "Form 11: Report of the Final Examination"
  - Please approve the "Form 9: Electronic Thesis Acceptance Form (ETAF)" after reviewing the thesis.
- **Other Professors**
  - Please sign "Form 11: Report of the Final Examination"
  - Please approve the "Form 9: Electronic Thesis Acceptance Form (ETAF)" after reviewing the thesis.

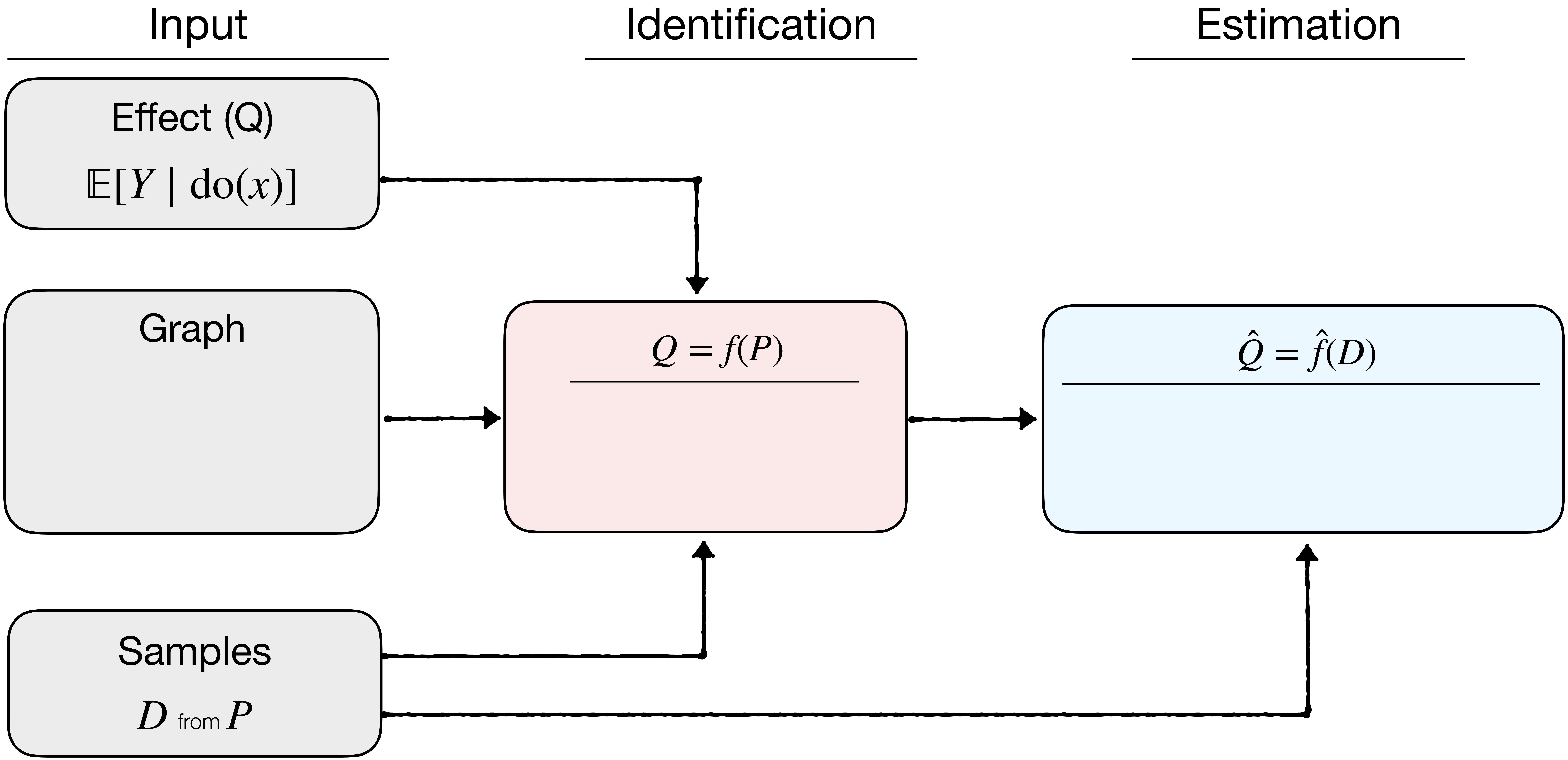
I kindly ask that you complete these by **June 12** to meet the PhD completion deadline for my next job appointment — Assistant Professor at UIUC's School of Information Sciences.

Logistics

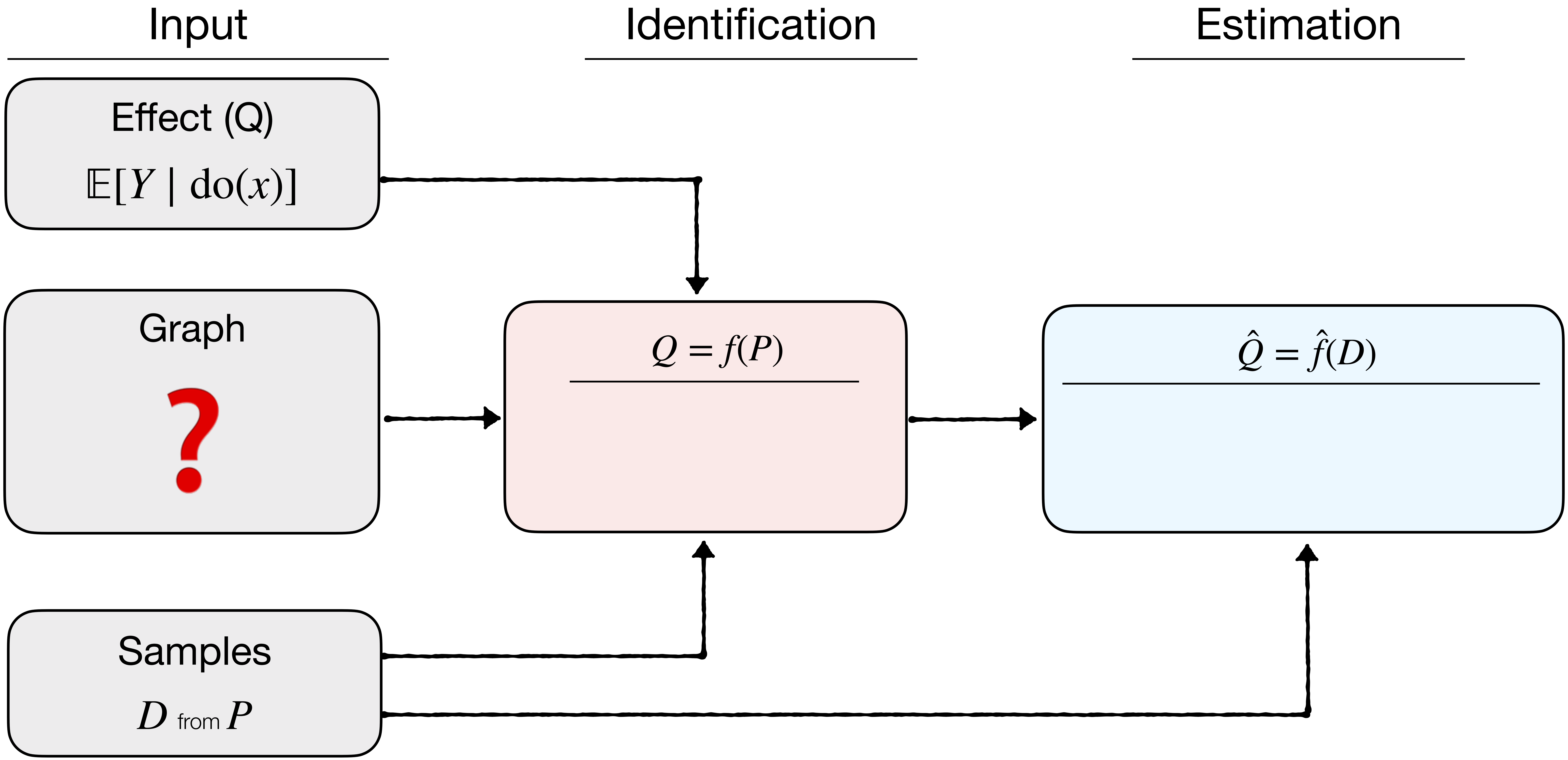
- Please initiate and sign "Form 11: Report of the Final Examination"
- Please approve the "Form 9: Electronic Thesis Acceptance Form (ETAF)" after reviewing the thesis.

Omitted Works

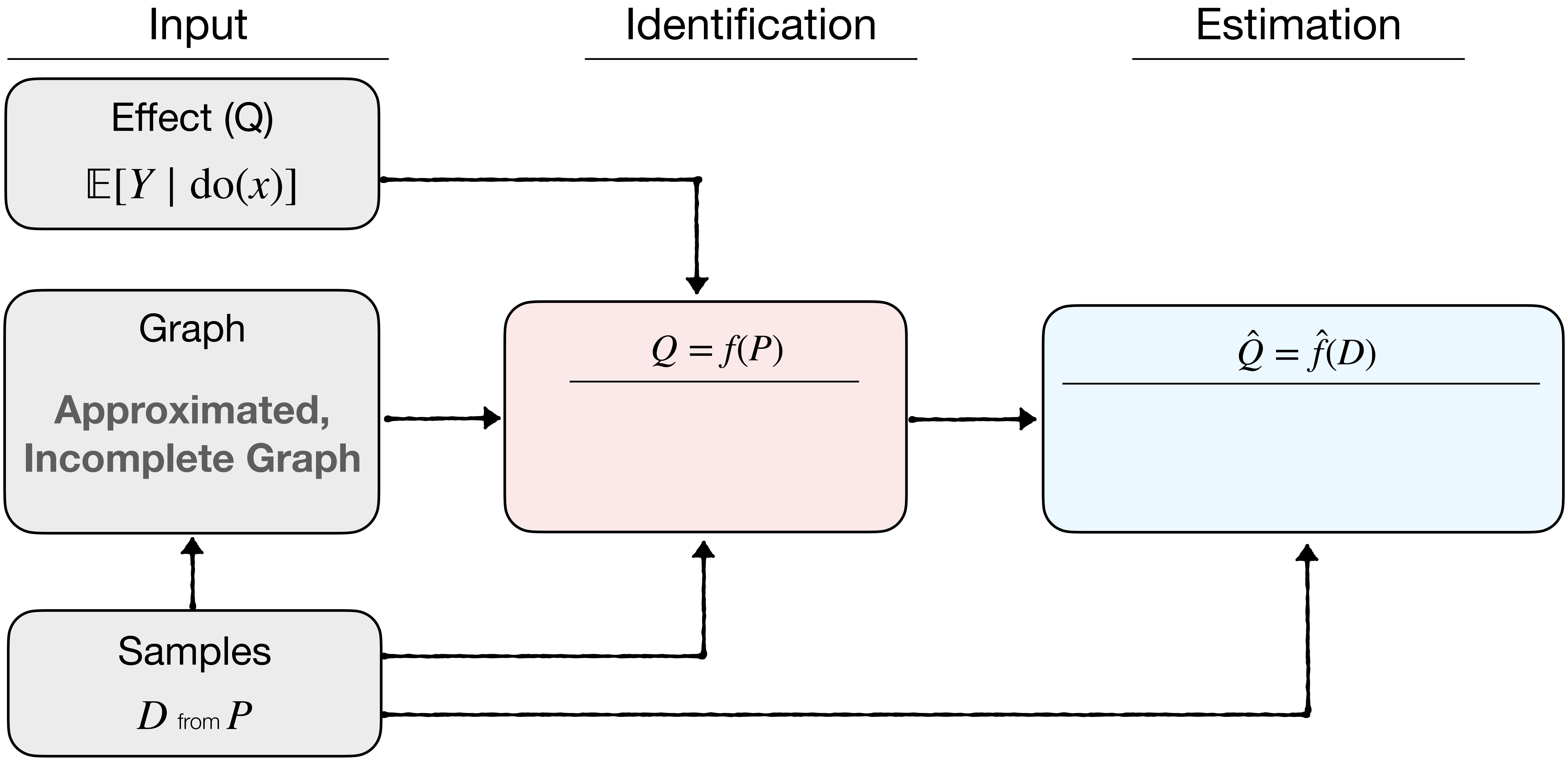
Other Work 1: Causal inference Without Graphs



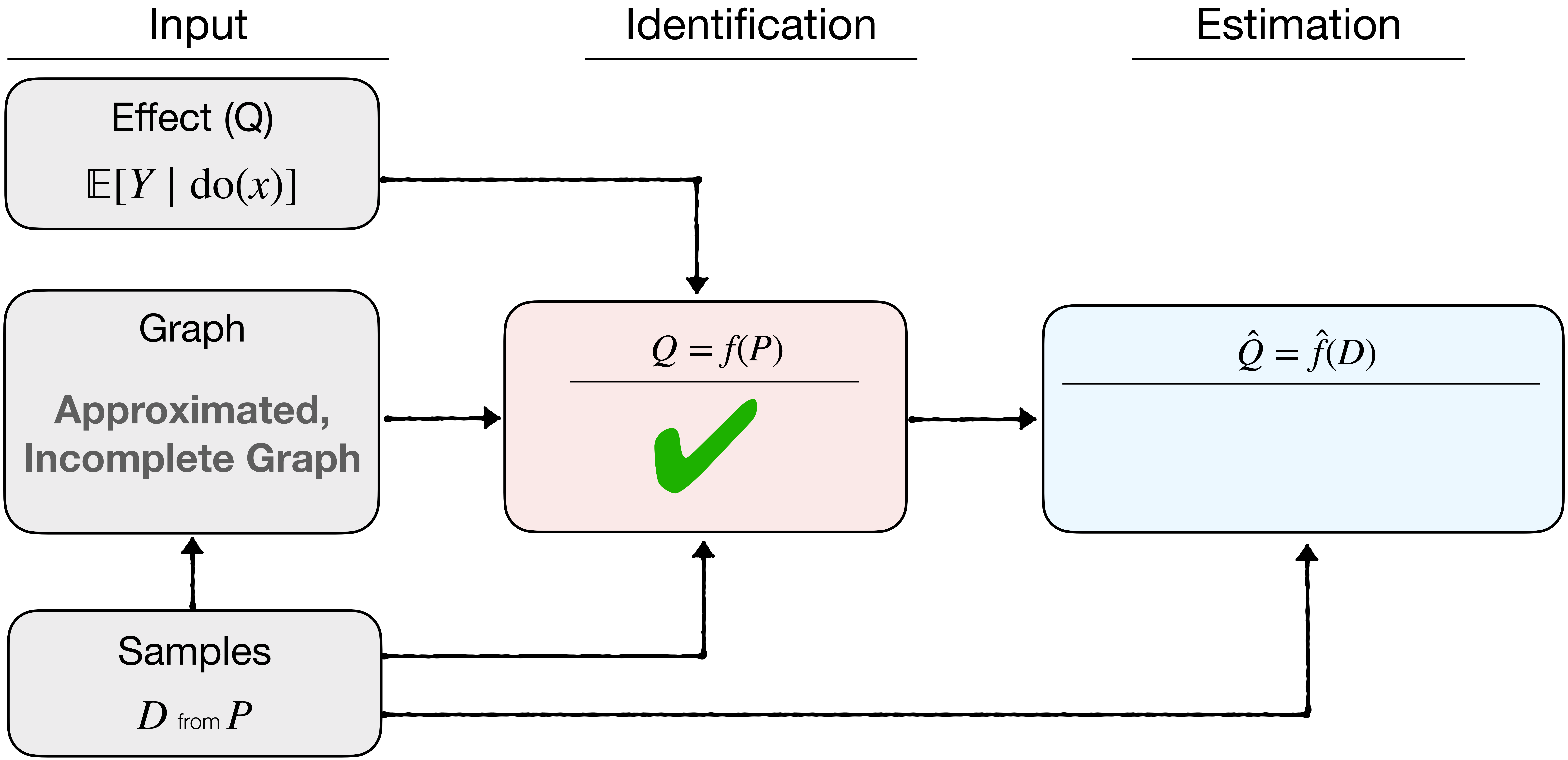
Other Work 1: Causal inference Without Graphs



Other Work 1: Causal inference Without Graphs



Other Work 1: Causal inference Without Graphs



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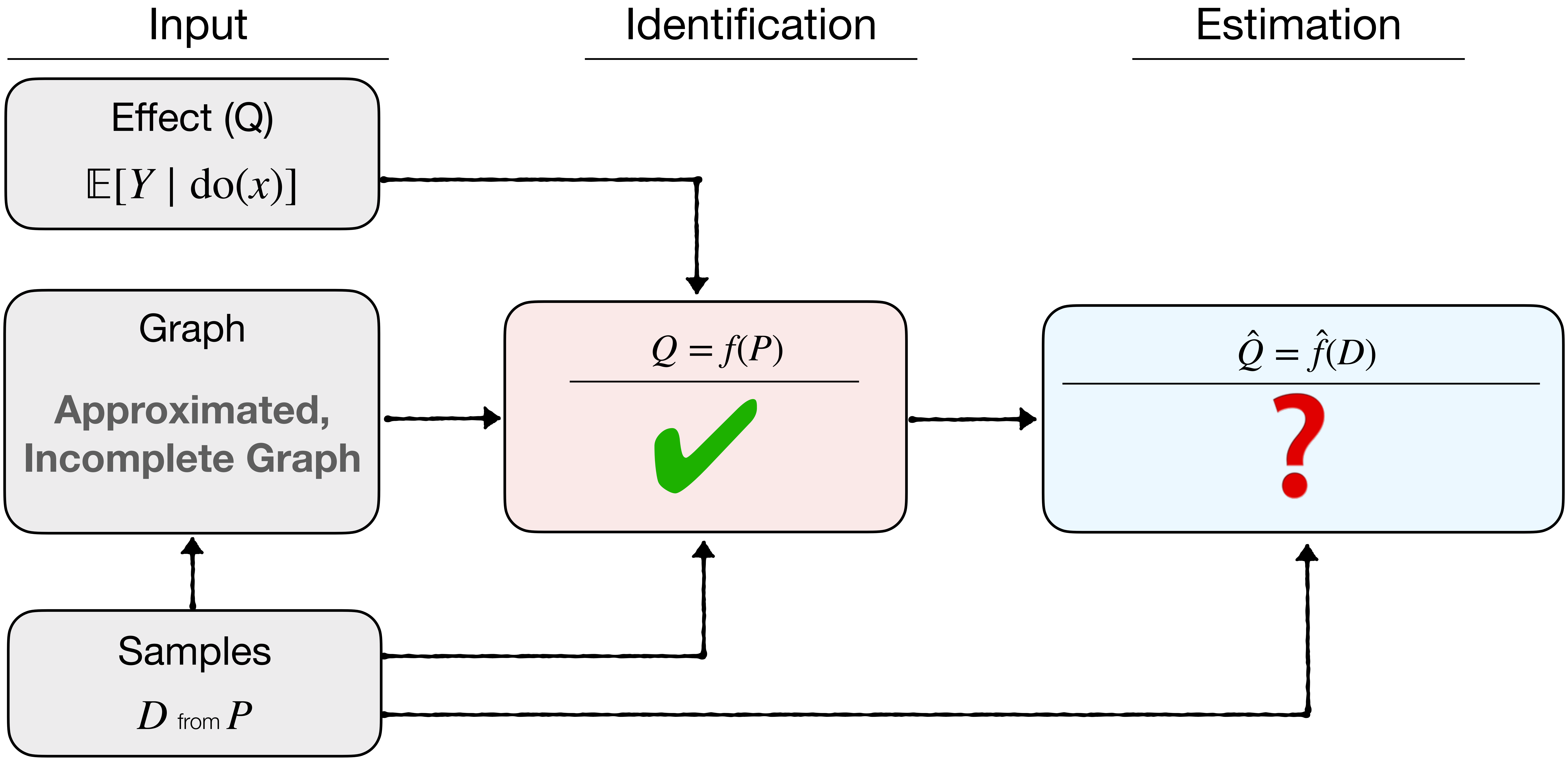

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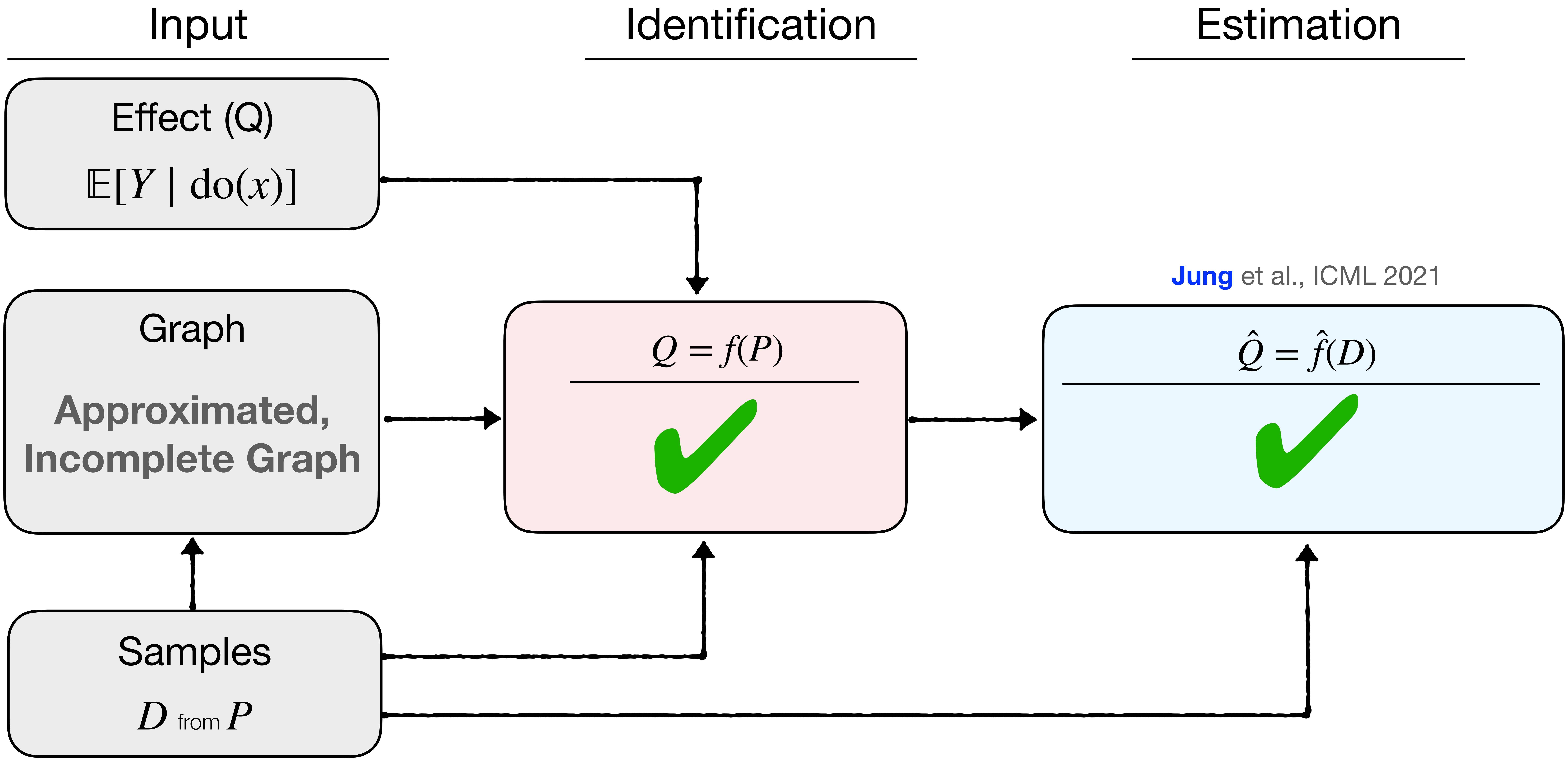


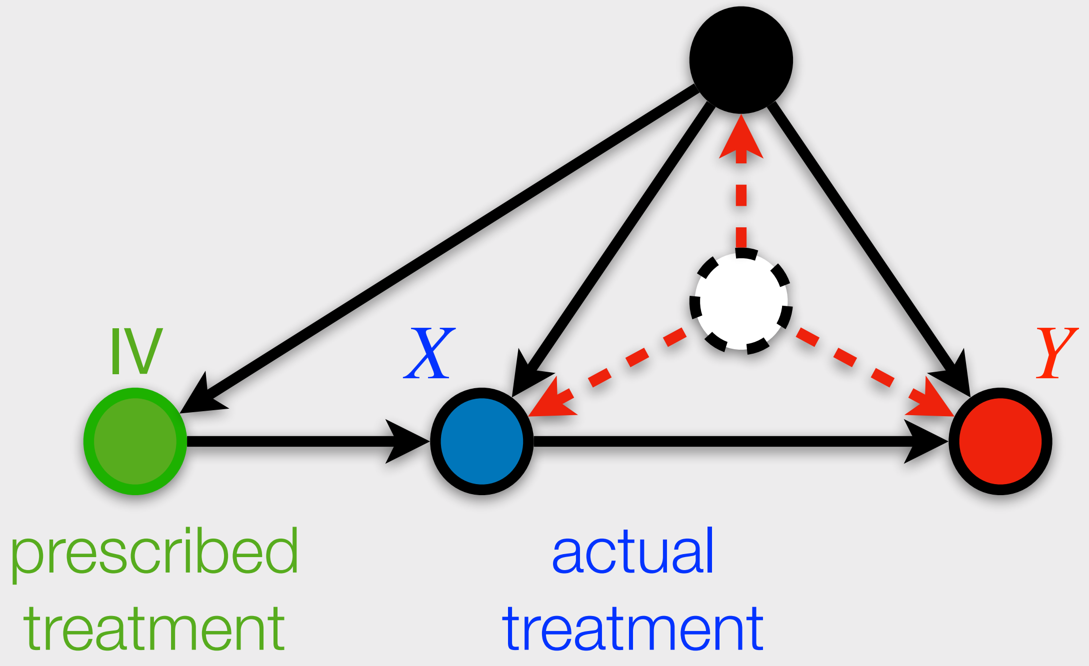
Other Work 2: Instrumental Variable (IV) Analysis

Input | Identification | Estimation

Effect (Q)

$\mathbb{E}[Y \mid \mathrm{do}(x), \mathrm{complier}]$

Graph



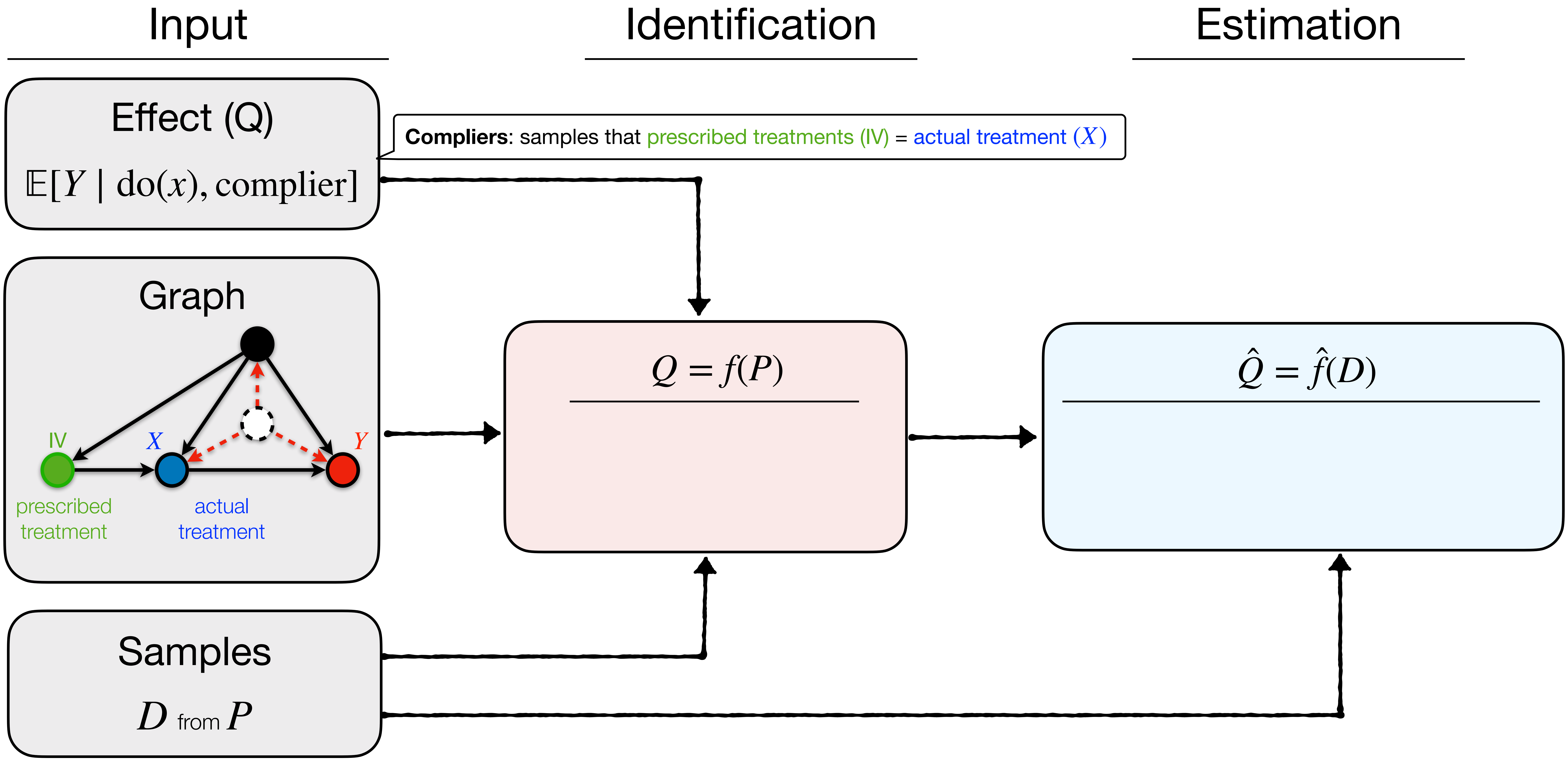
Samples

D from P

$Q = f(P)$

$\hat{Q} = \hat{f}(D)$

Other Work 2: Instrumental Variable (IV) Analysis



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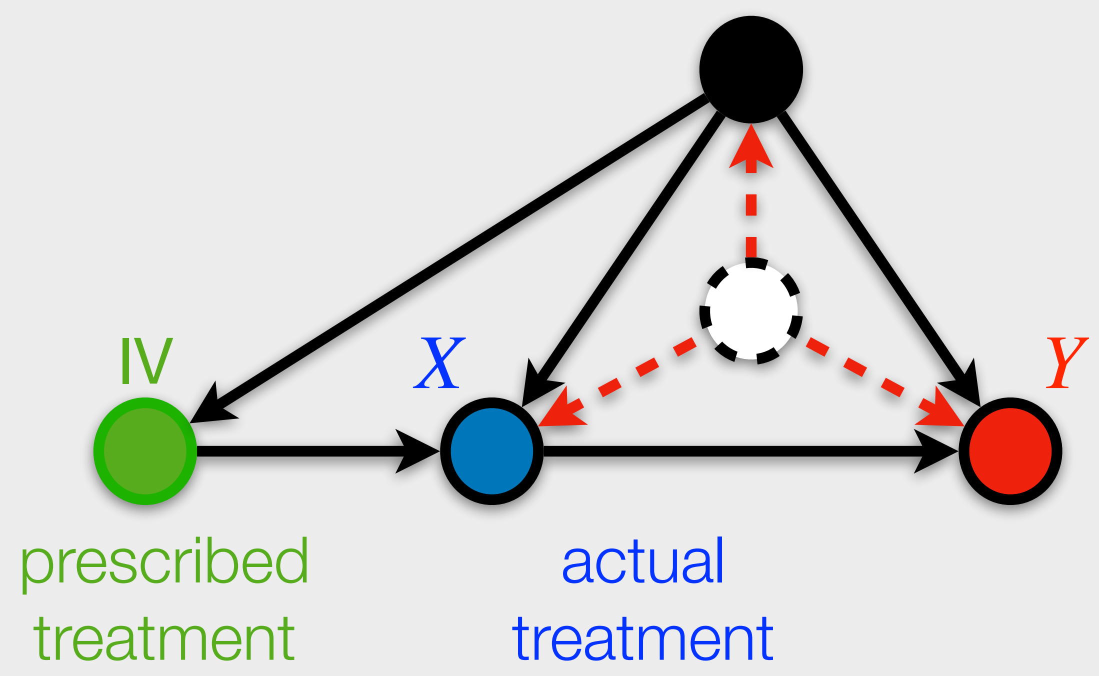

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Imbens & Angrist won the **2021 Nobel Prize in Economics** for their achievements in IV analysis.

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✔ with additional assumptions

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✔

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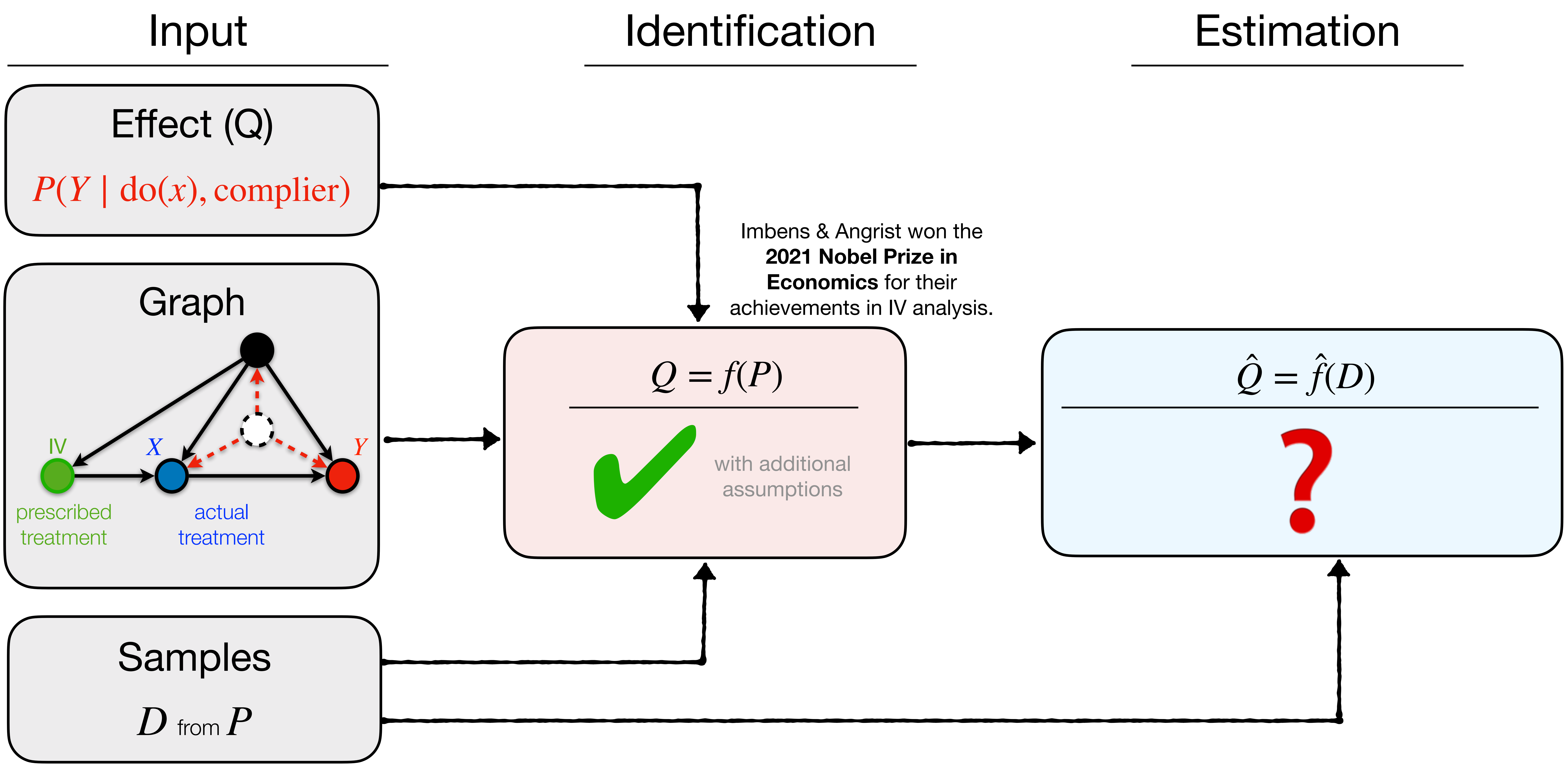

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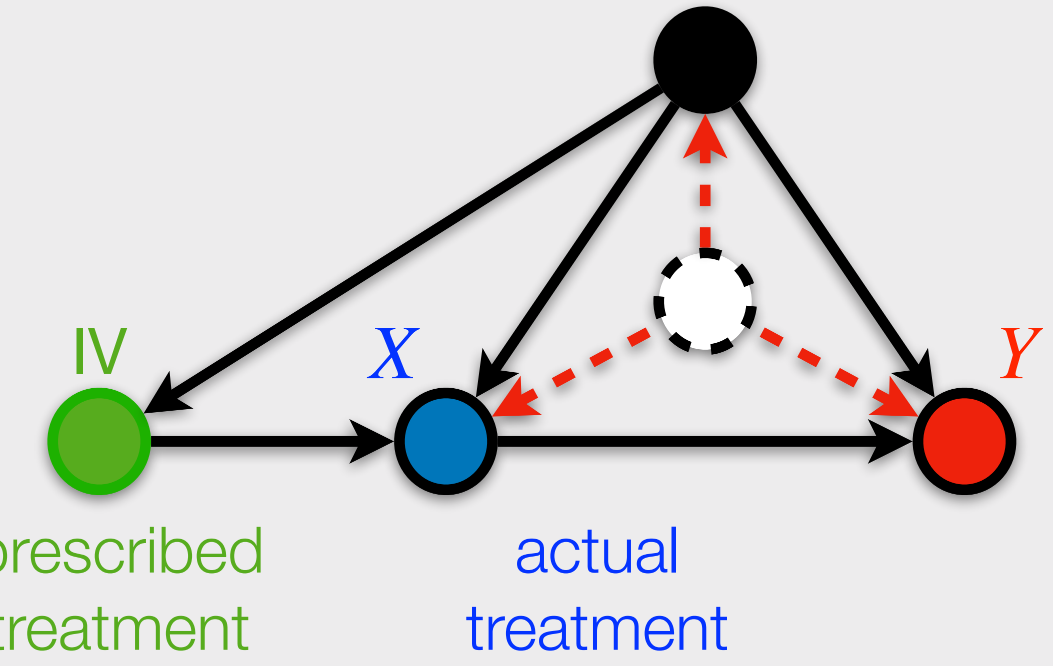

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Jung et al., NeurIPS 2021 (Spotlight)

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✔

Application 1. Healthcare Science

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RCT

- ➕ Gold standard in causal inference
- ➖ Expensive
- ➖ Selection bias

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RCT	EHR MIMIC-IV, OpenMRS eICU, …
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➖ Expensive	➕ Easy to collect
➖ Selection bias	➕ Generalizable

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Best of Both Worlds

Emulating RCT from EHR

Application 1. Emulating RCT from EHR

Application 1. Emulating RCT from EHR

Input

Effect (Q)

$\mathbb{E}[Y \mid \mathrm{do}(x)]$

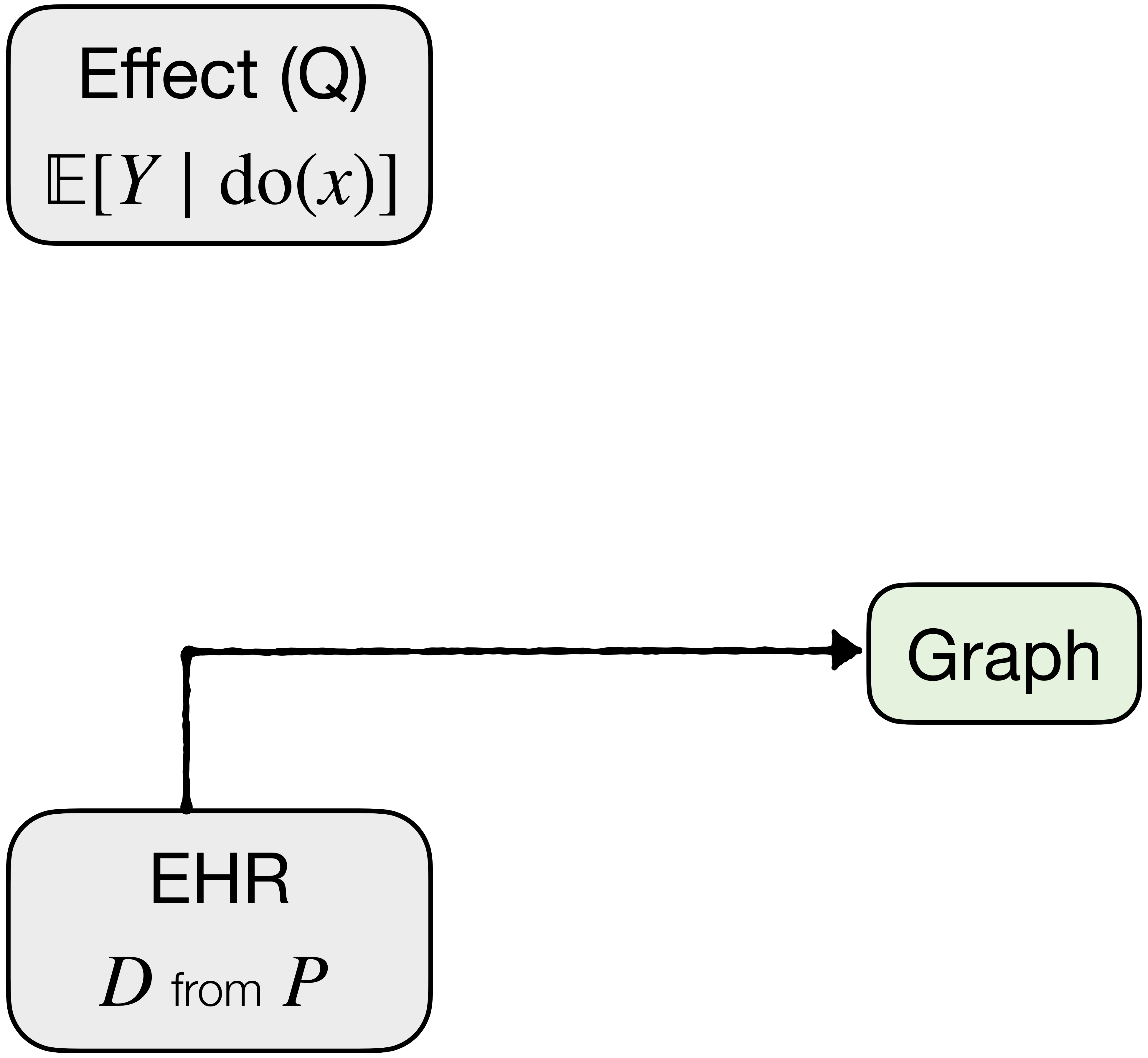
EHR

D from P

Application 1. Emulating RCT from EHR

Input

Graph Discovery



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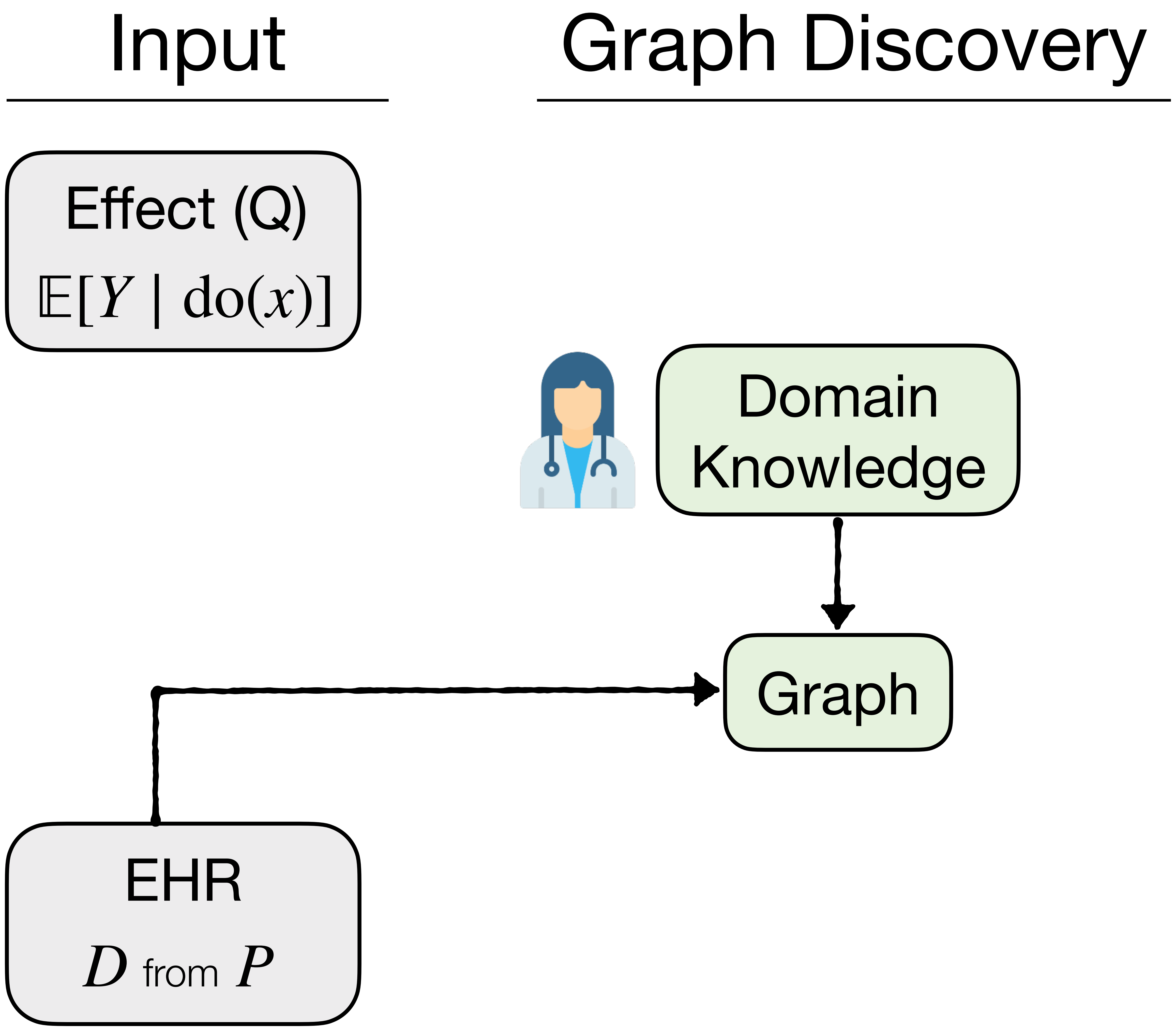

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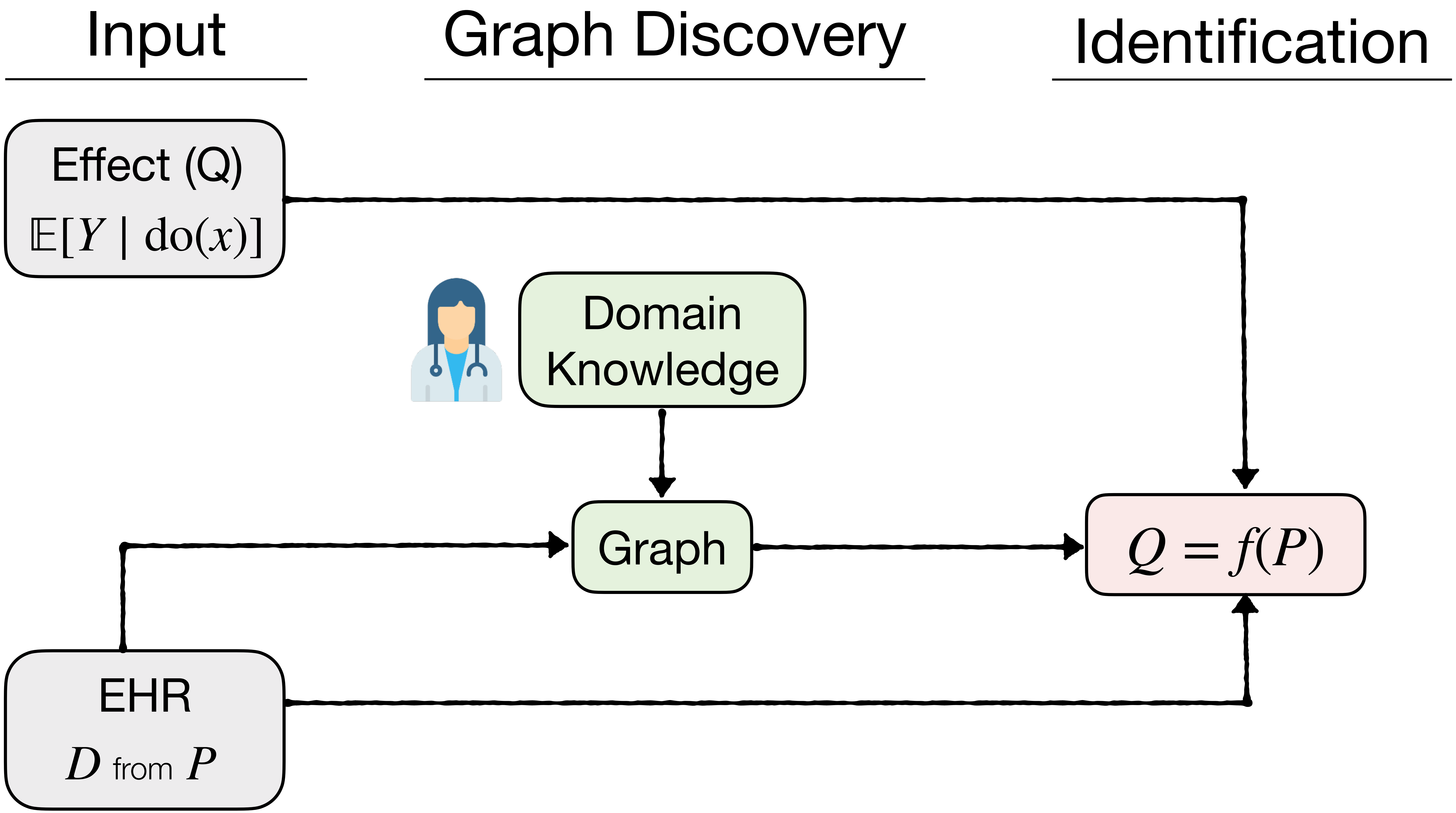

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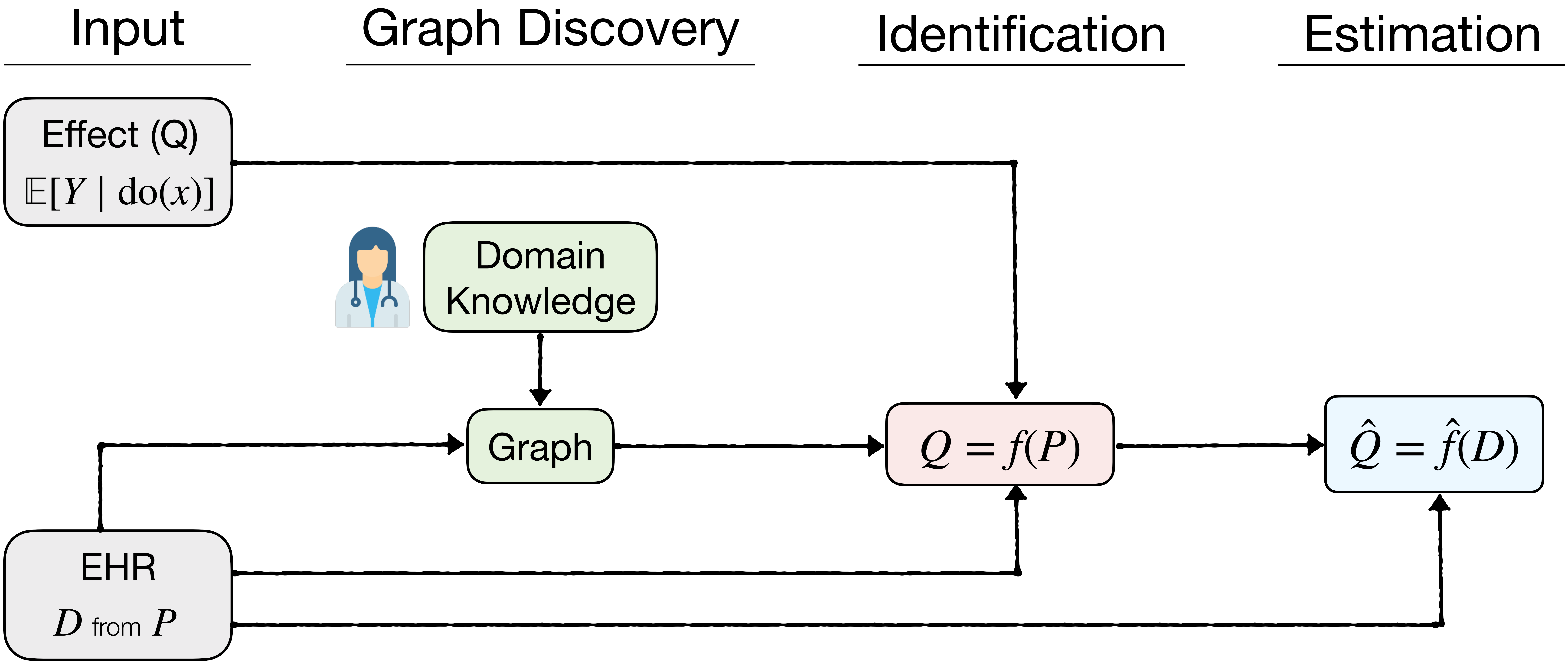

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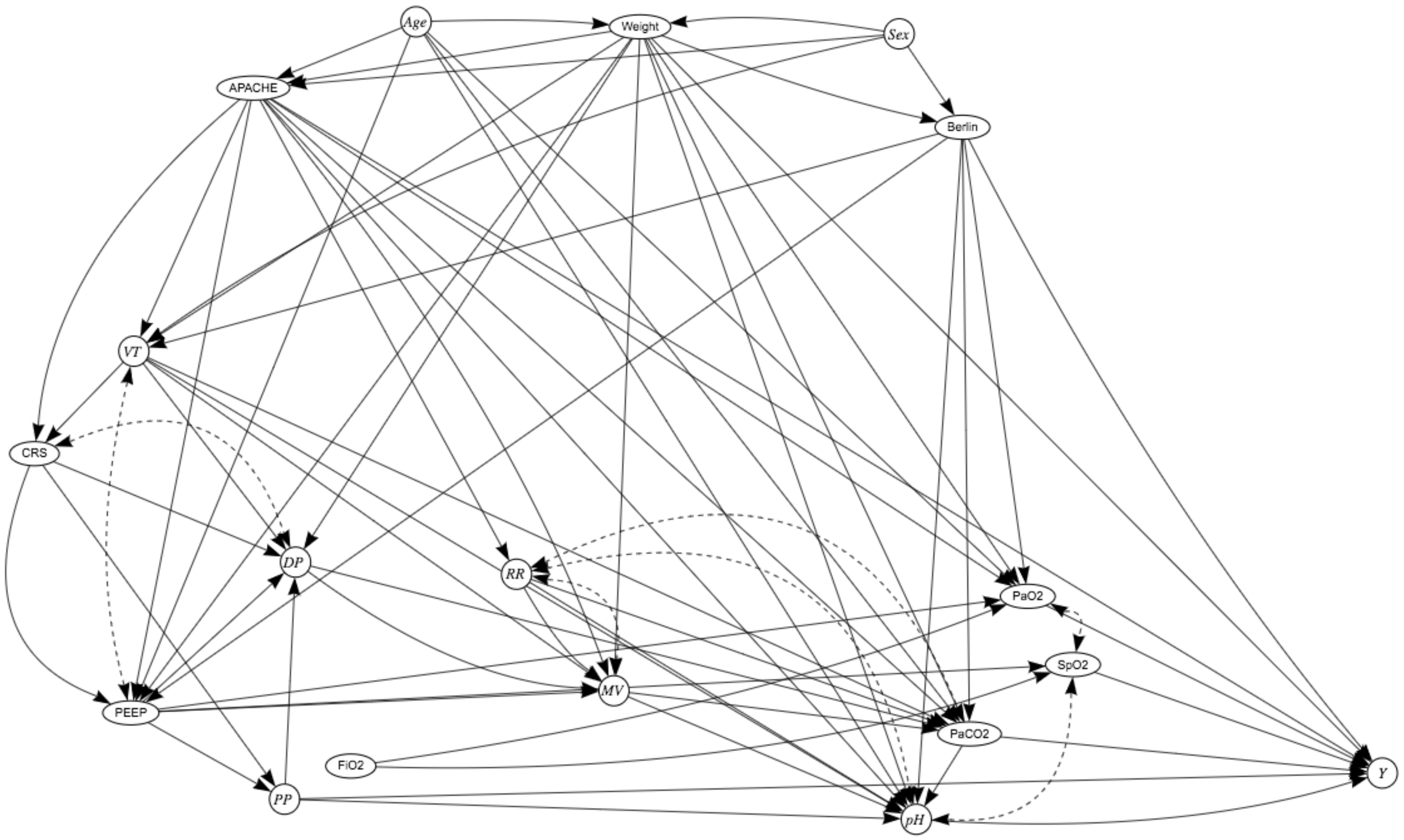

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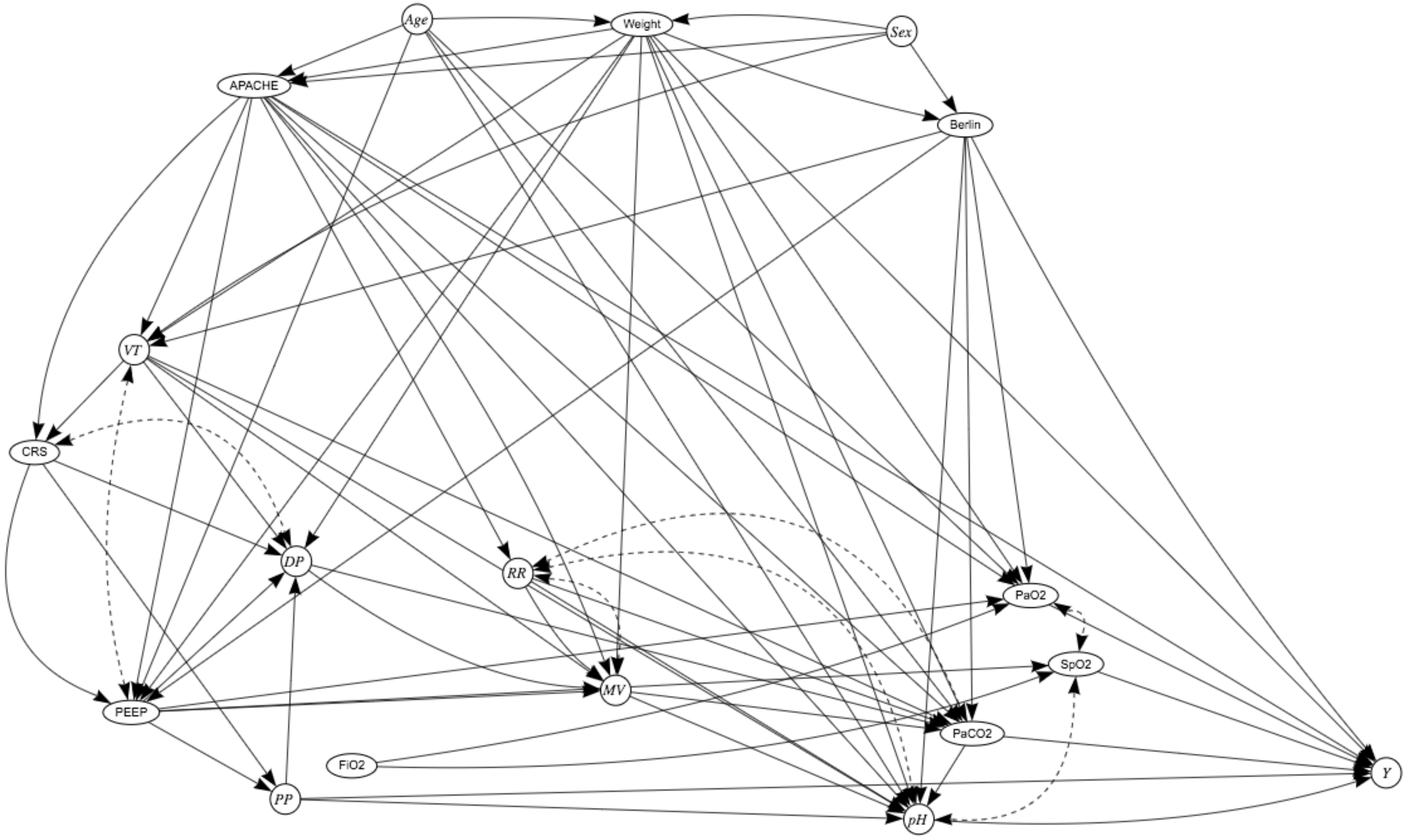
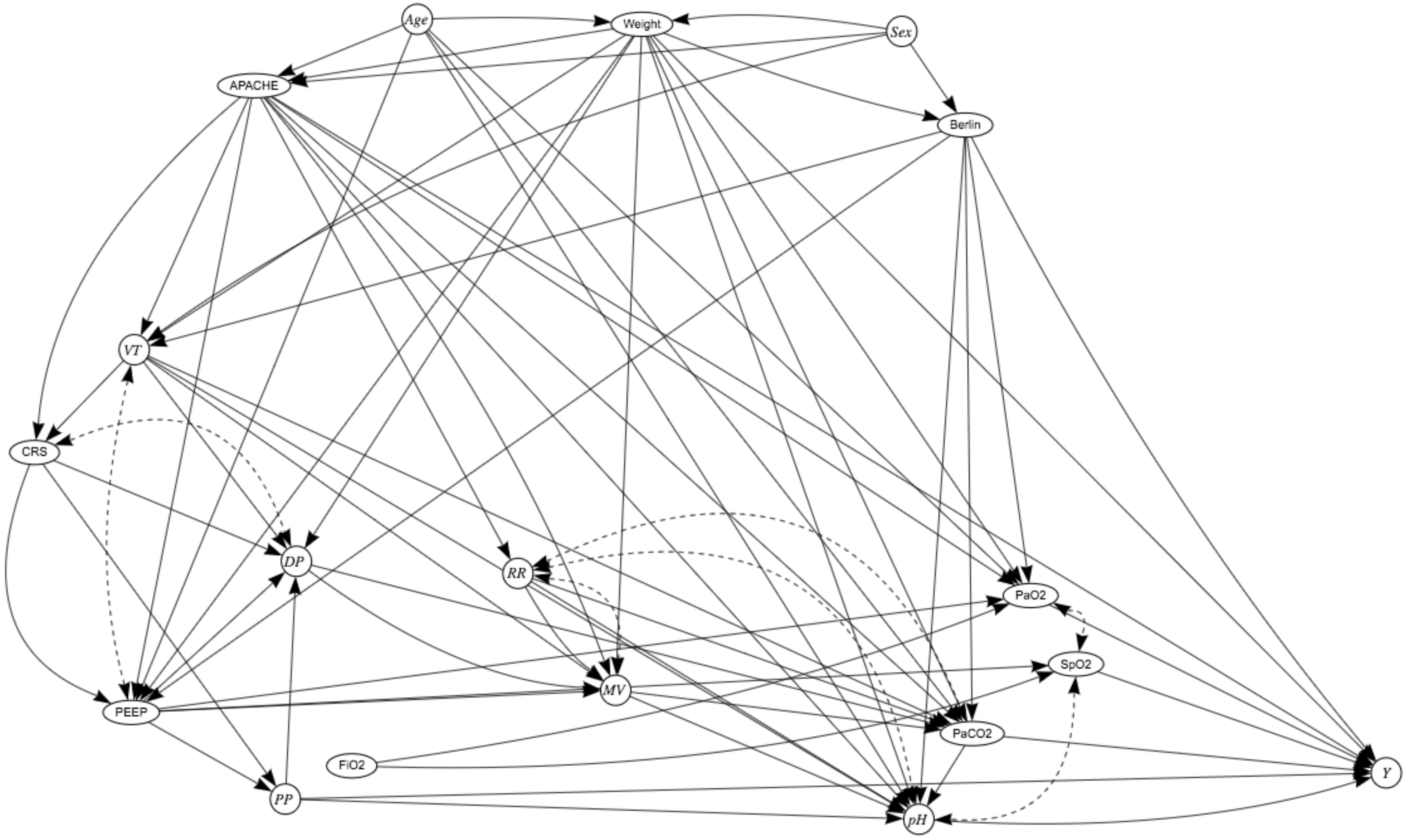


Causal graph on Acute Respiratory Distress Syndrome (ARDS)

Application 1. Emulating RCT from EHR

Jung et al., American Thoracic Society, 2018



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Result

For seminal RCTs,

Our treatment recommendation

= Trials' treatment recommendation

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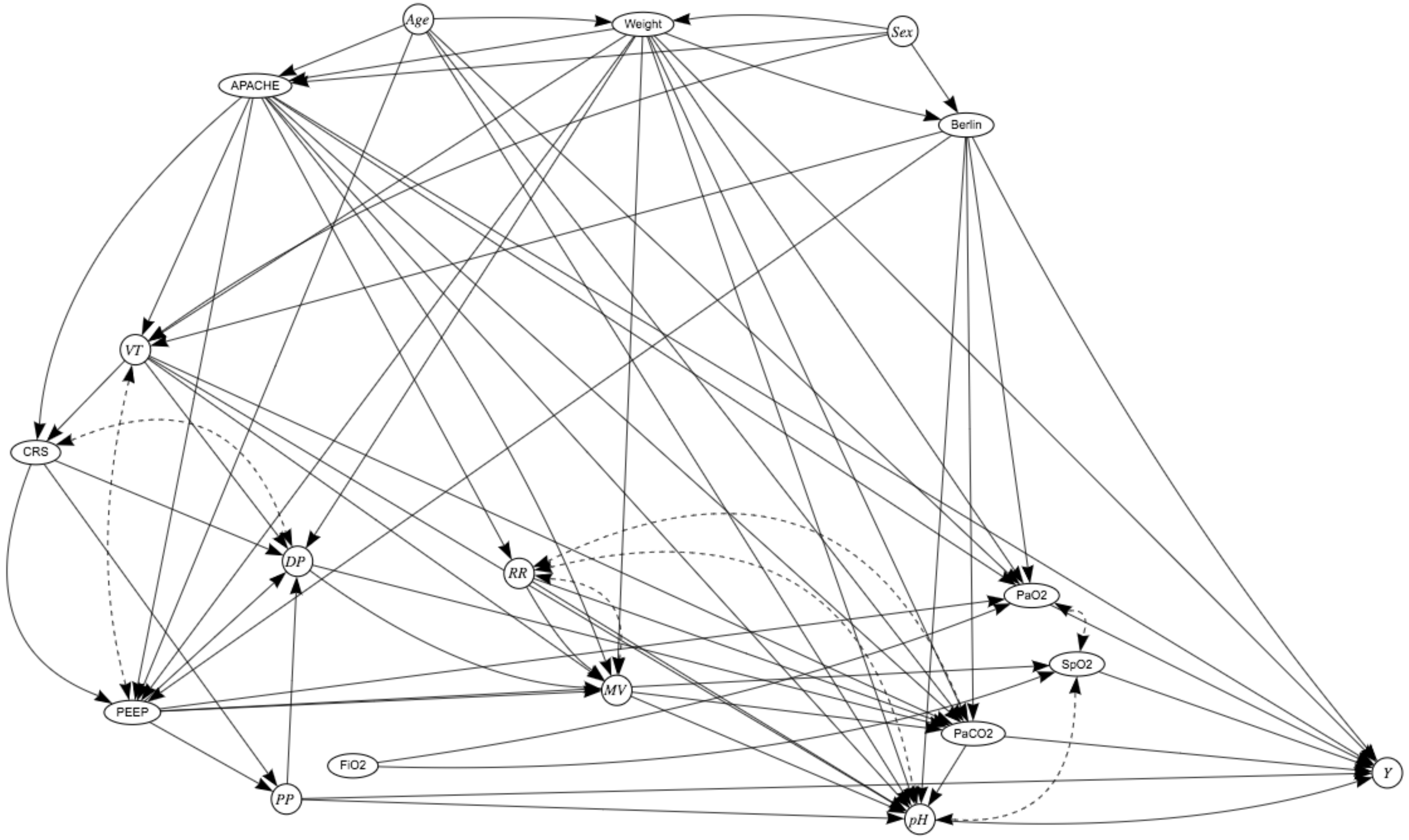

Causal graph on Acute Respiratory Distress Syndrome (ARDS)

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Impact

Our method can be used to construct an initial hypothesis before conducting trials.

Application 2. Explainable AI

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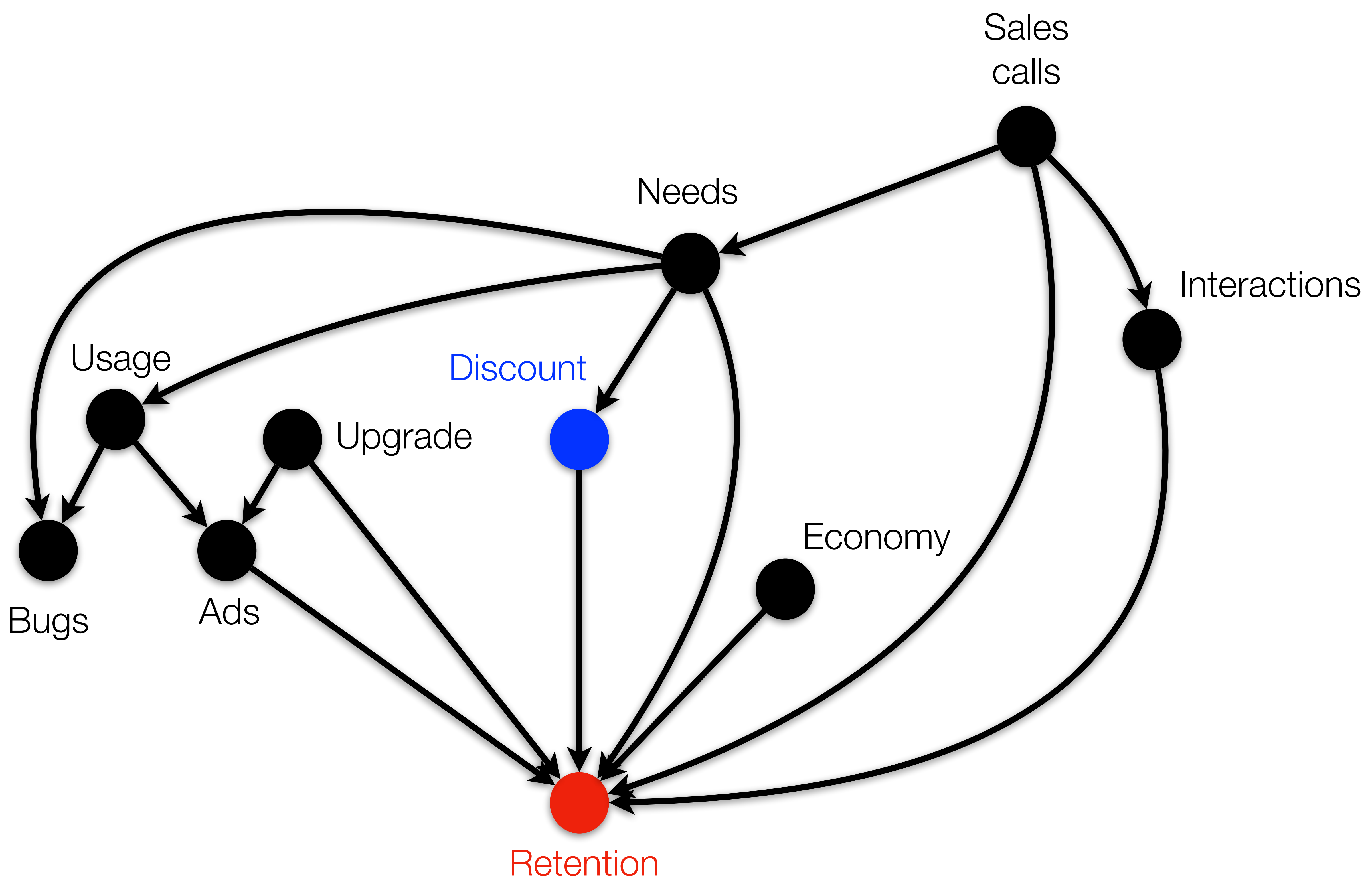
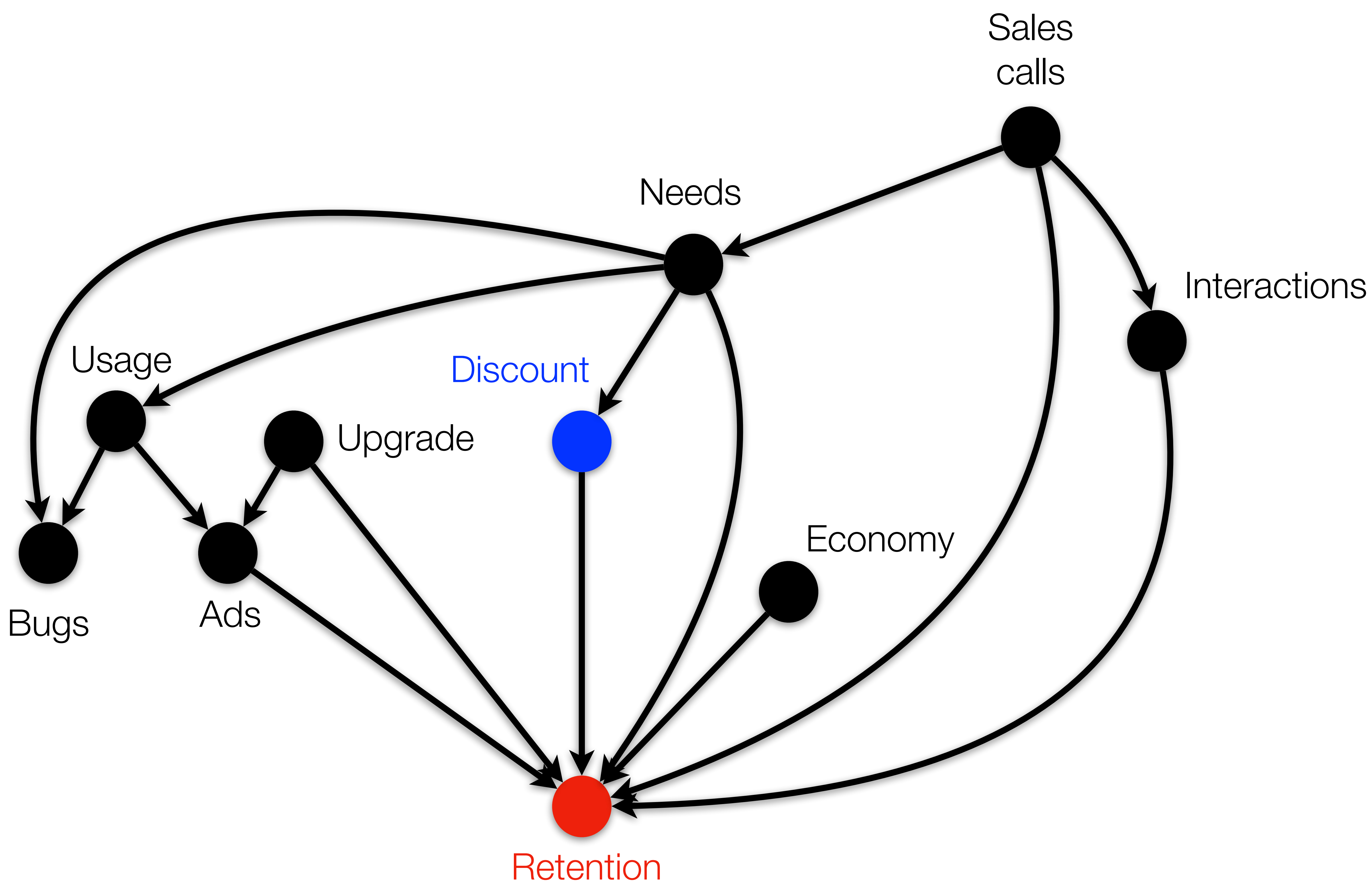


Contribution of Discount to the Retention?

Application 2. Explainable AI



Contribution of Discount to the Retention?

- SHAP value: one of the most cited measure for the feature importance

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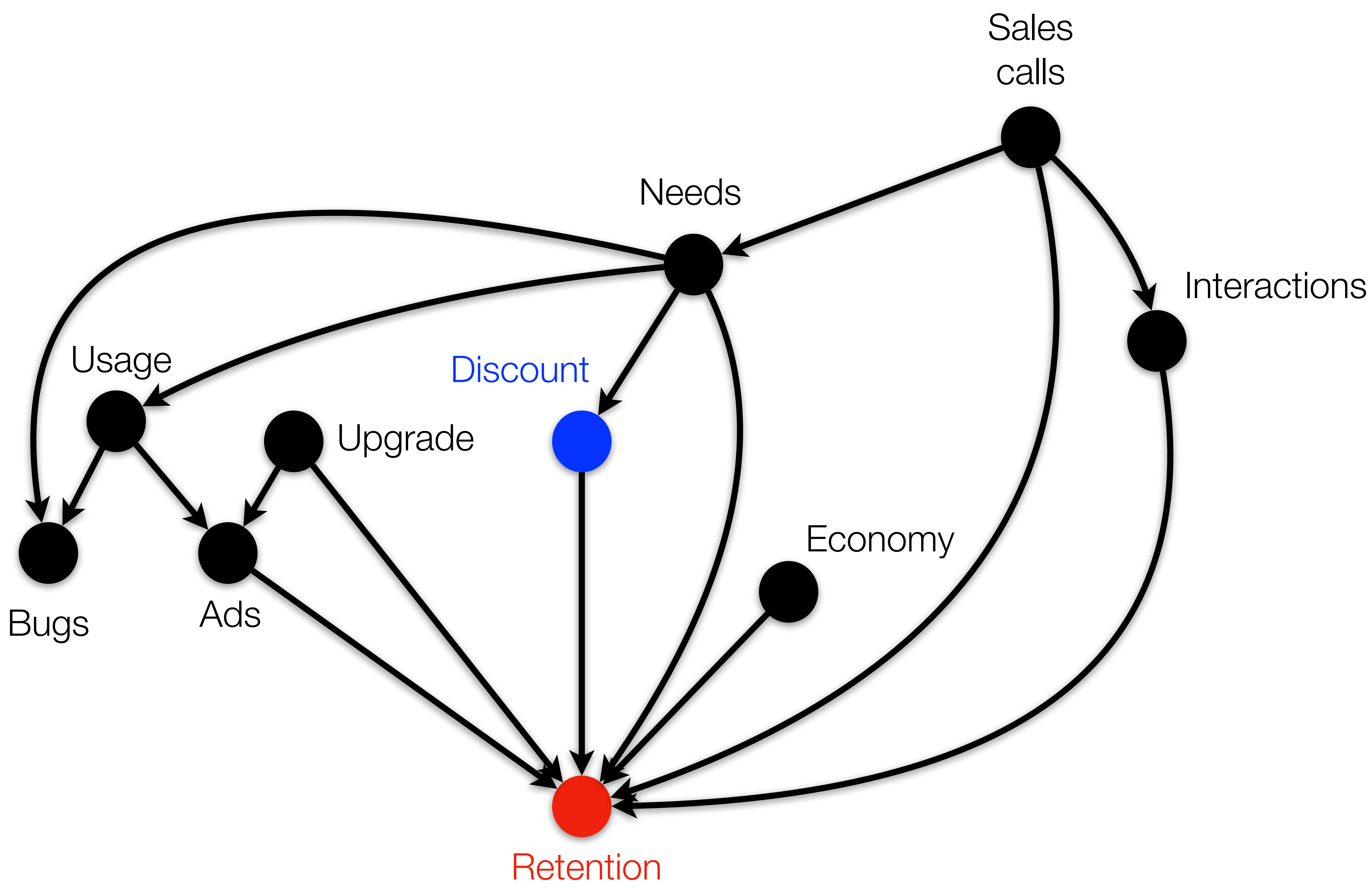

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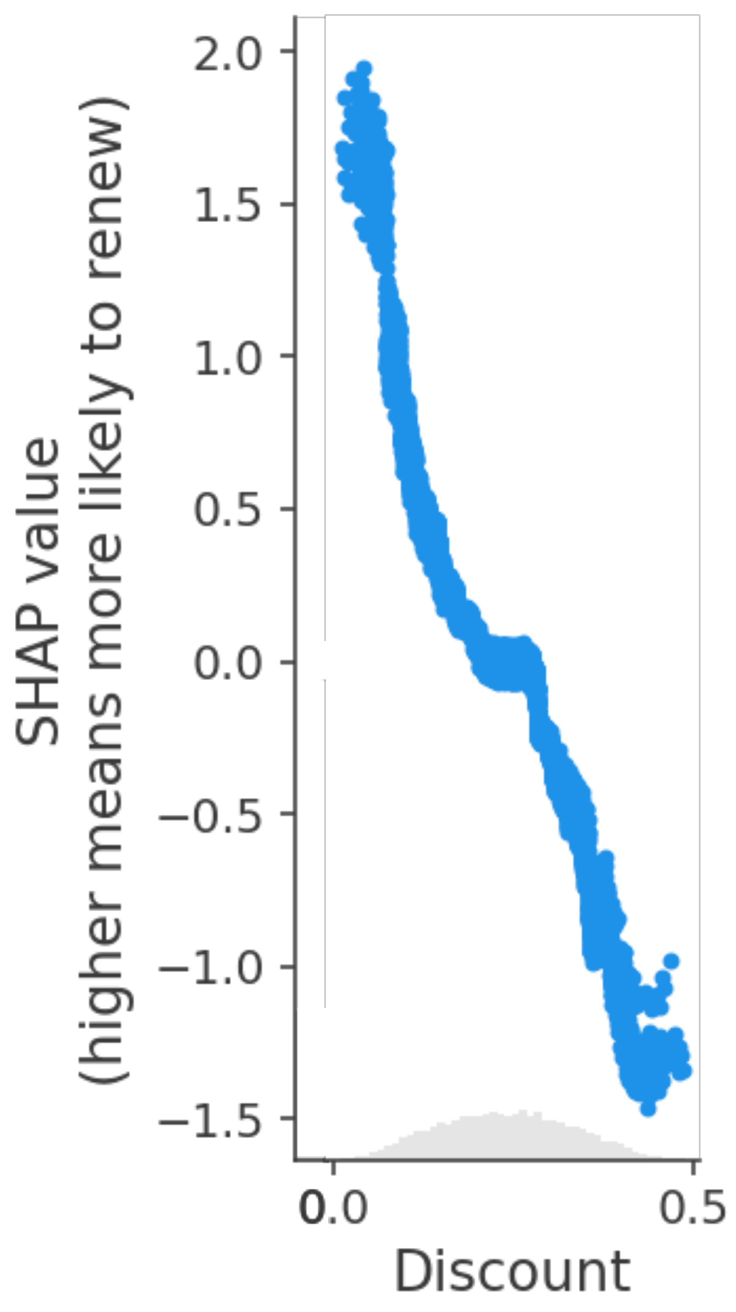

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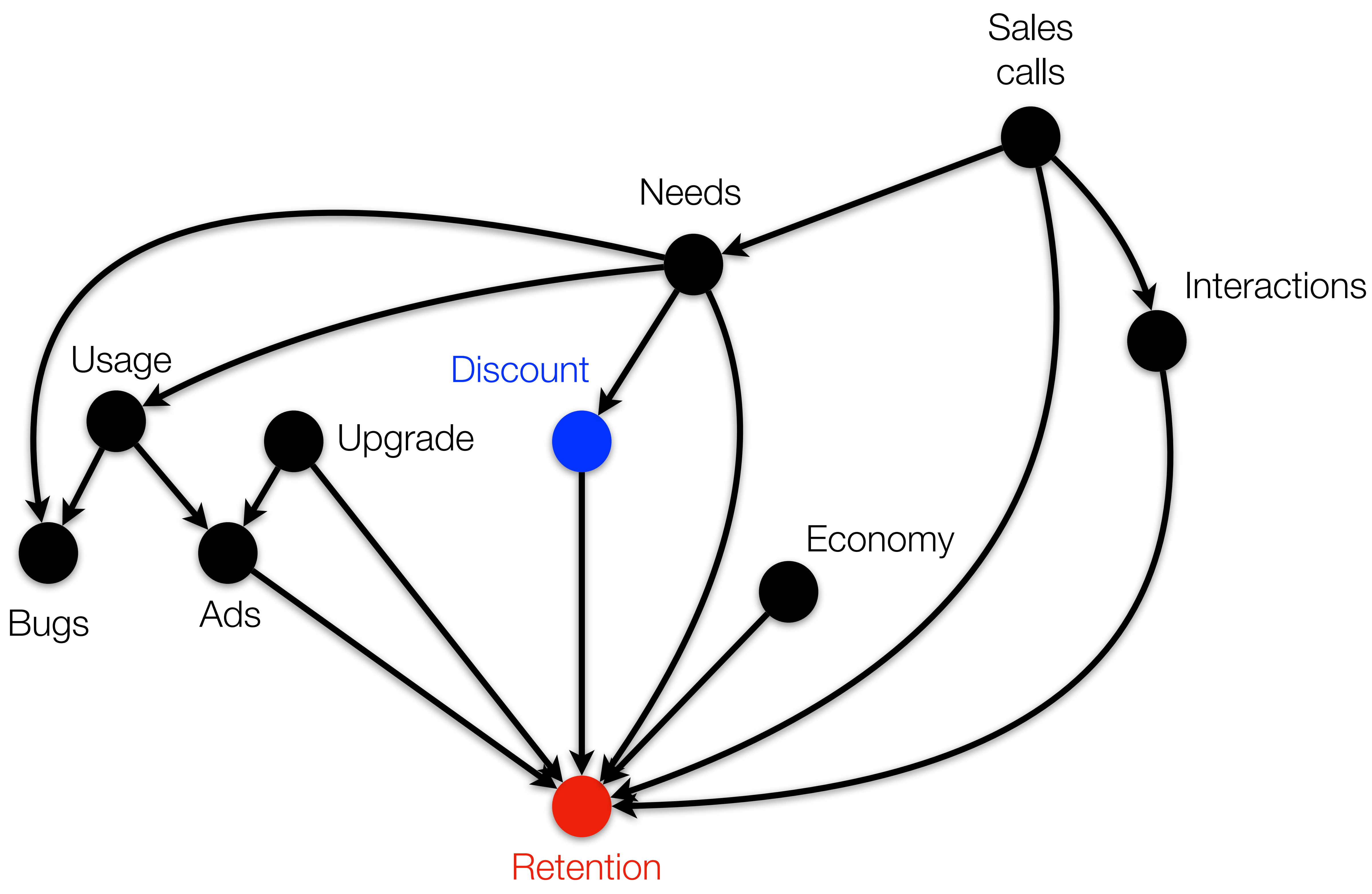

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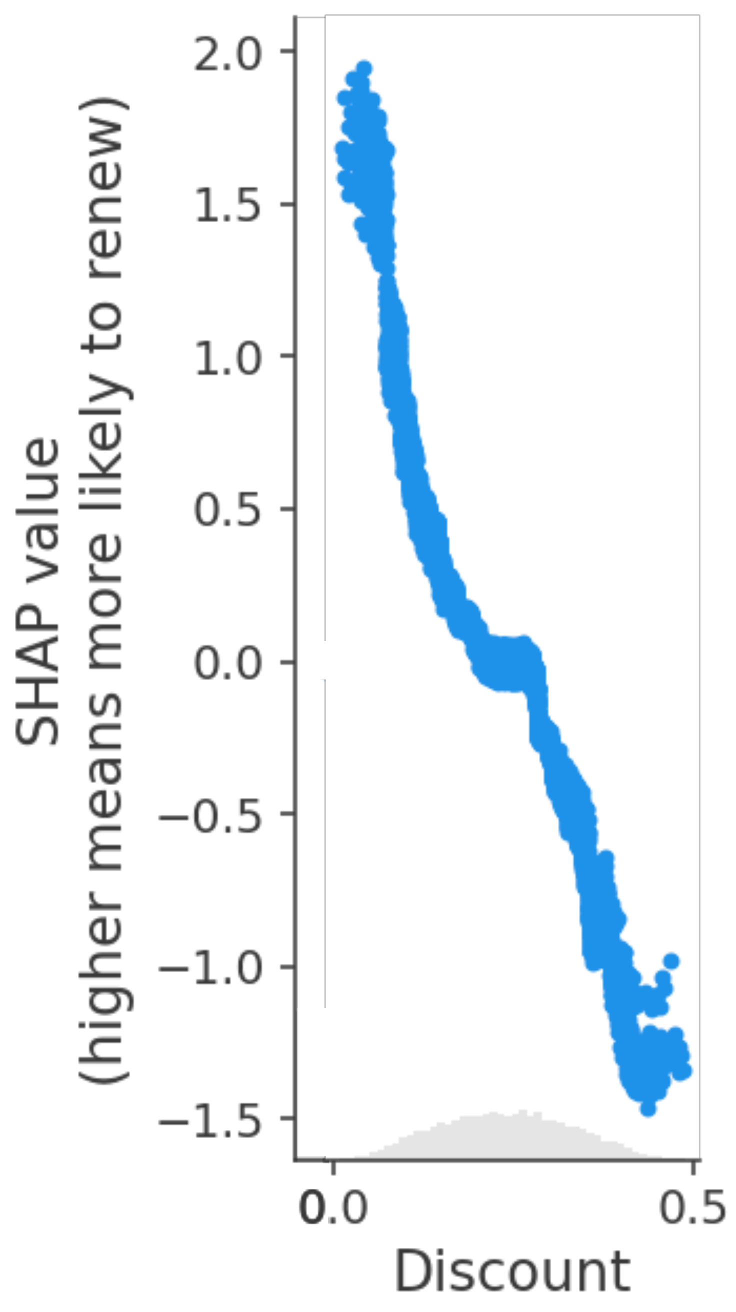

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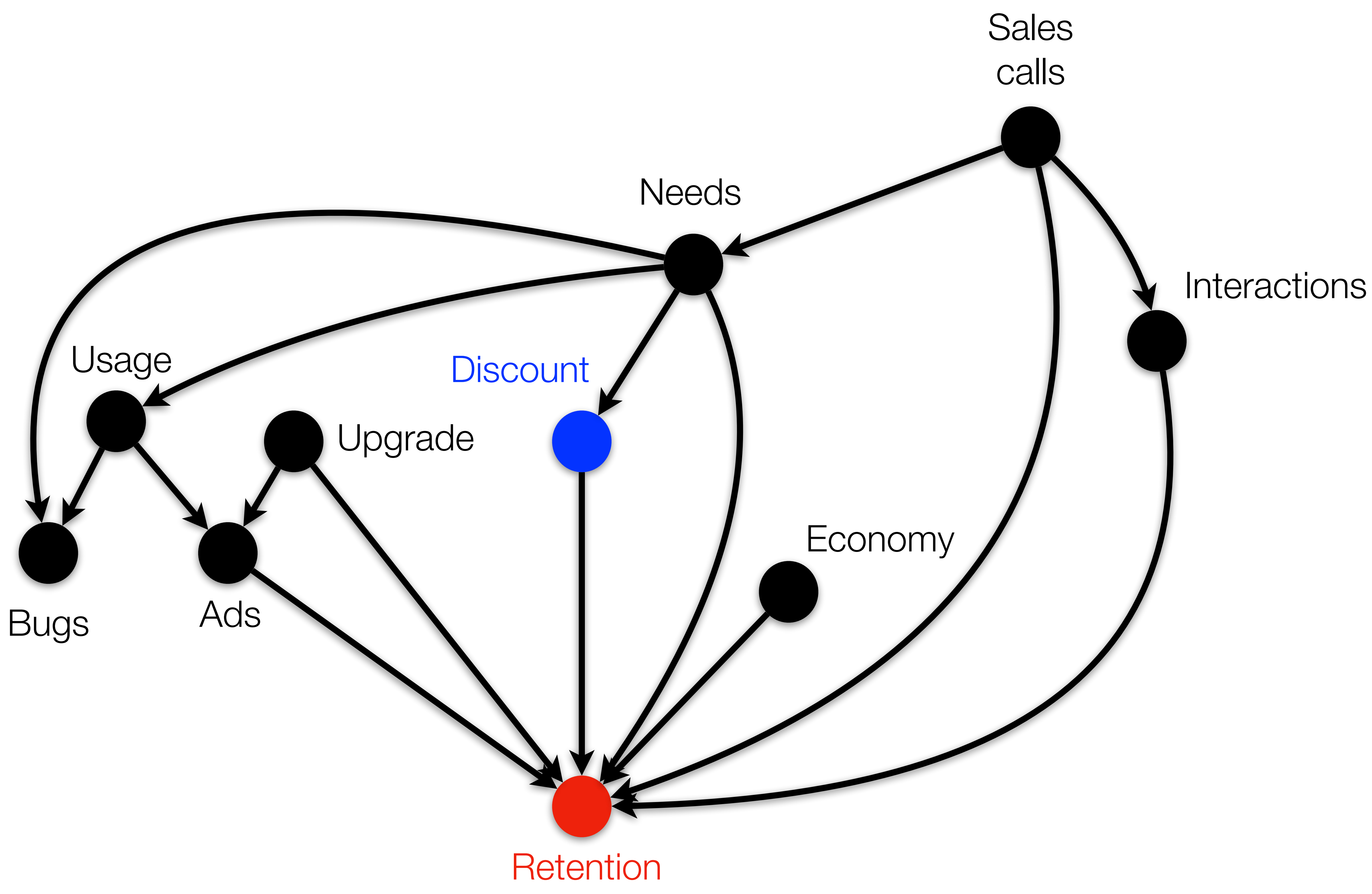

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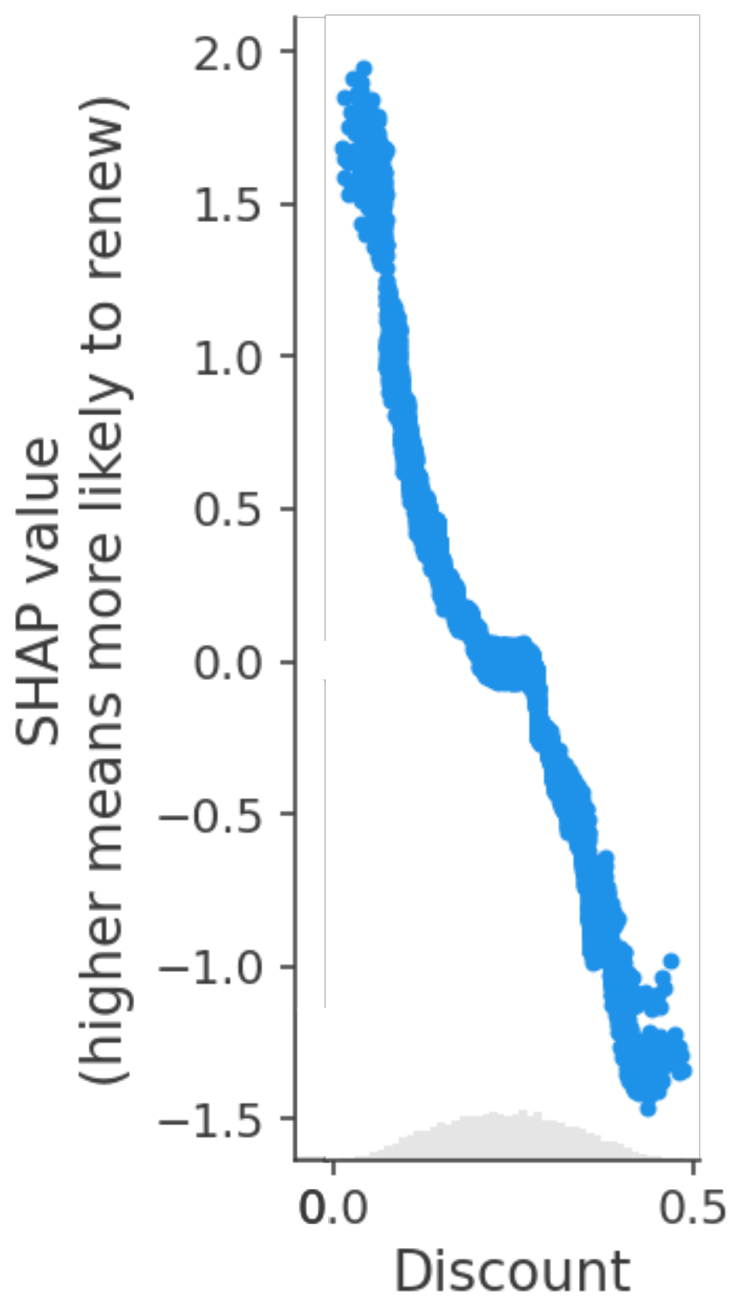

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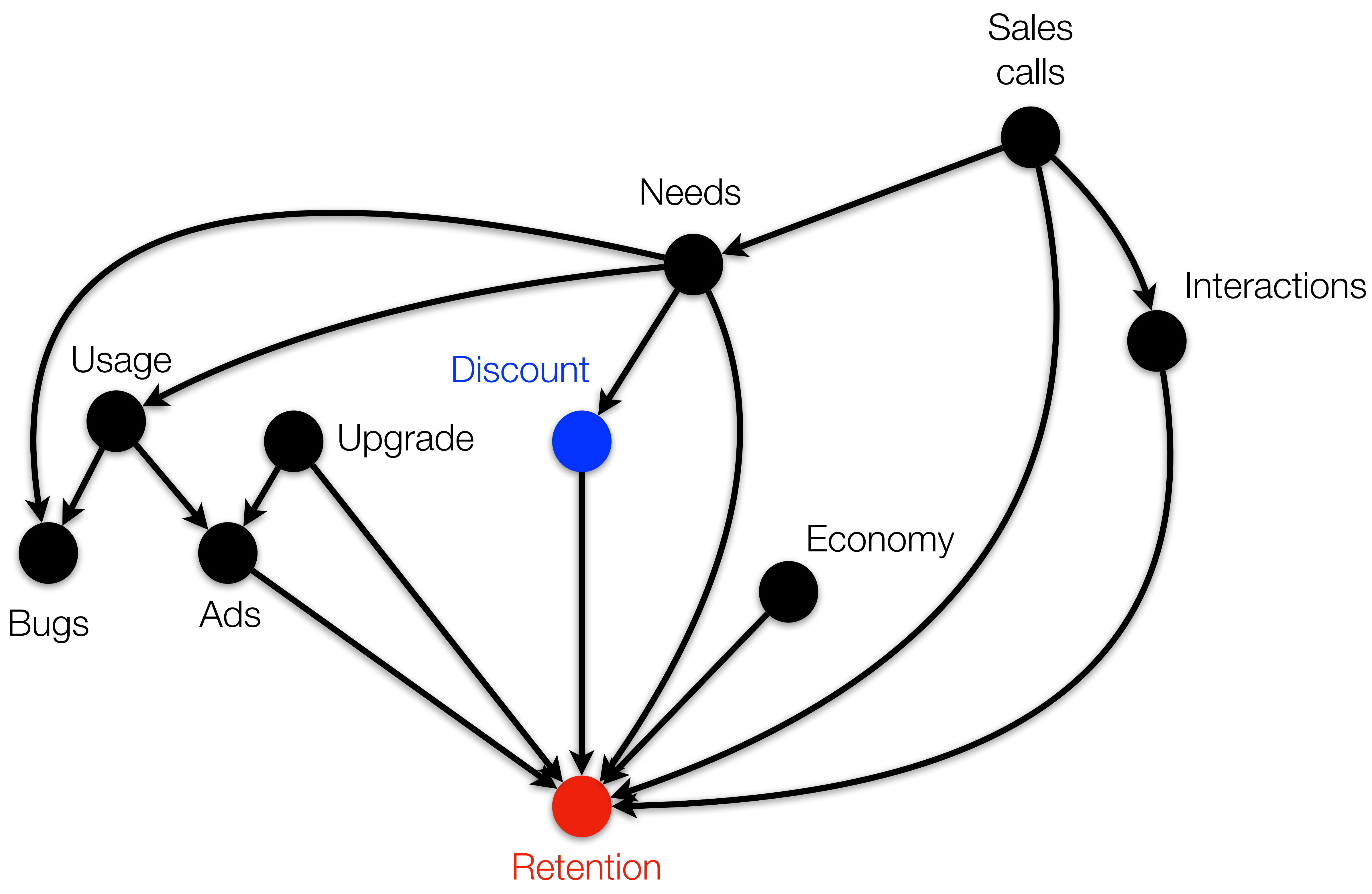

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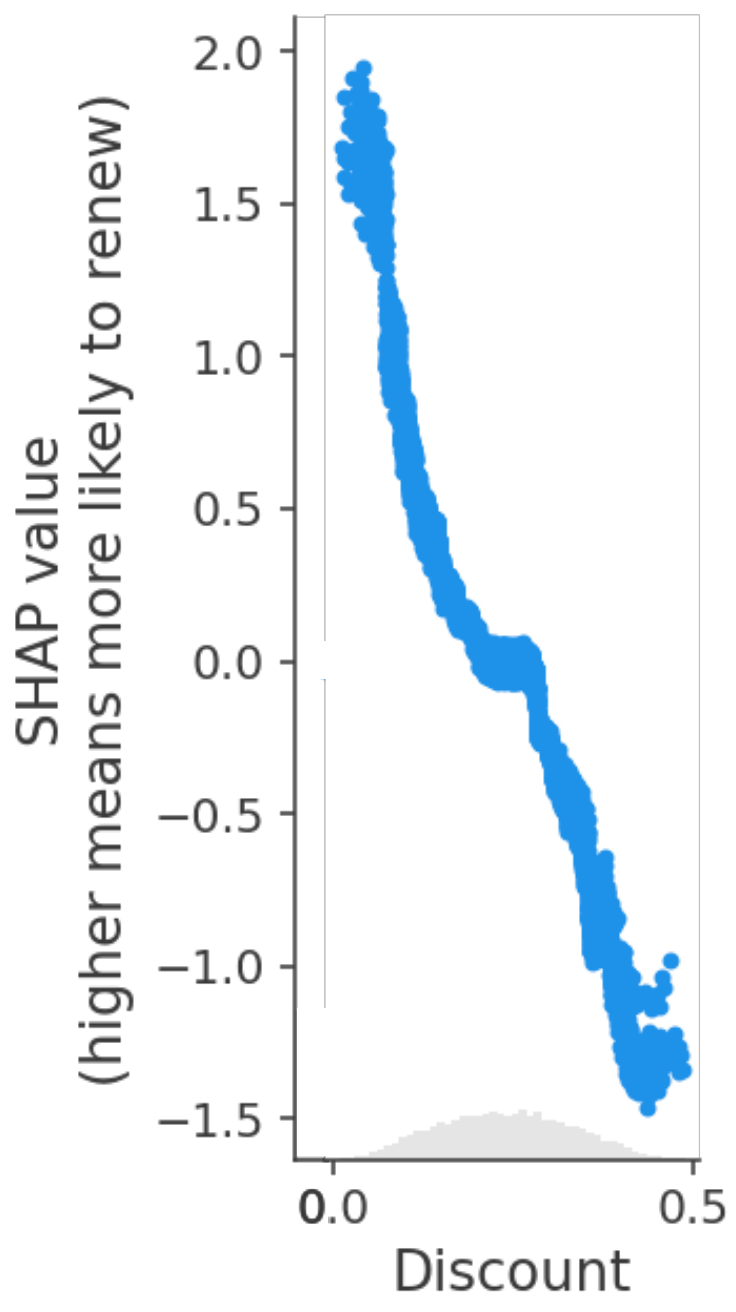

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Causality-based feature importance measure is essential

do-Shapley: Causality-based Feature Attribution

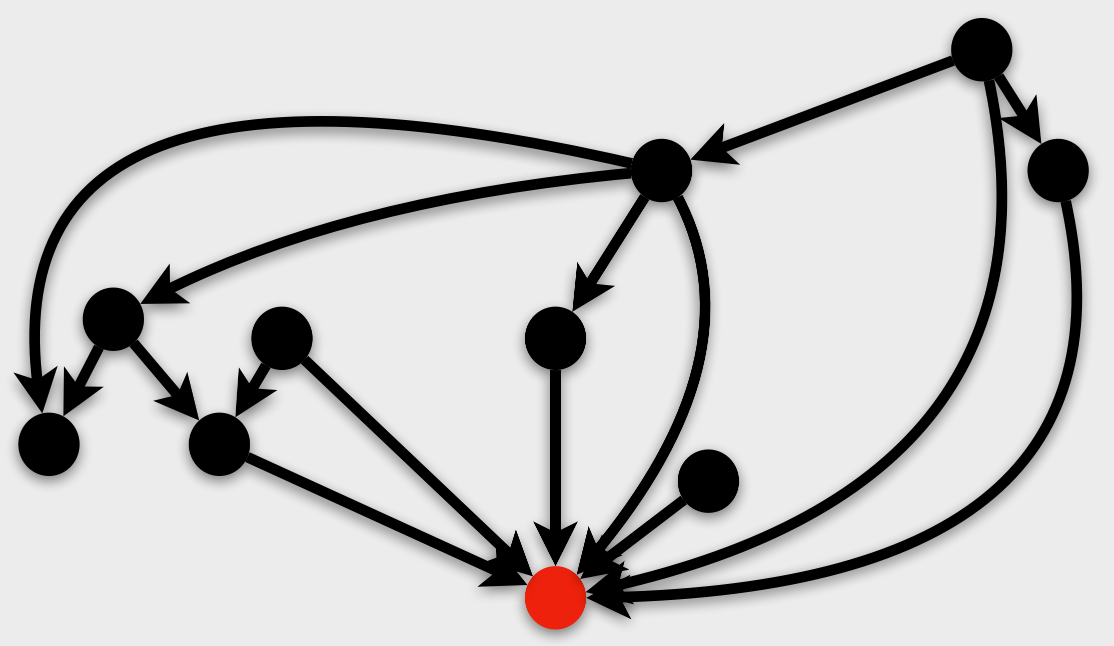
Jung et al.,ICML 2022

do-Shapley: Causality-based Feature Attribution

Jung et al.,ICML 2022

Input

Graph



Data

- Input: $(X_1, \cdots, X_m)$
- output: $f(X_1, \cdots, X_m)$

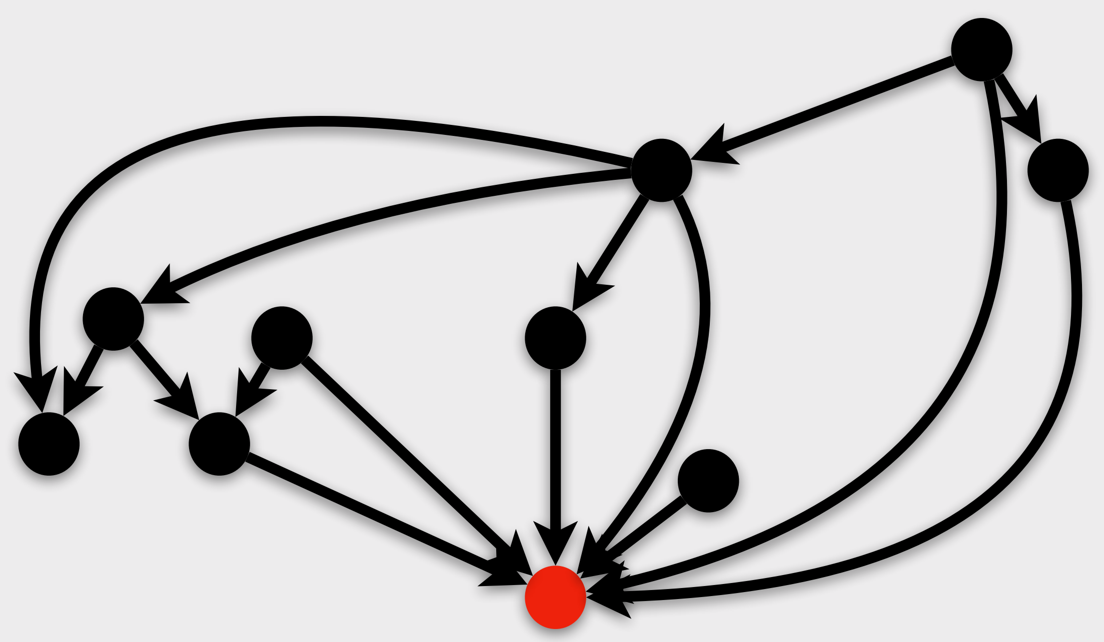
do-Shapley: Causality-based Feature Attribution

Jung et al.,ICML 2022

Input

Attribution

Graph



do-Shapley

$(\phi_1, \cdots, \phi_m)$

Causality-based feature attribution

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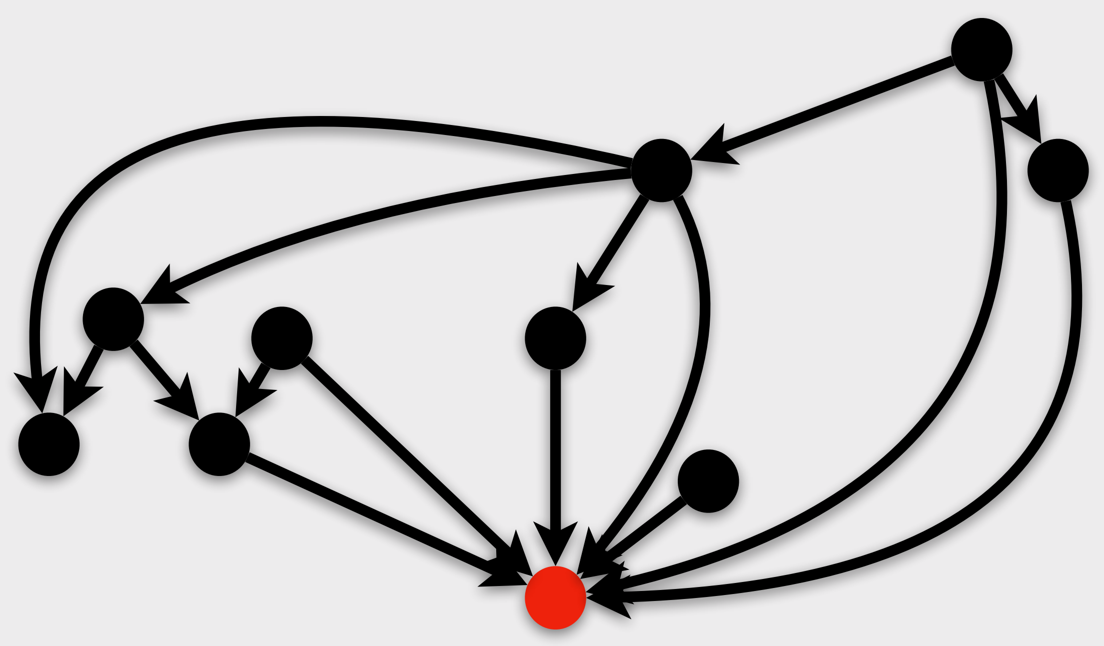

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Attribution

do-Shapley

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Causality-based feature attribution

$$\phi_i = \frac{1}{n}\sum_{S\subseteq[n]\setminus\{i\}} \binom{n-1}{|S|}^{-1} \{\mathbb{E}[Y\,|\,do(\mathbf{x}_S, x_i)] - \mathbb{E}[Y\,|\,do(\mathbf{x}_S)]\}$$

do-Shapley: Causality-based Feature Attribution

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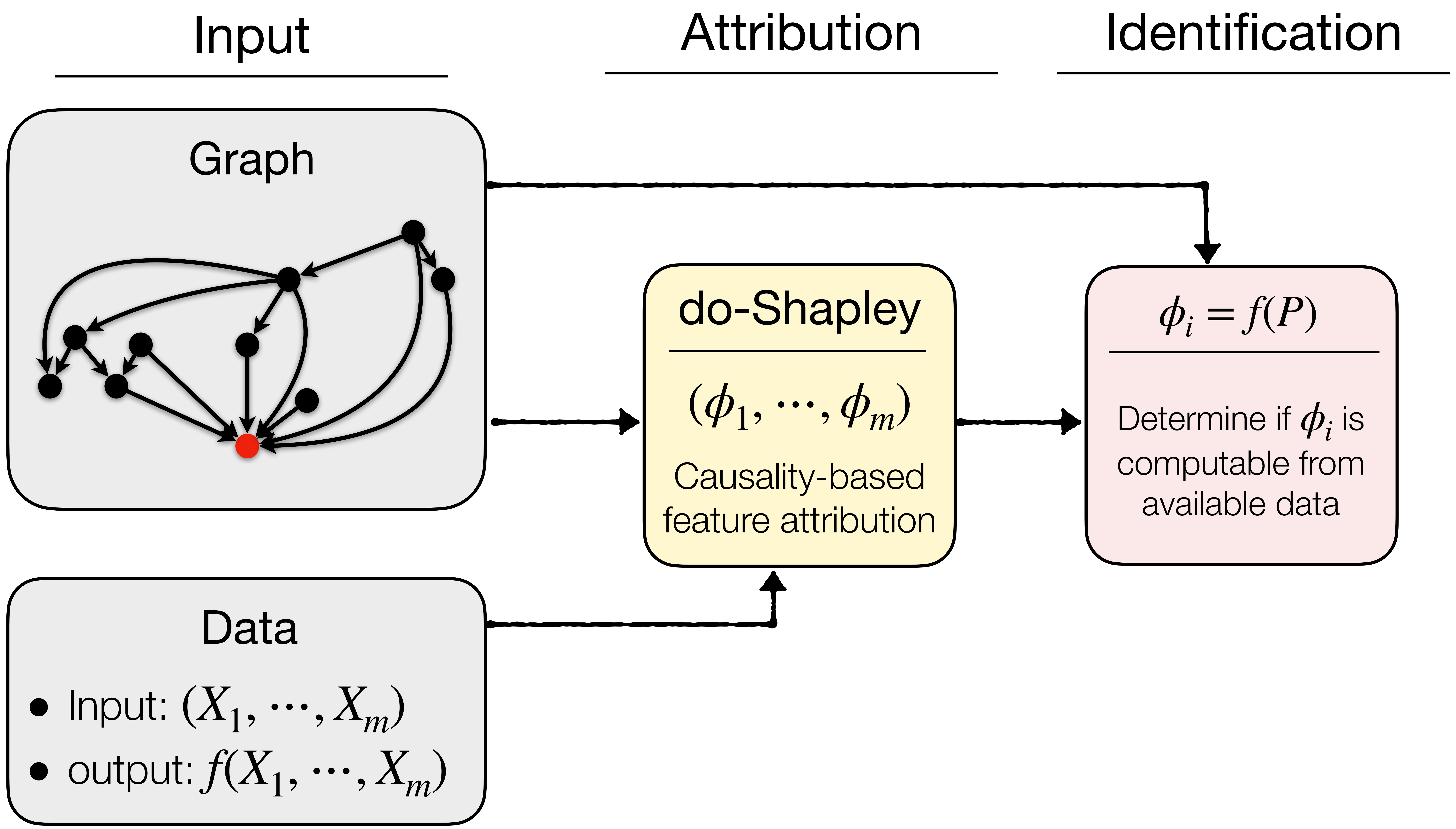

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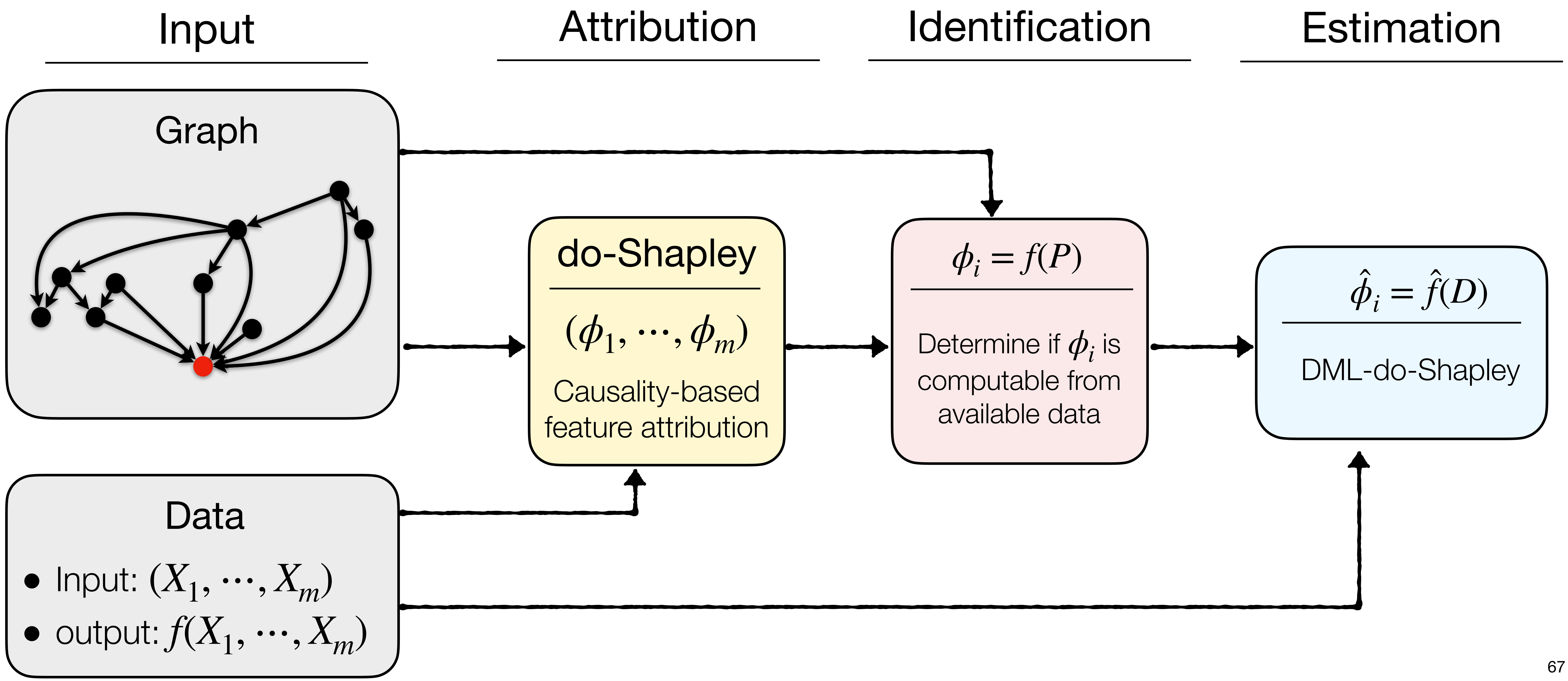

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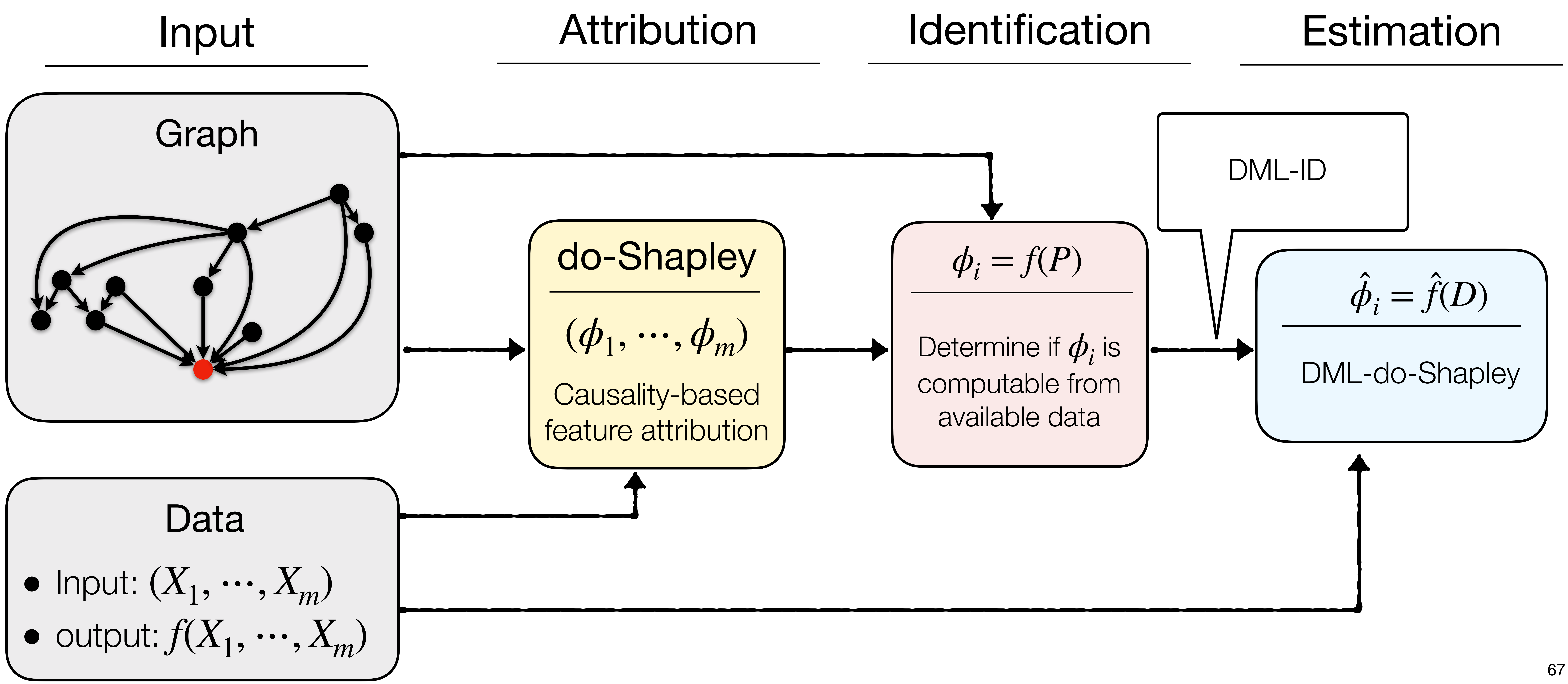

Simulation: Better Interpretability

Estimator	Rank Correlation with True Importances	Implication
DML-do-Shapley	**1.0**	
SHAP	**-0.28**	

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Impact on Explainable AI

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Unique causality-based feature importance measure that aligns with human intuition:

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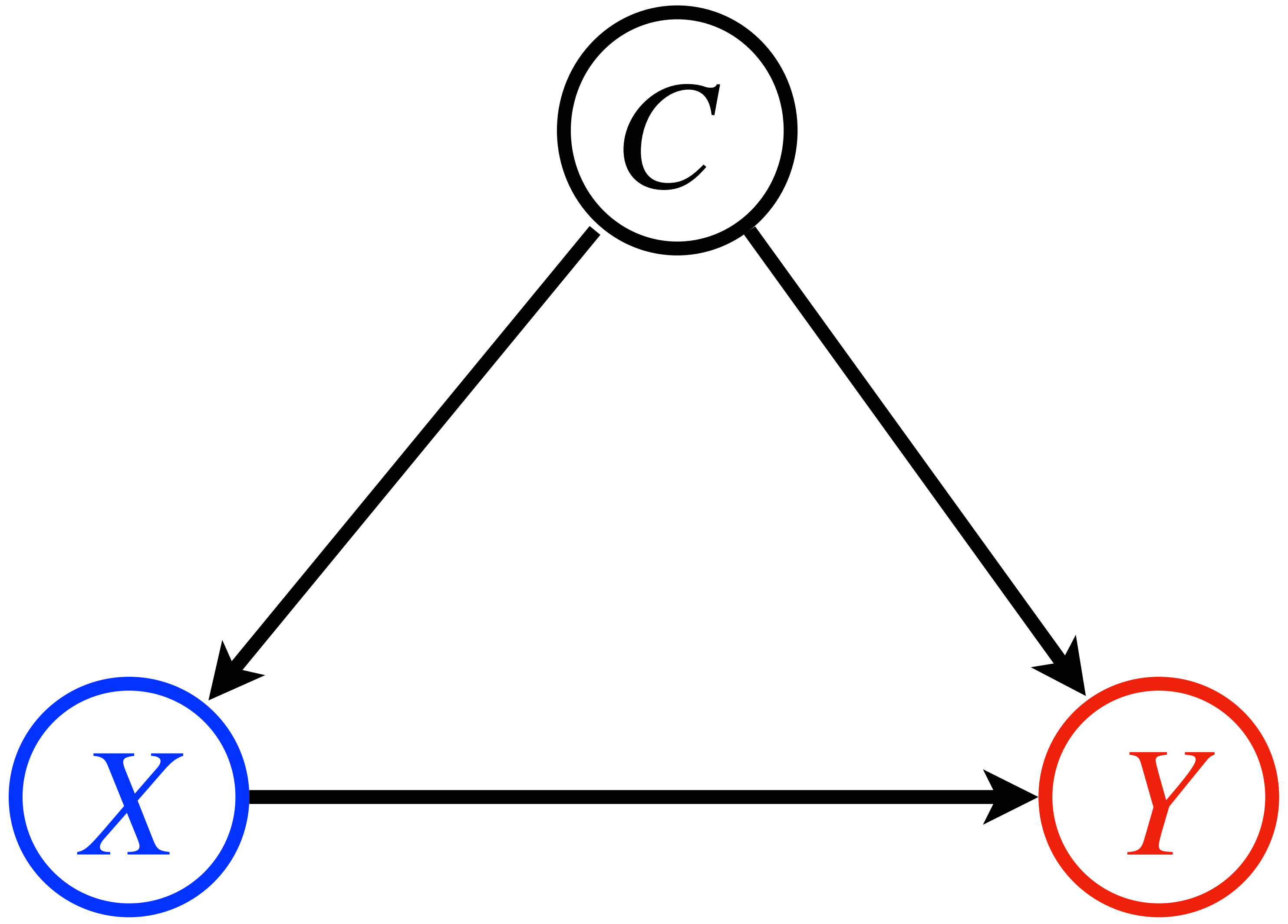
- Feature contributions closely approximate their causal effects on the outcome
- The sum of feature contributions = The outcome $f(X_1, \cdots, X_m)$

Future Direction

Future 1: Inference with Multi-modal Data



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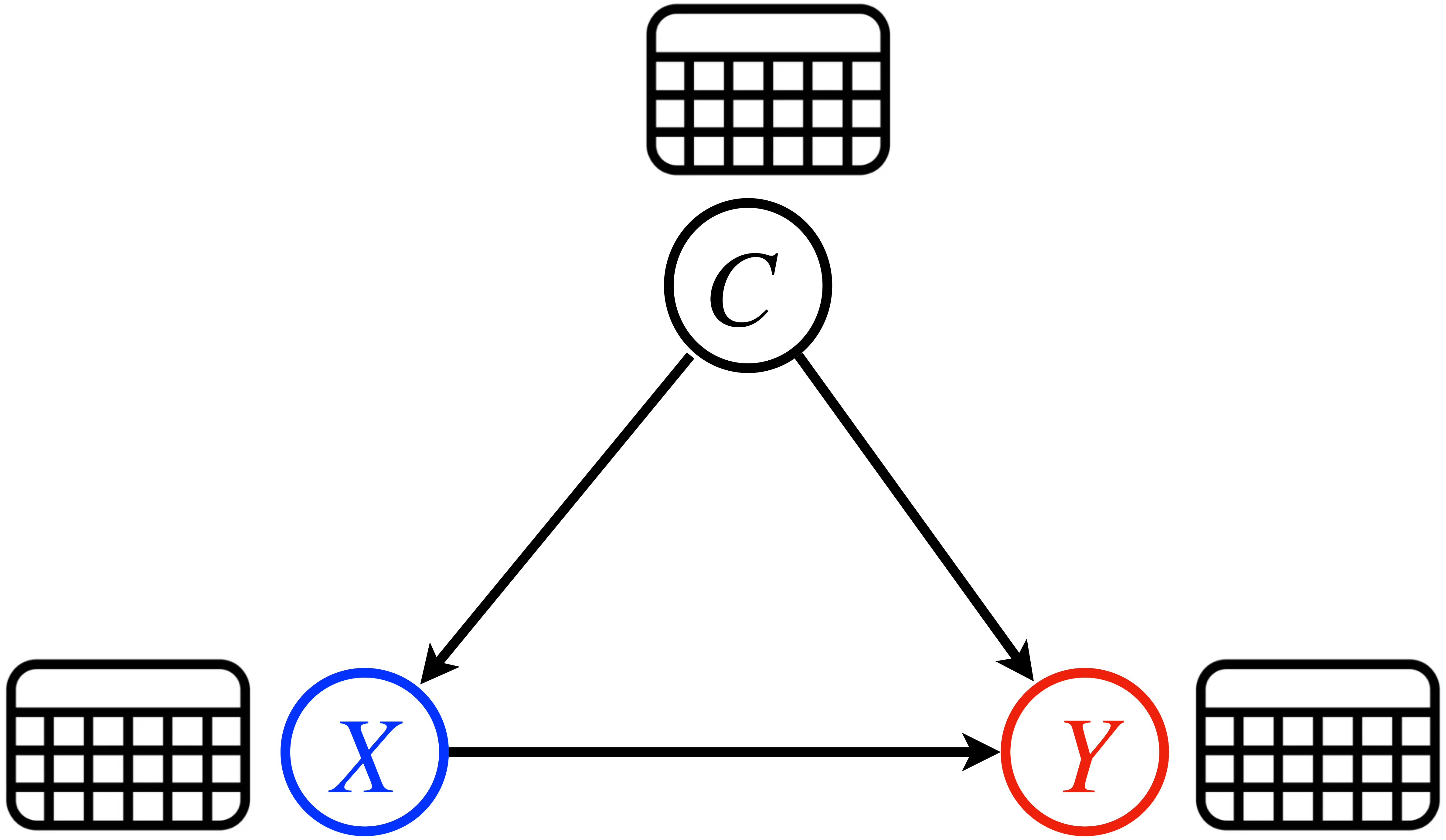

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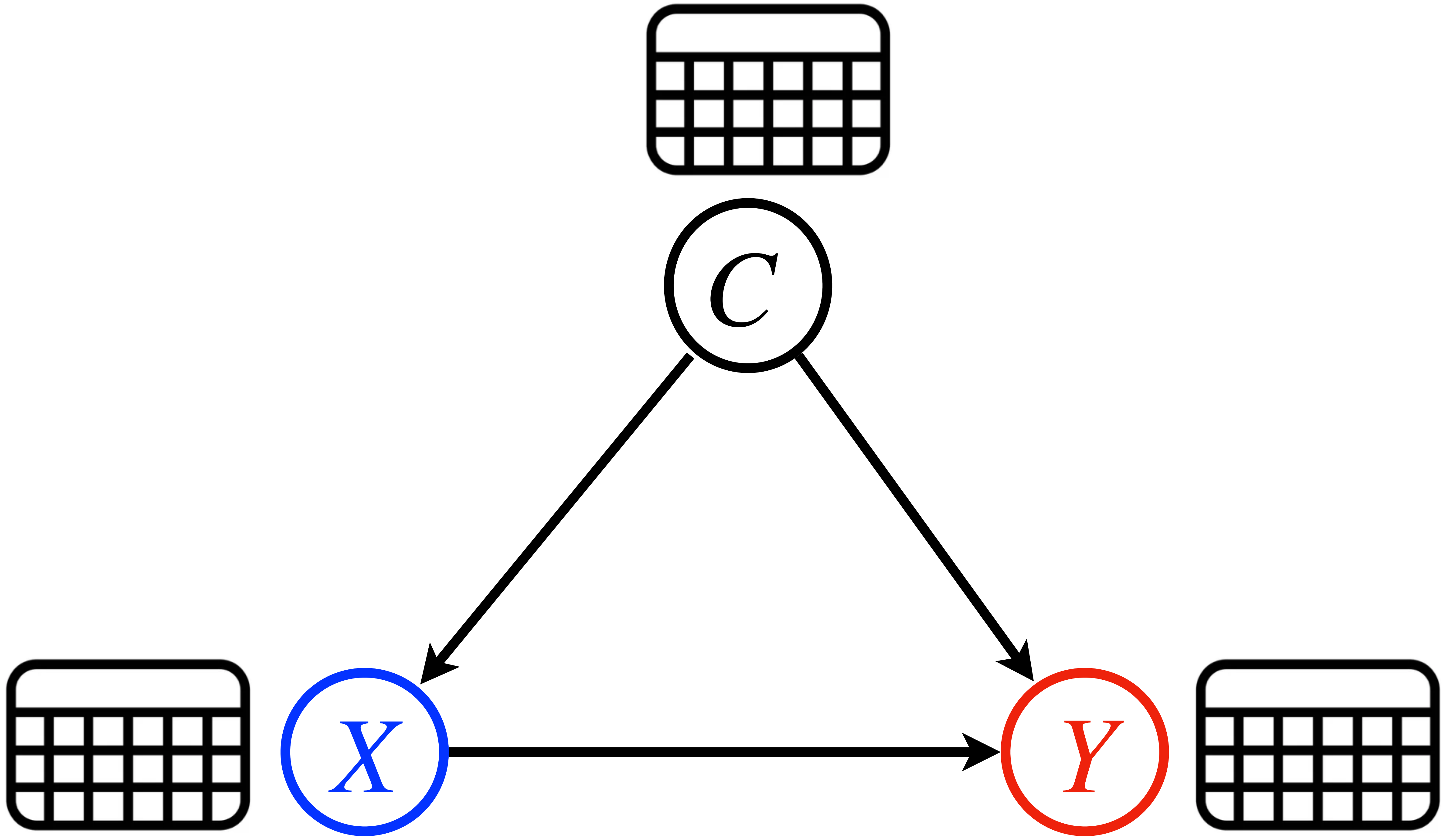

$$\mathbb{E}[Y \mid \text{do}(x)] = \sum_c \mathbb{E}[Y \mid x, c]\, P(c)$$

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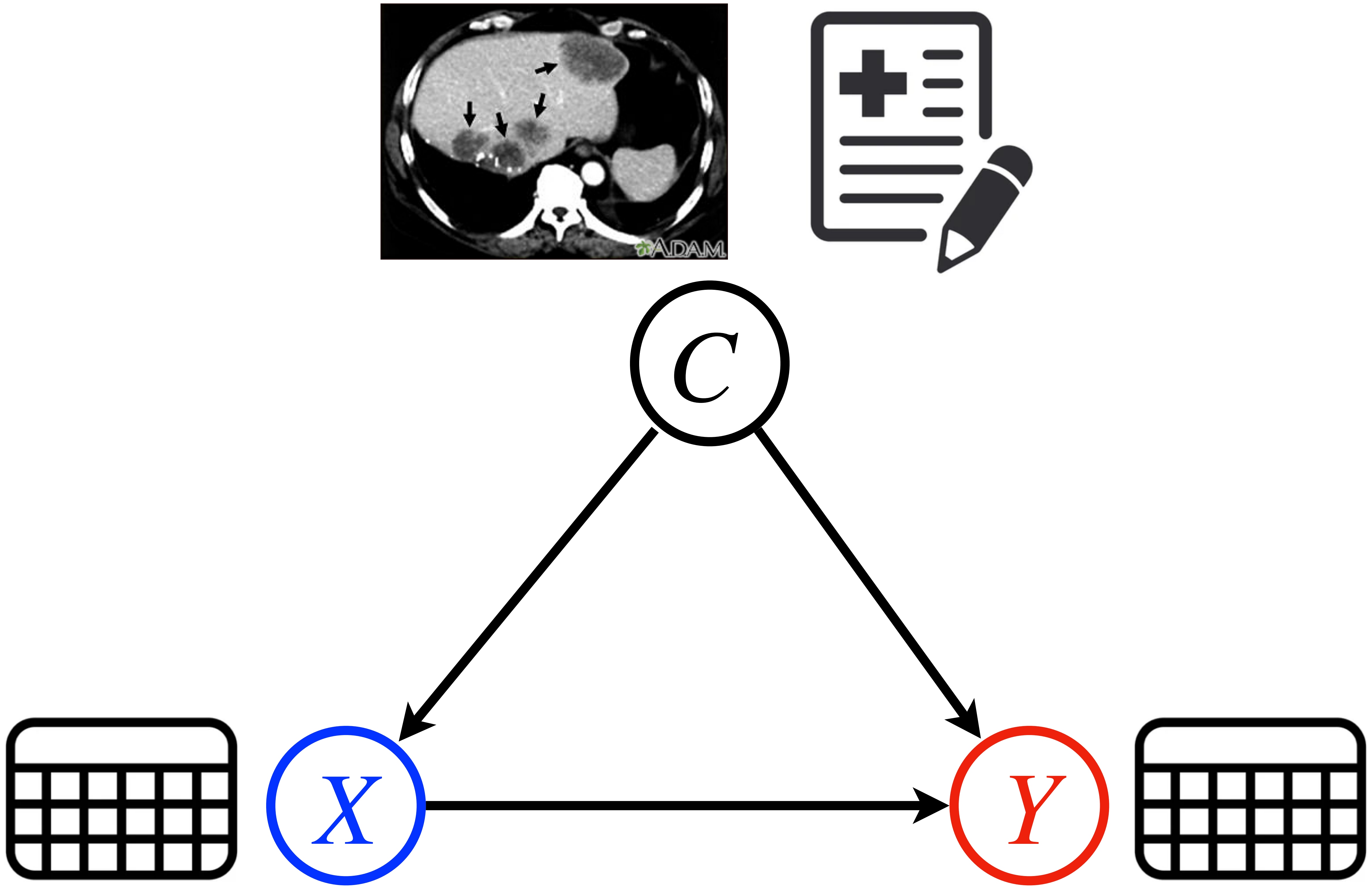

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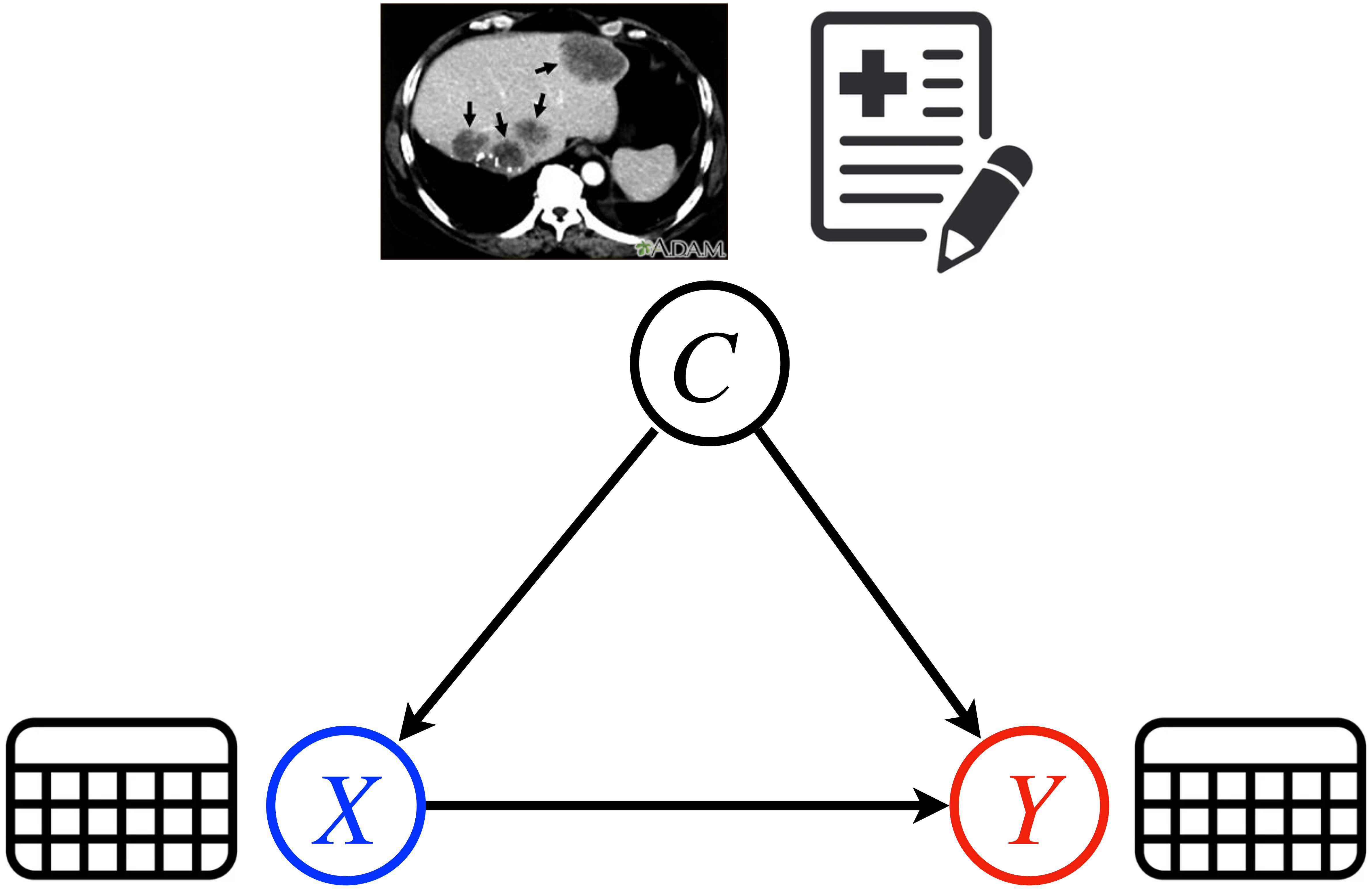

$$\mathbb{E}[Y \mid \mathrm{do}(x)] = \quad ?$$

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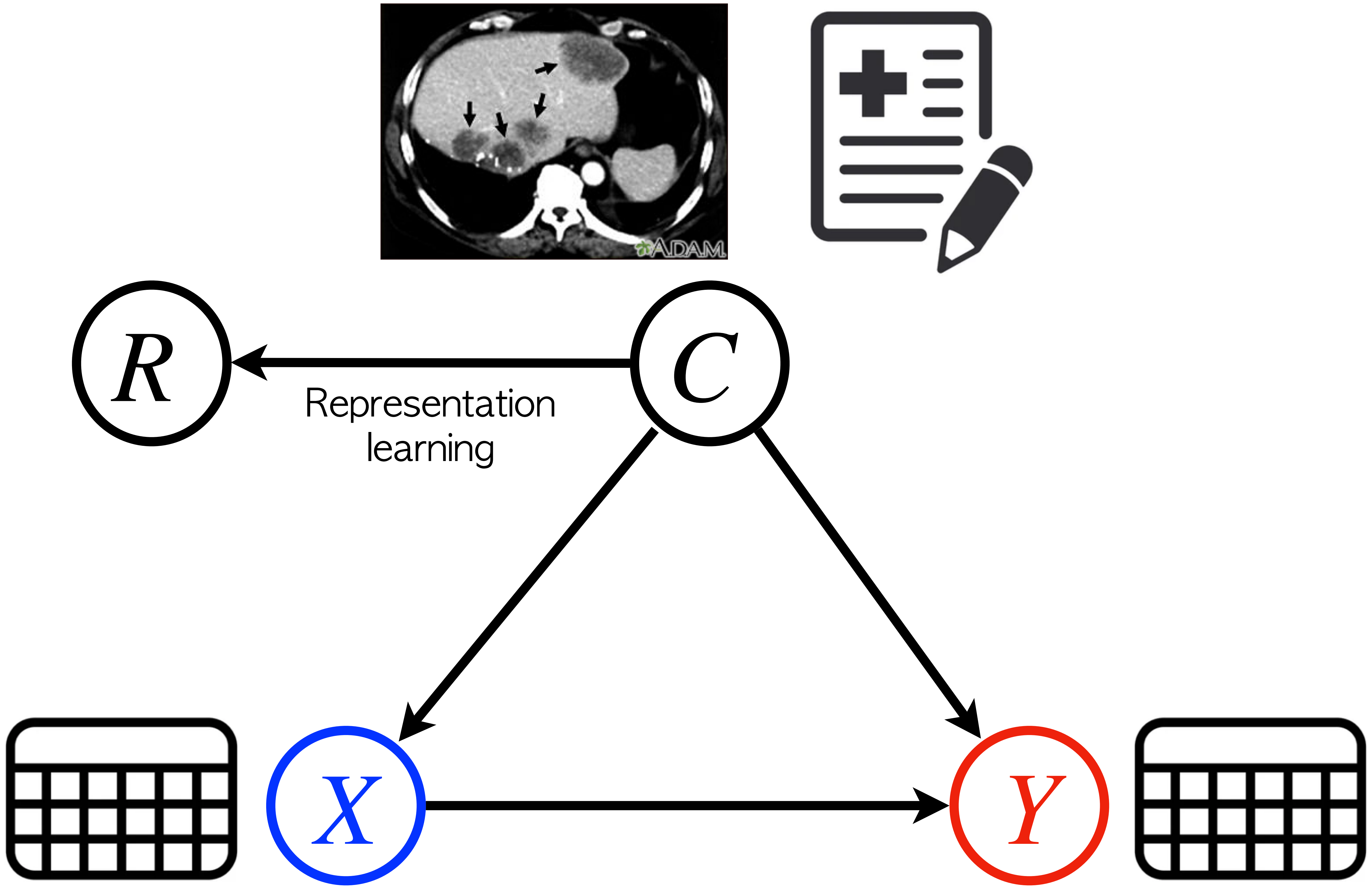

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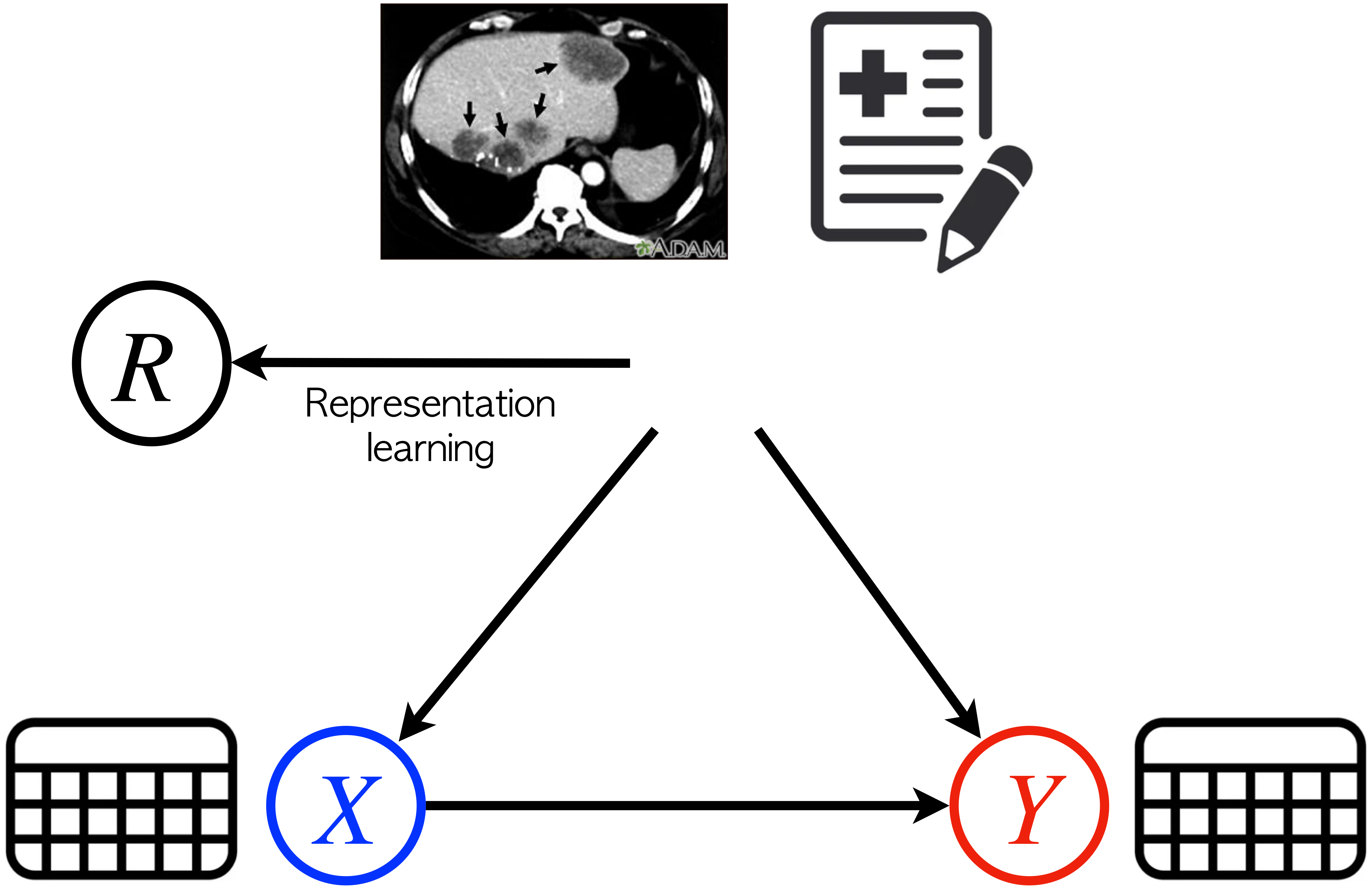

$$\mathbb{E}[Y \mid do(x)] = \sum_r \mathbb{E}[Y \mid x, r] \; P(r)$$

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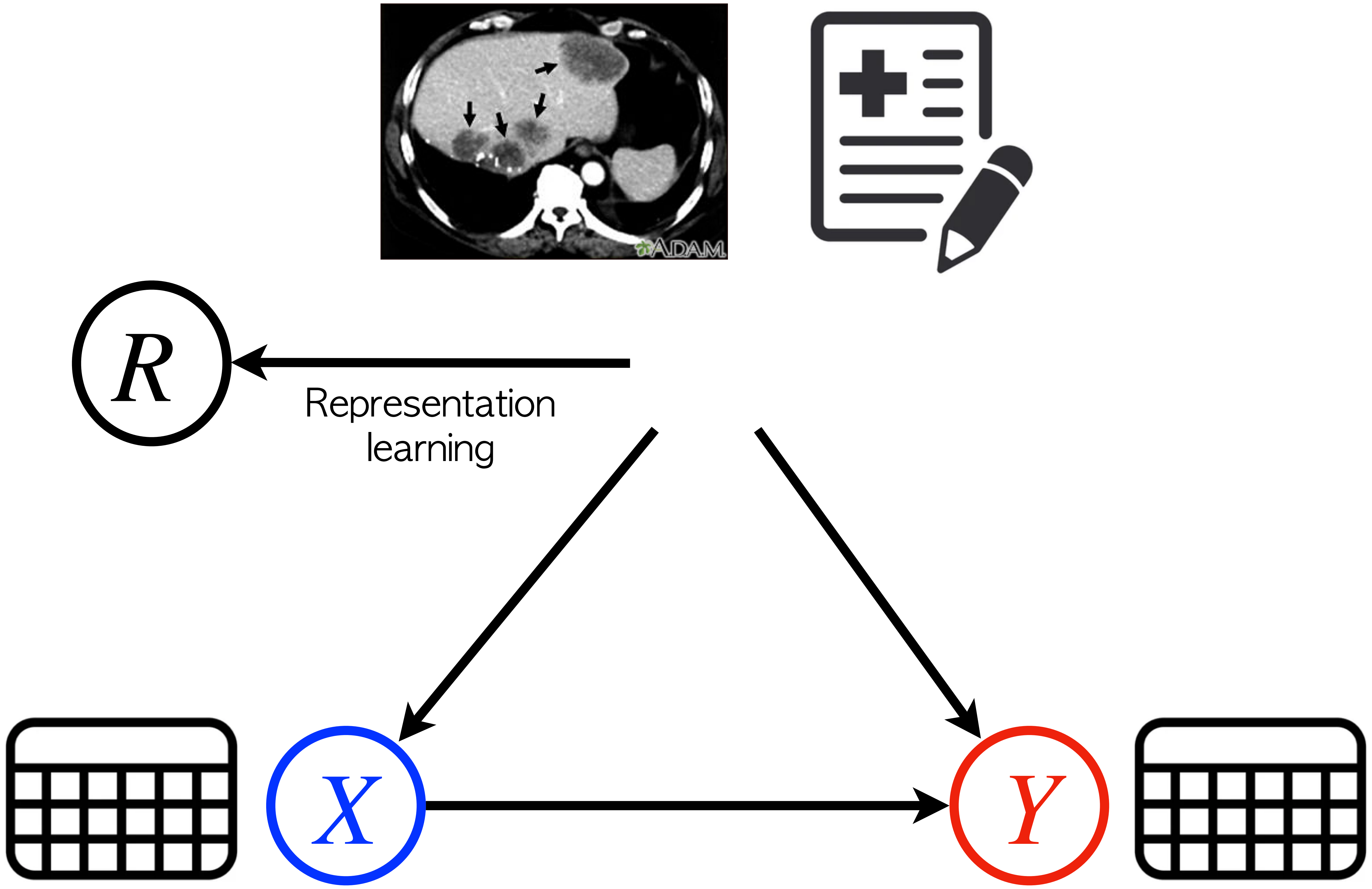

$$\mathbb{E}[Y \mid \mathrm{do}(x)] \neq \sum_r \mathbb{E}[Y \mid x, r]\; P(r)$$

R doesn't satisfy the BD criterion

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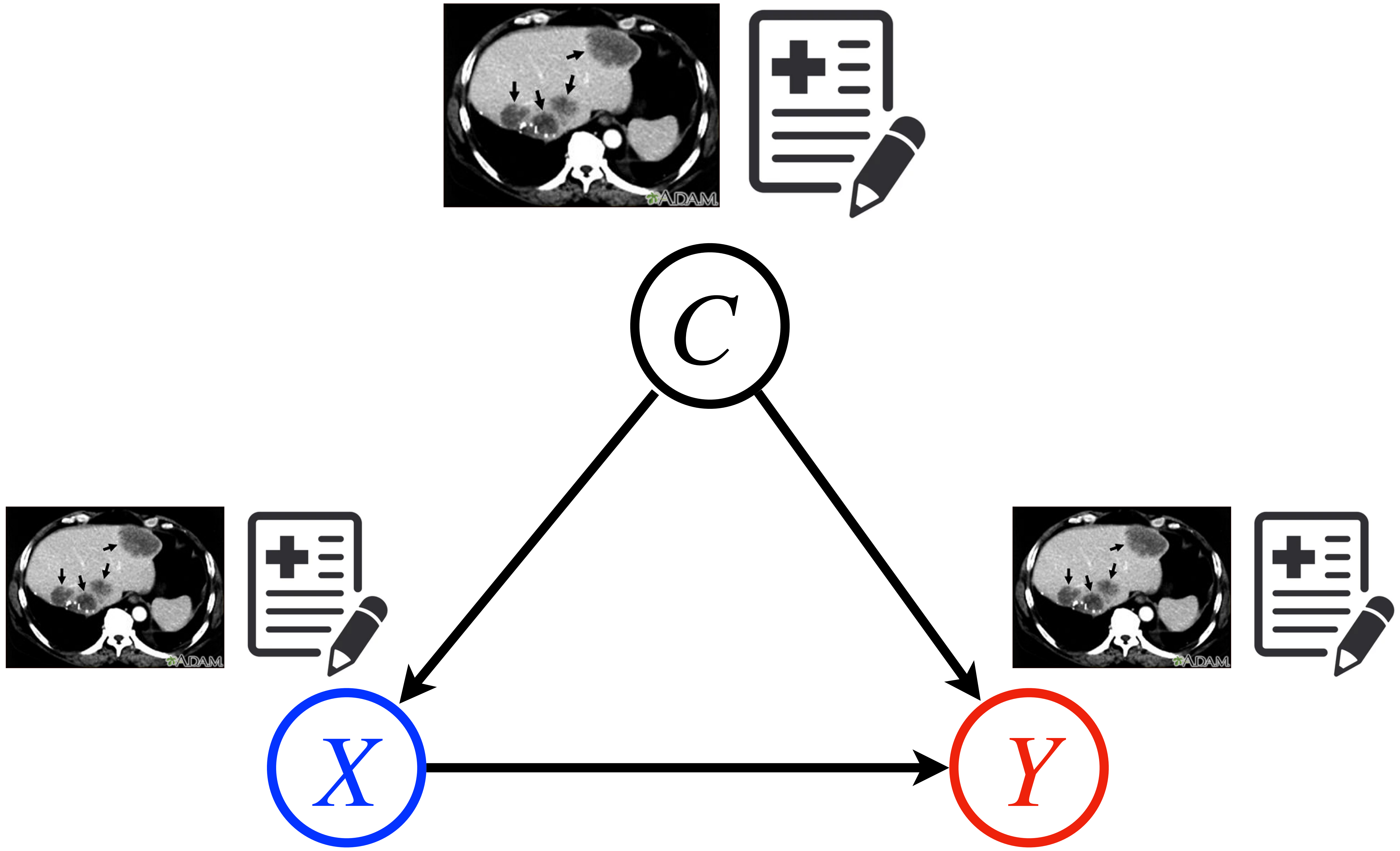

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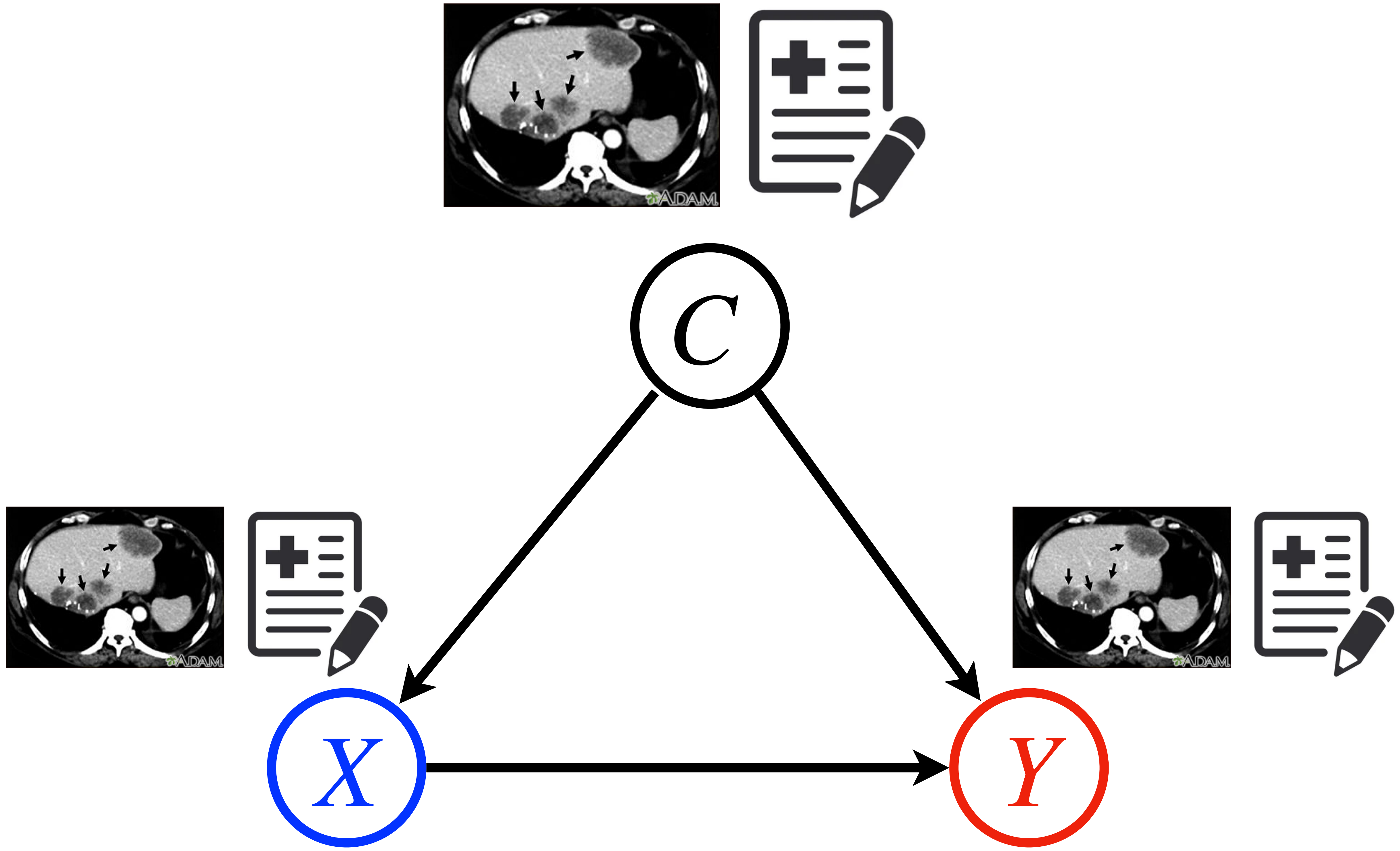

$$\mathbb{E}[Y \mid do(x)] = \text{?}$$

Approach

- Representation learning taking account of causal dependencies
- New causal inference methods that allows us to use existing representation learning models

Future 2: Causal Inference with Spatiotemporal Data

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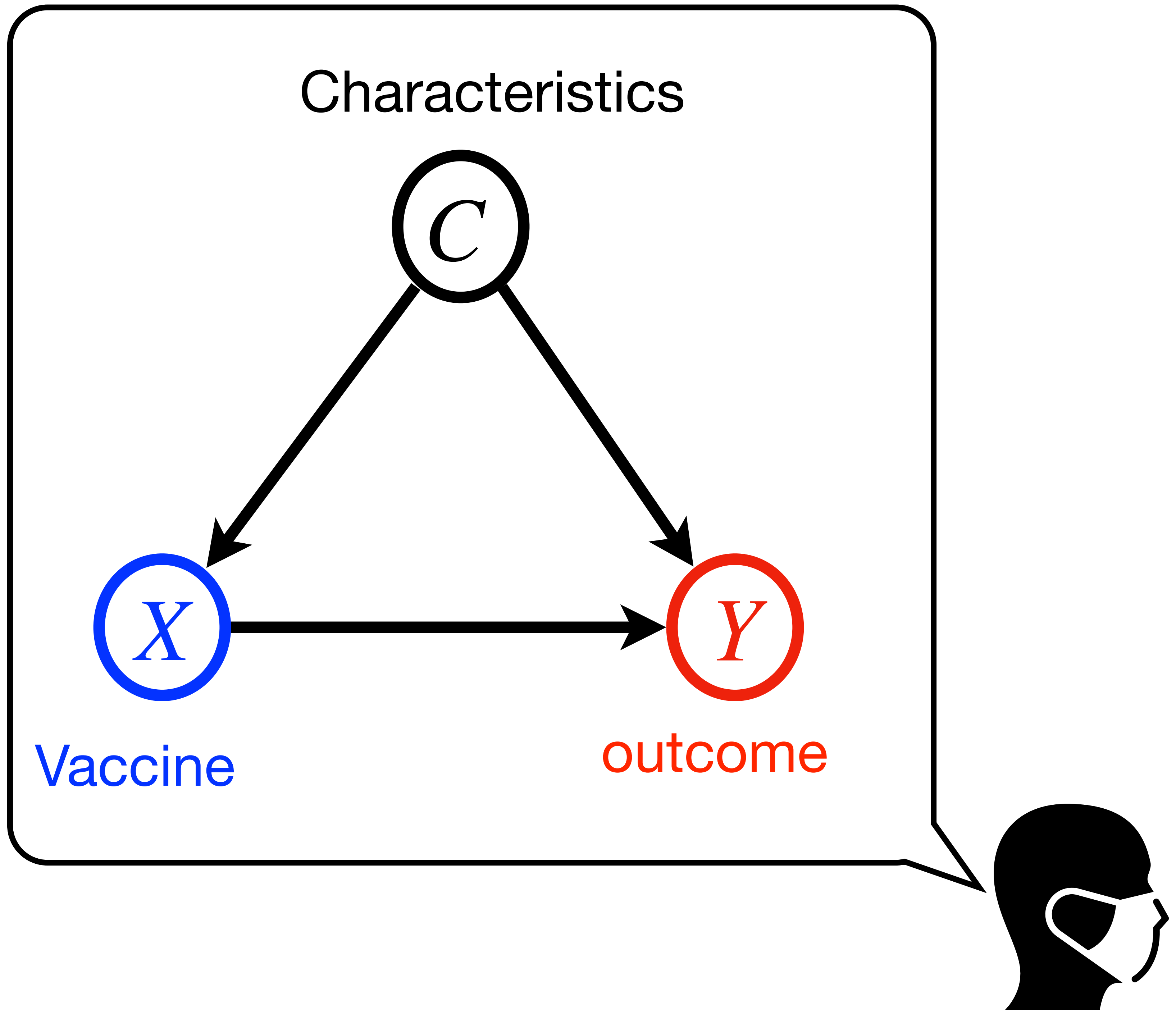

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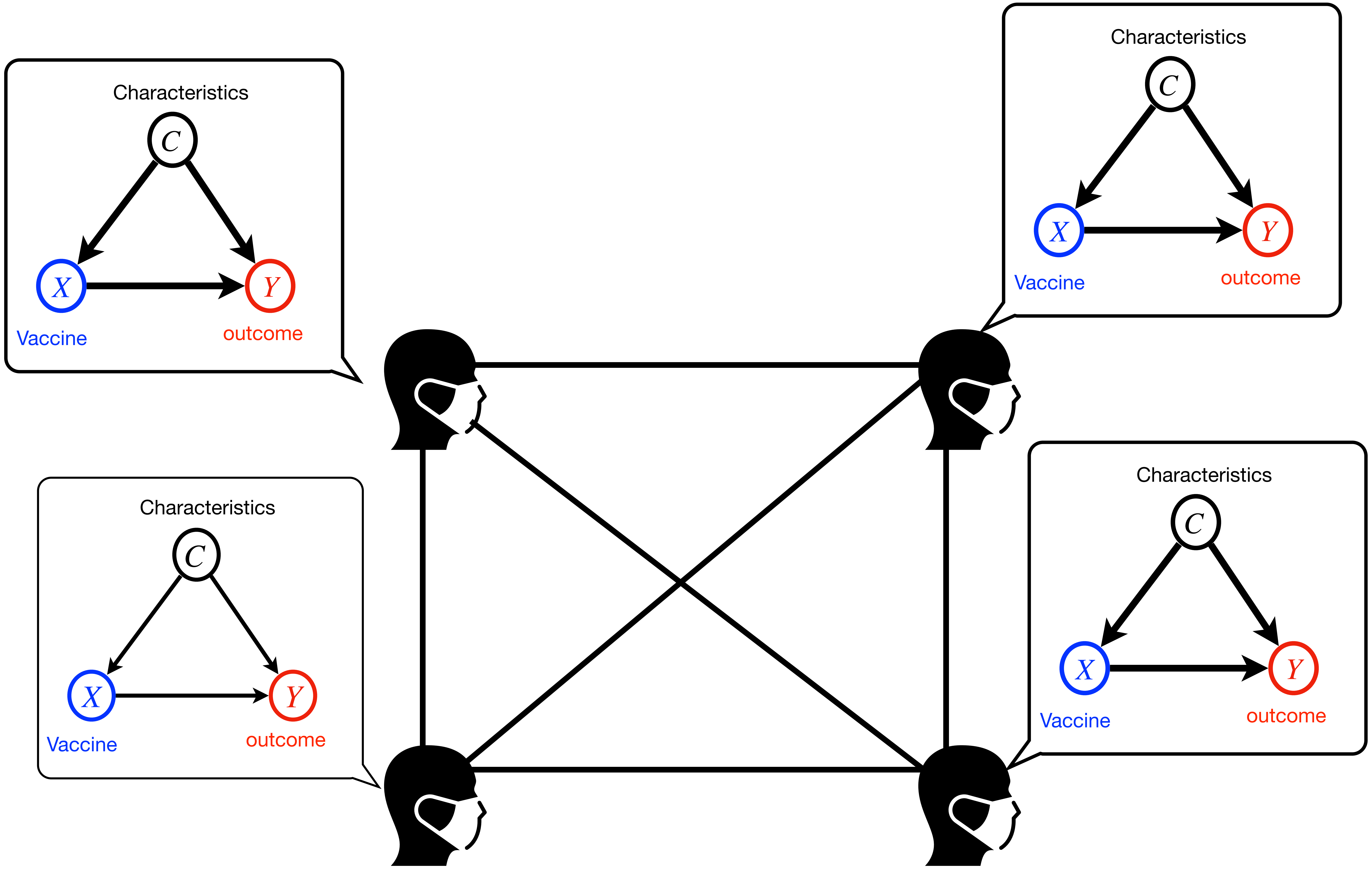

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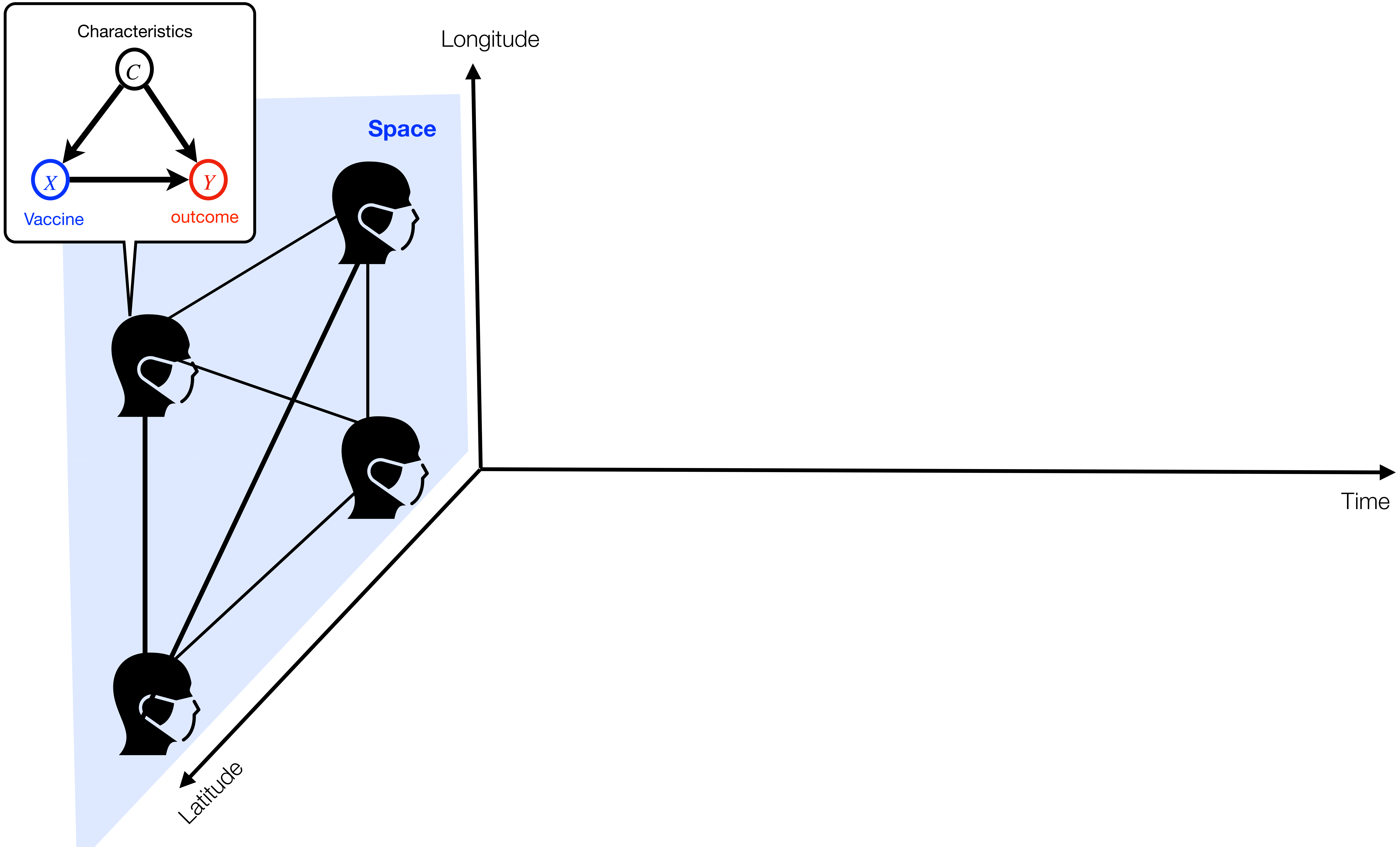

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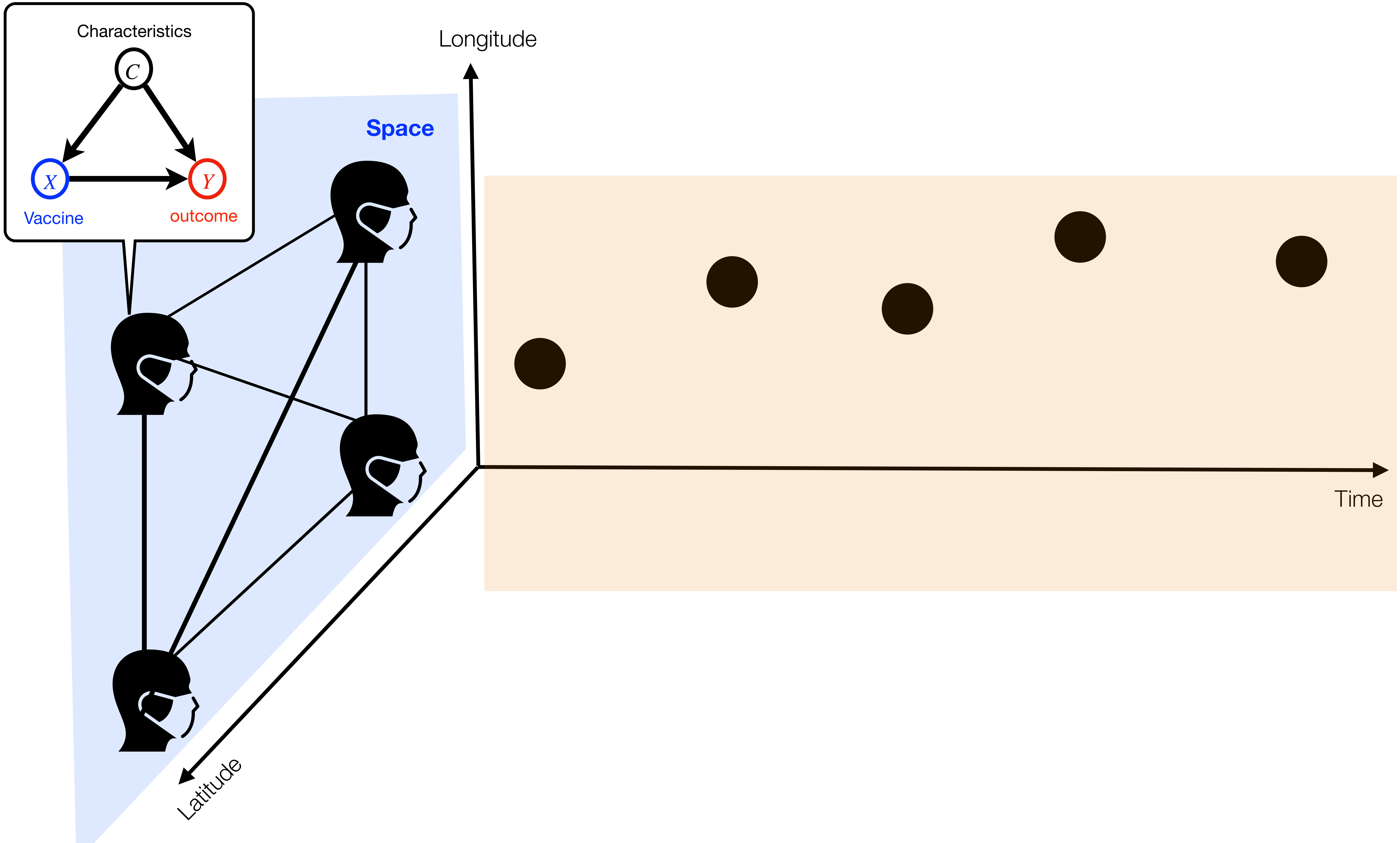

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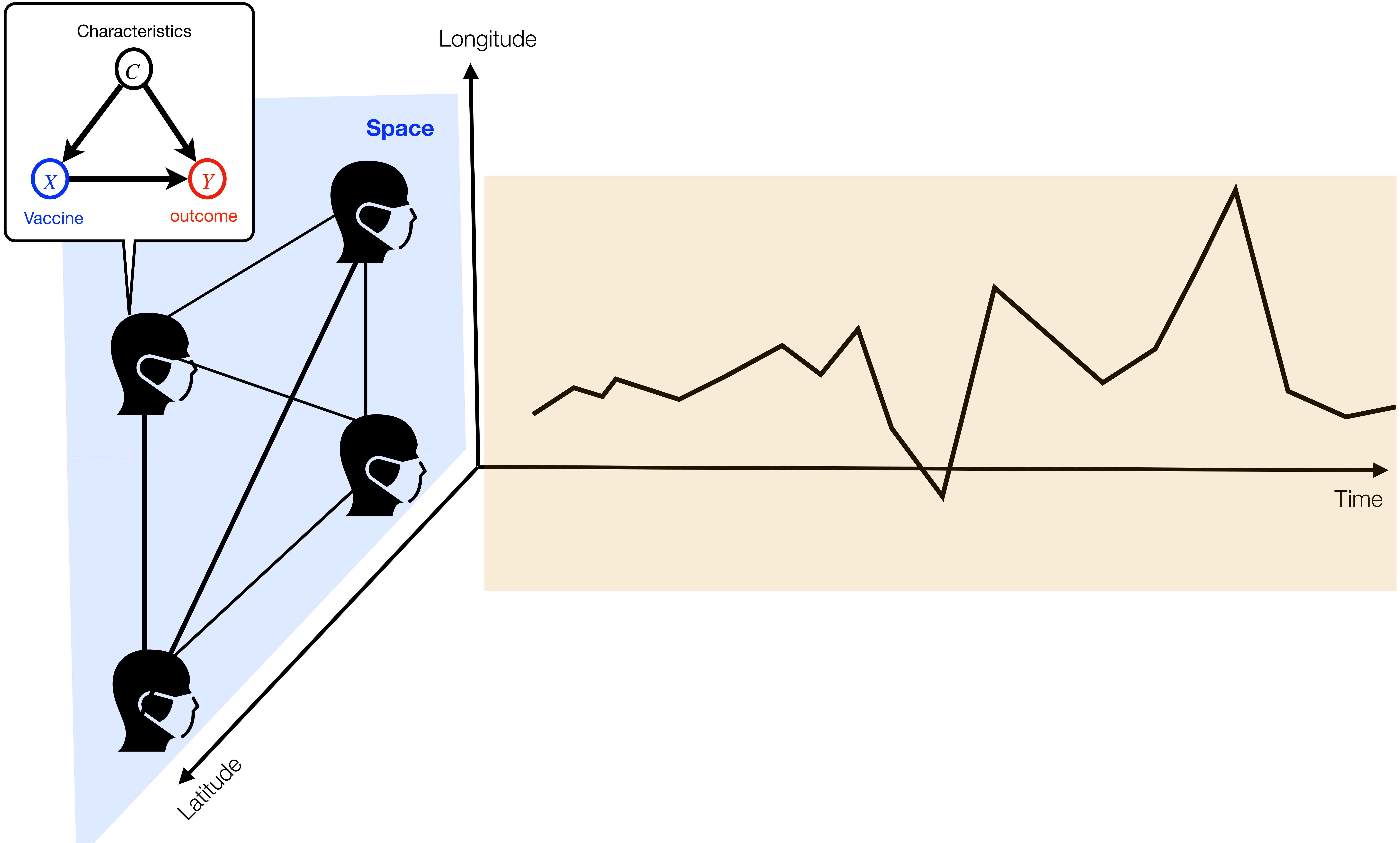

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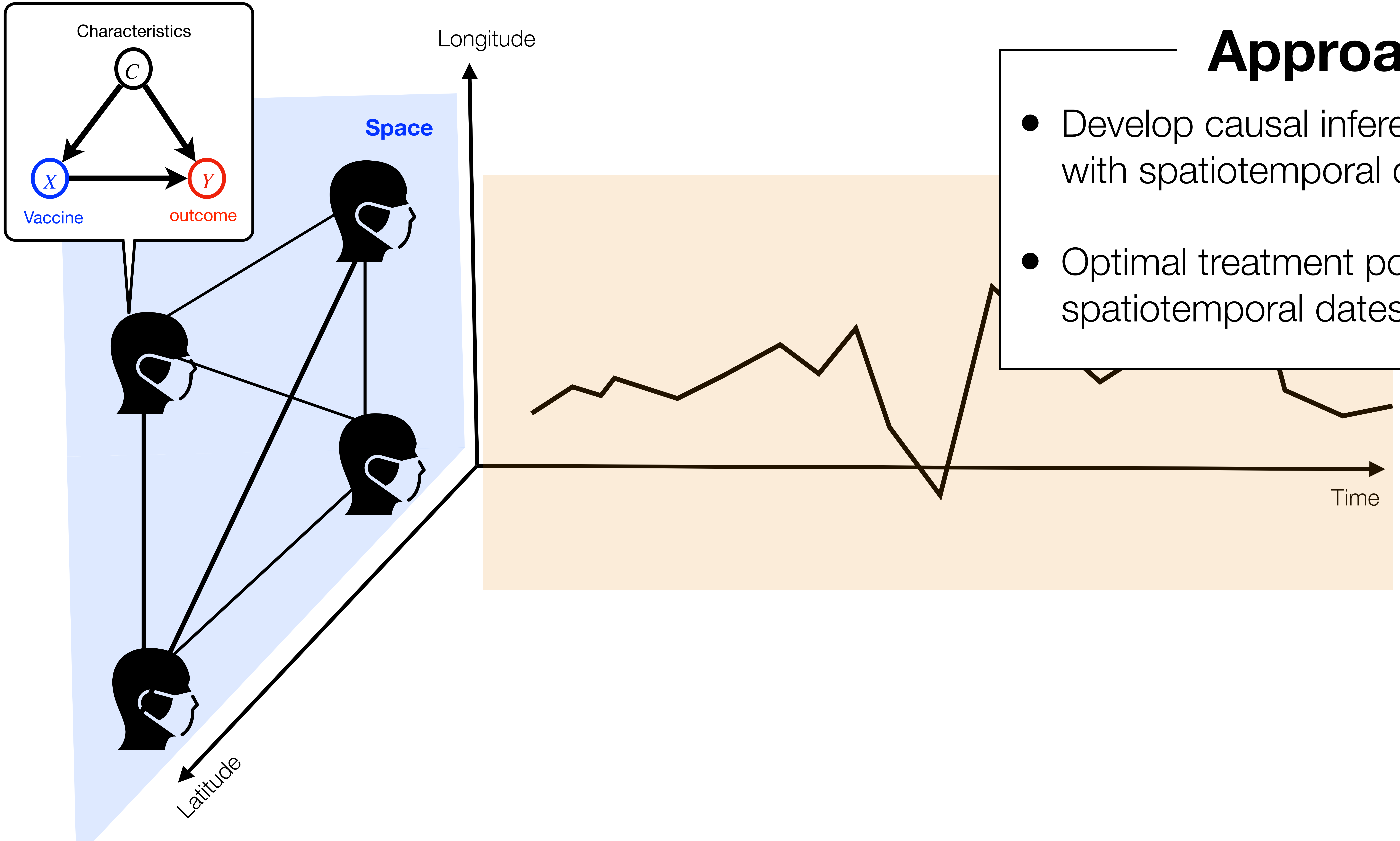

Approach

- Develop causal inference methods with spatiotemporal dataset
- Optimal treatment policy with spatiotemporal dates

Future 3: Causal Inference Loop with Uncertainty

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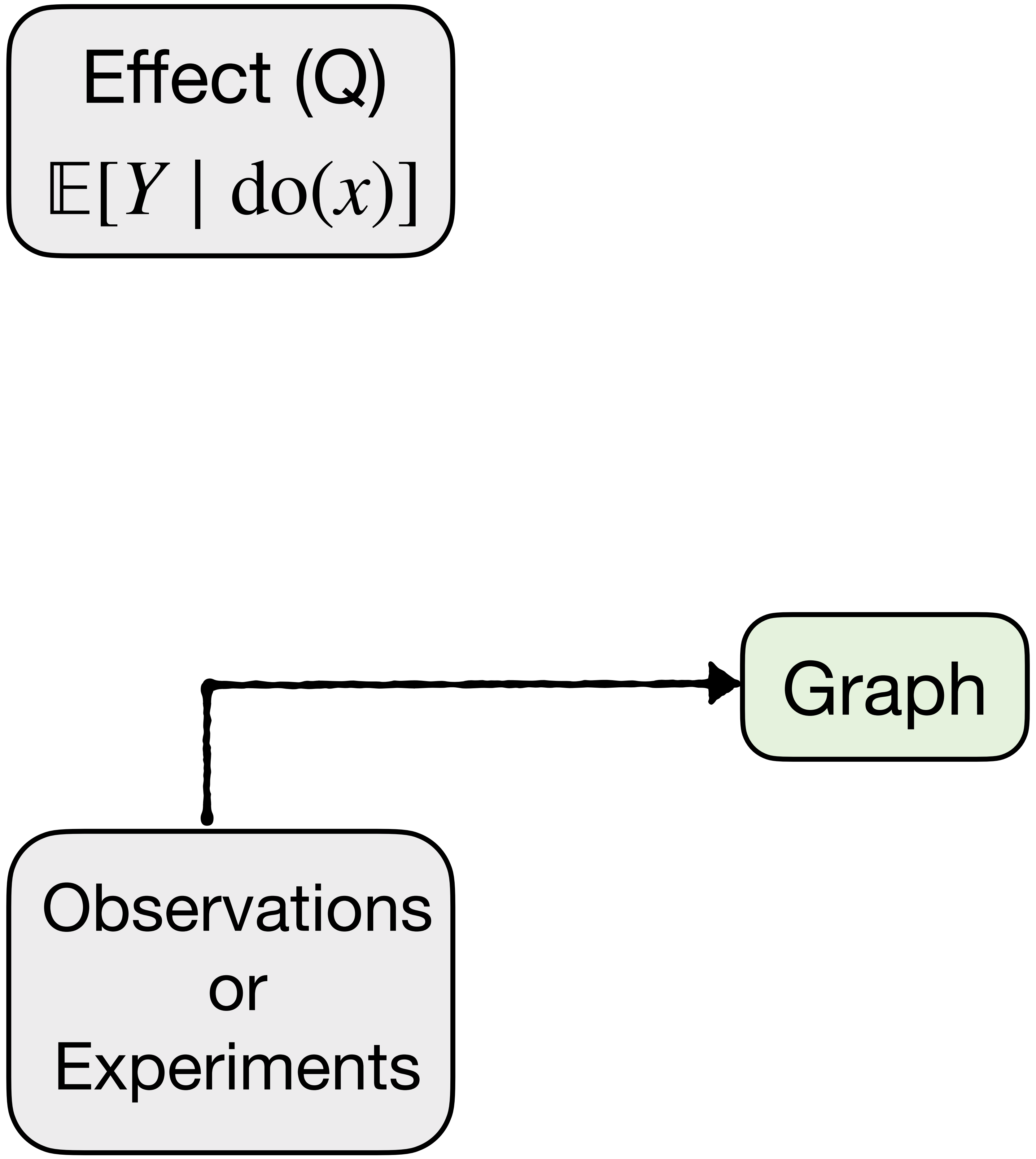
Effect (Q)

$\mathbb{E}[Y \mid \mathrm{do}(x)]$

Observations
or
Experiments

Future 3: Causal Inference Loop with Uncertainty



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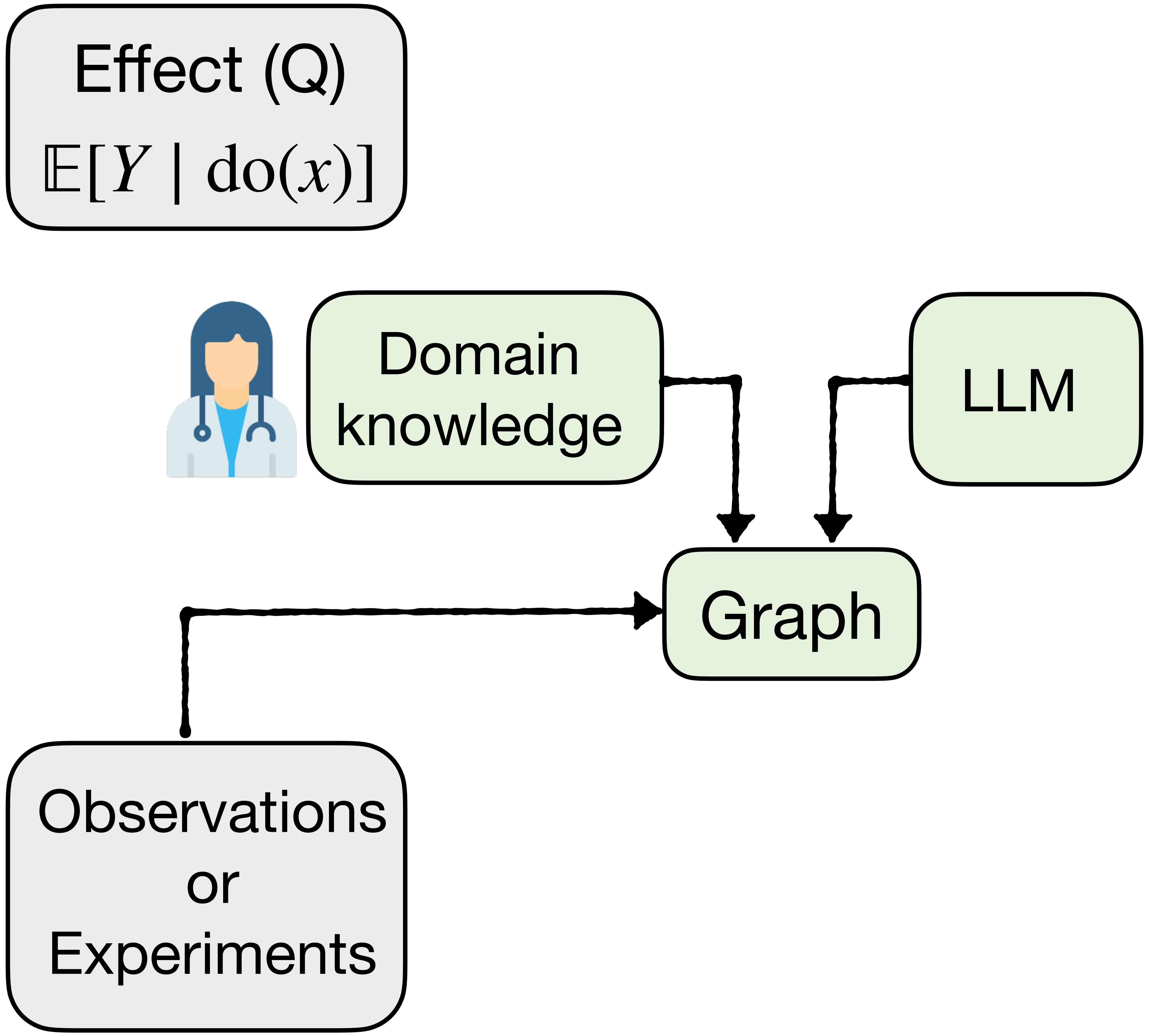
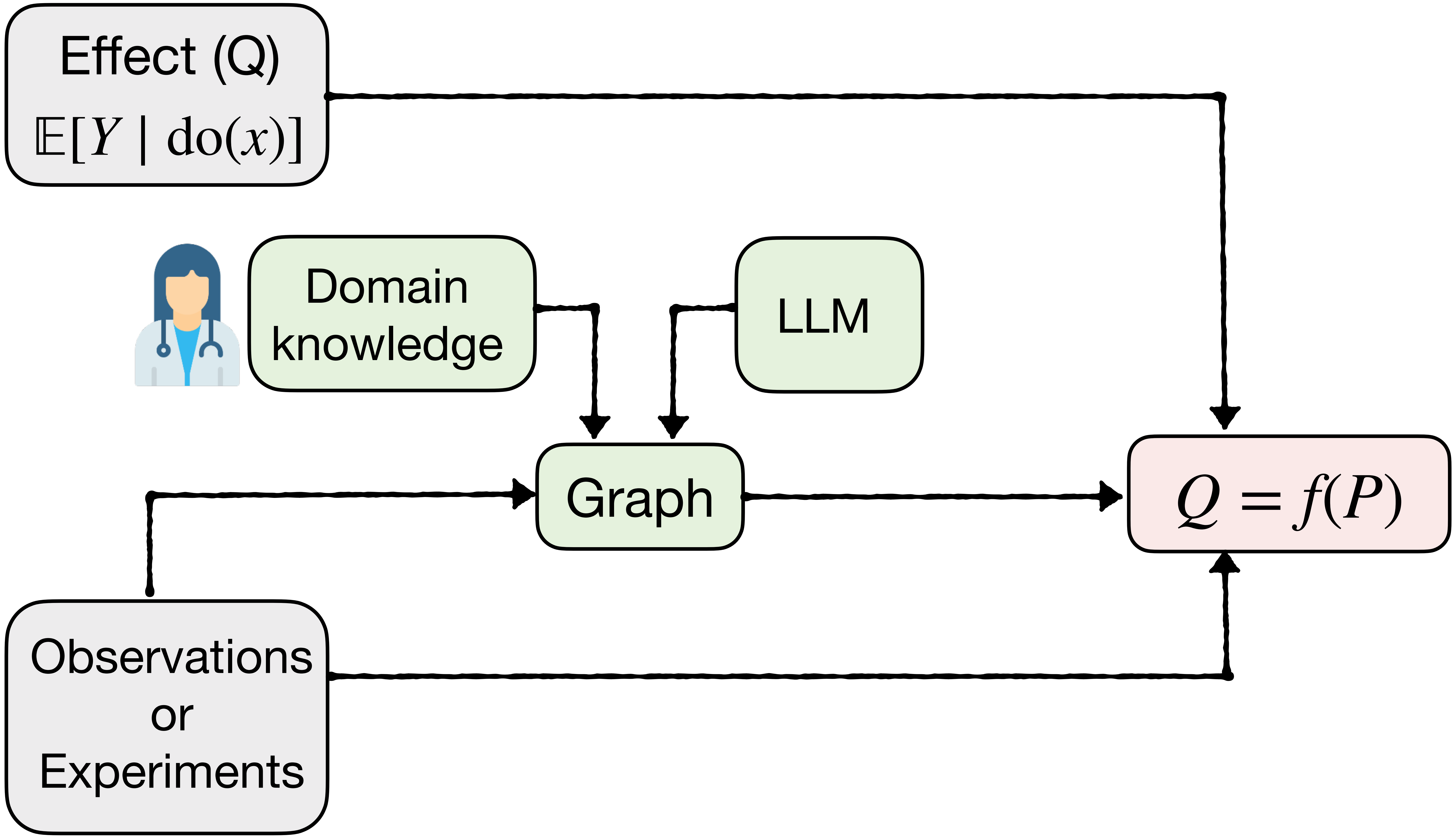
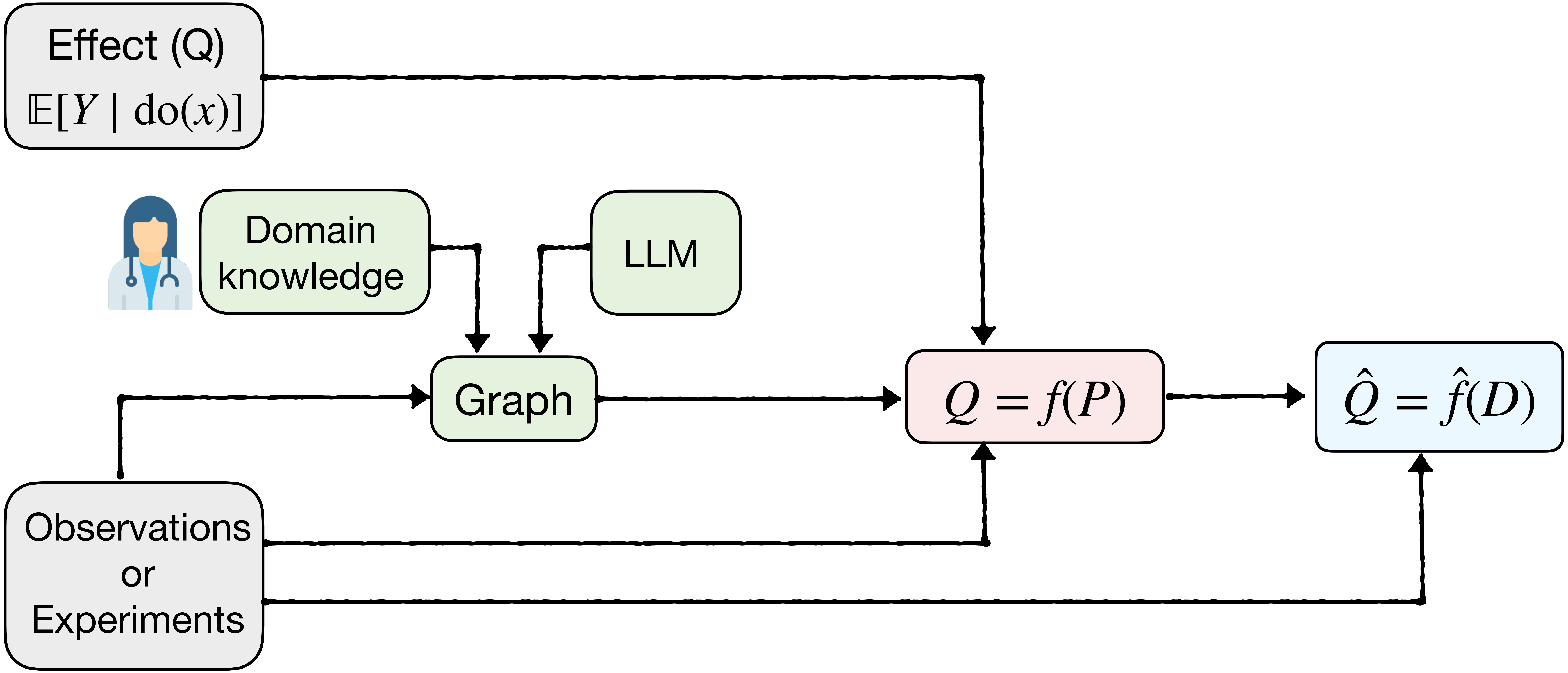


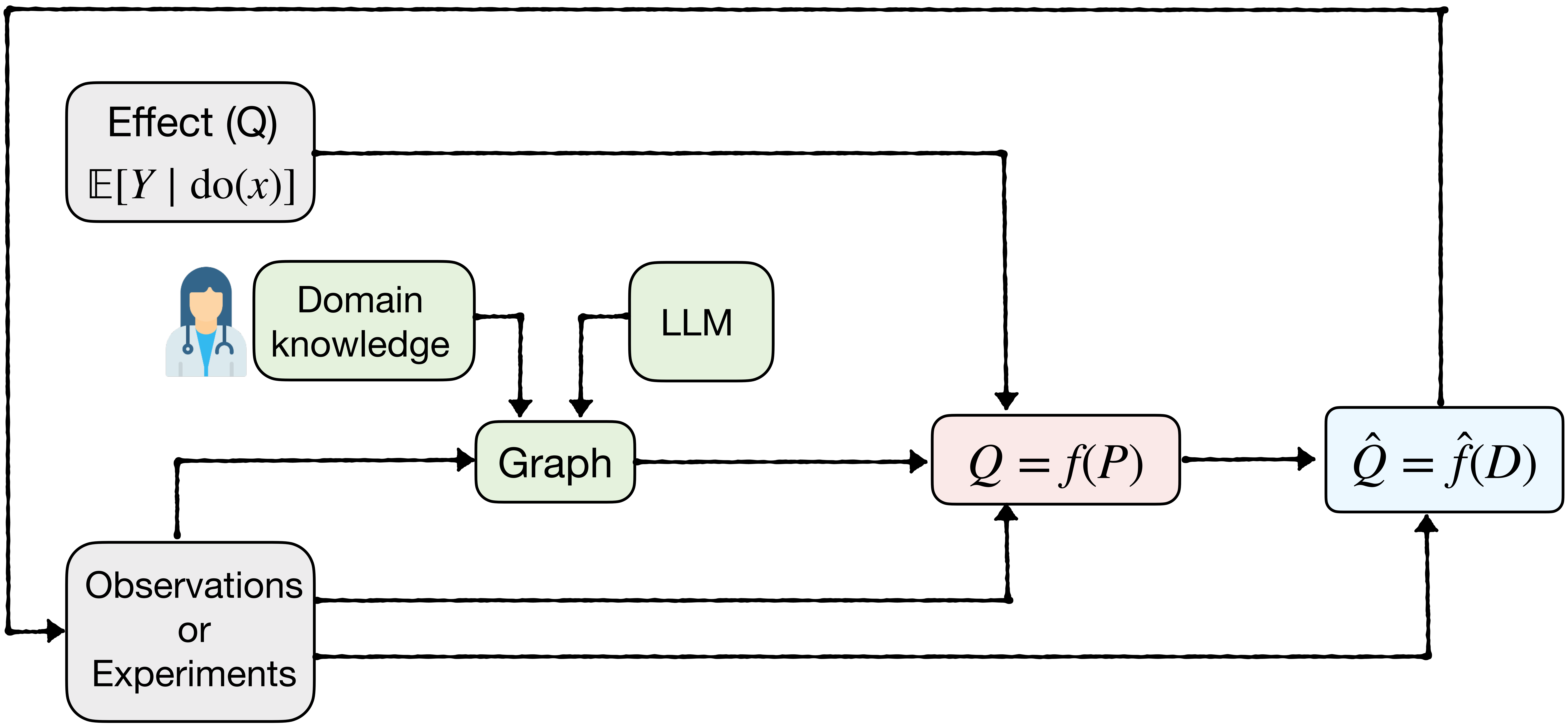
Future 3: Causal Inference Loop with Uncertainty



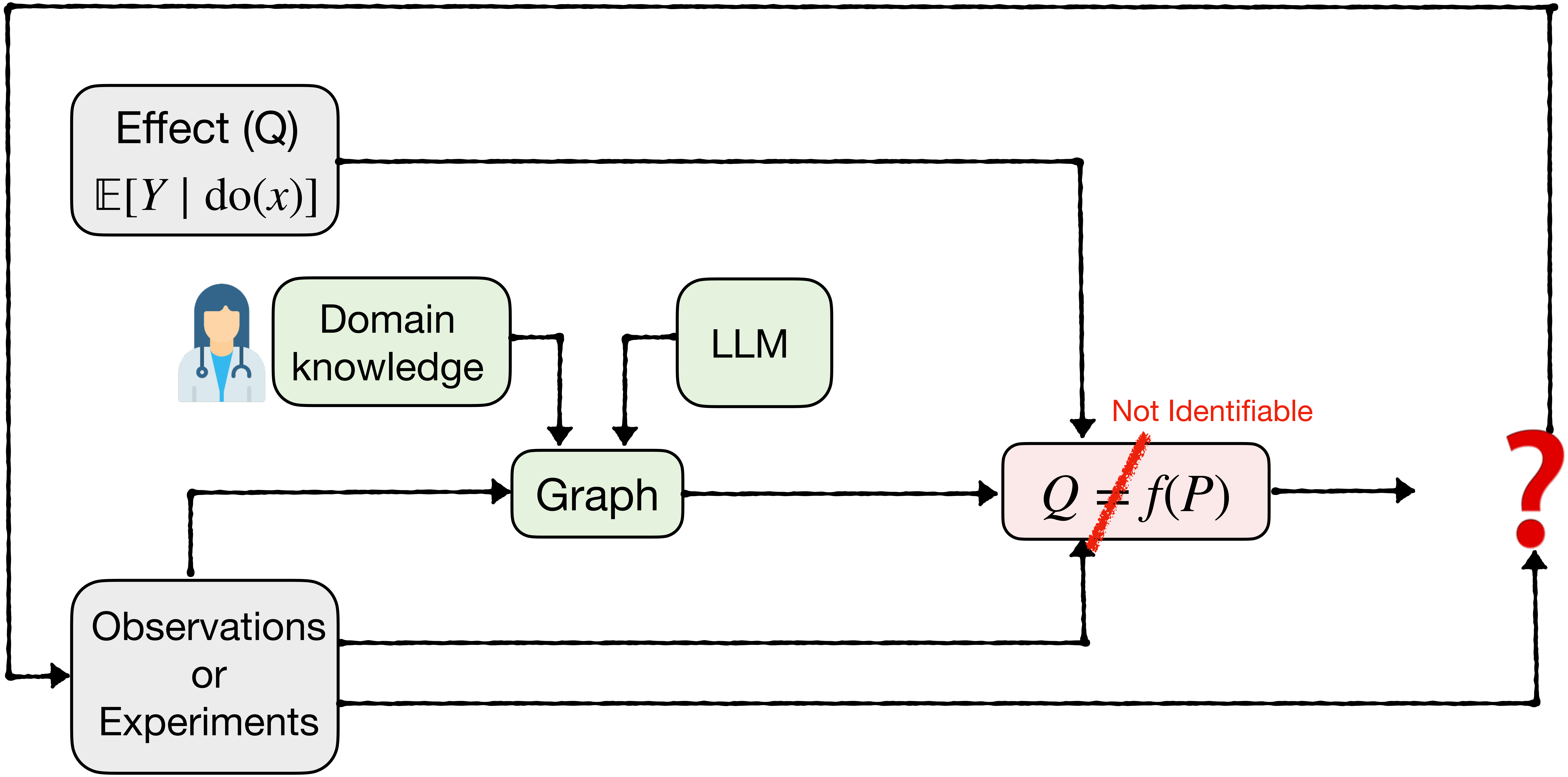
Future 3: Causal Inference Loop with Uncertainty



Future 3: Causal Inference Loop with Uncertainty



Future 3: Causal Inference Loop with Uncertainty



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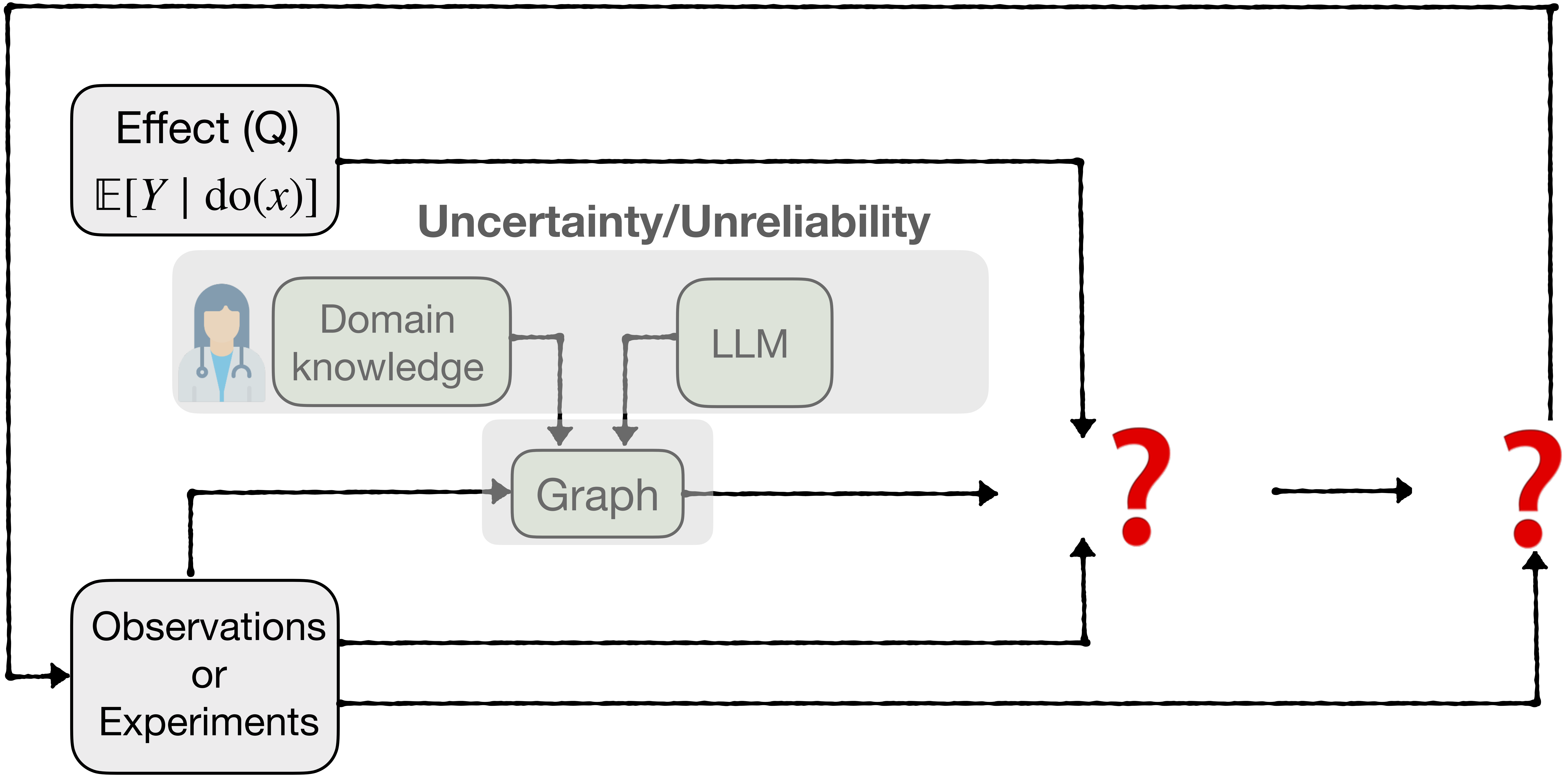

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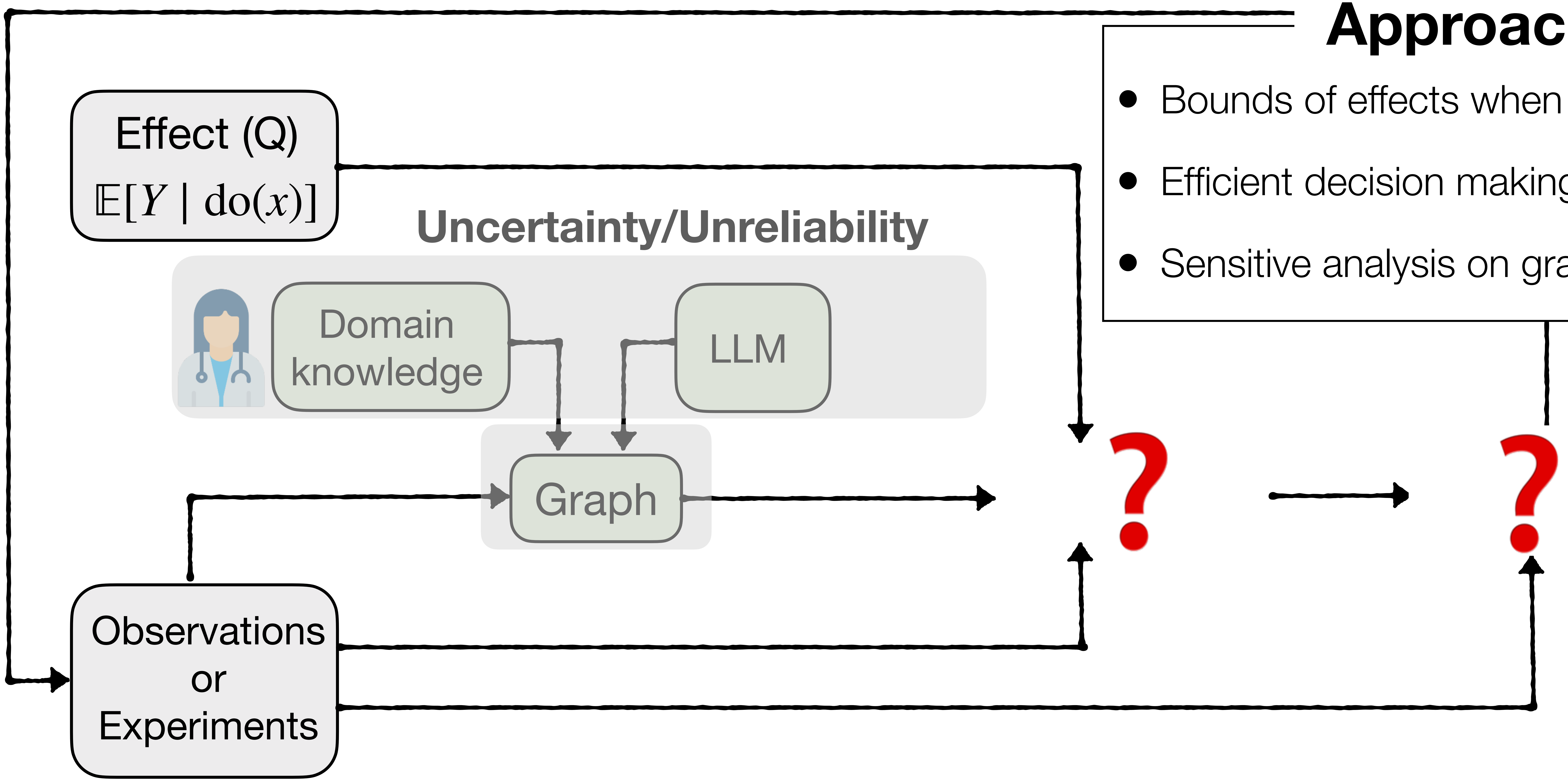

Approach

- Bounds of effects when unidentifiable
- Efficient decision making with bounds
- Sensitive analysis on graphs