NEURAL INFORMATION PROCESSING SYSTEMS

Double Machine Learning Density Estimation for Local Treatment Effects with Instruments

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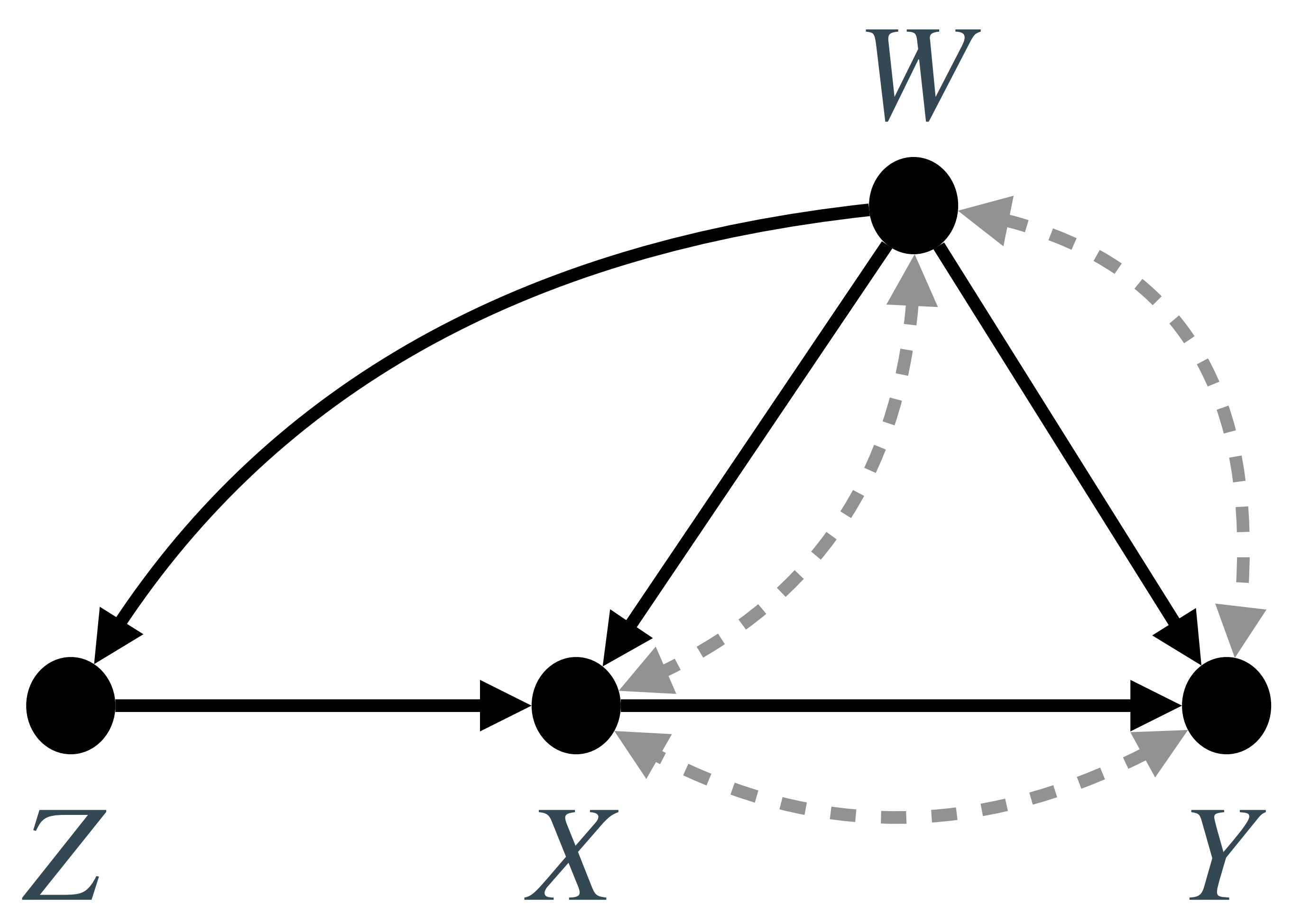
Example: 401(k) on the financial asset

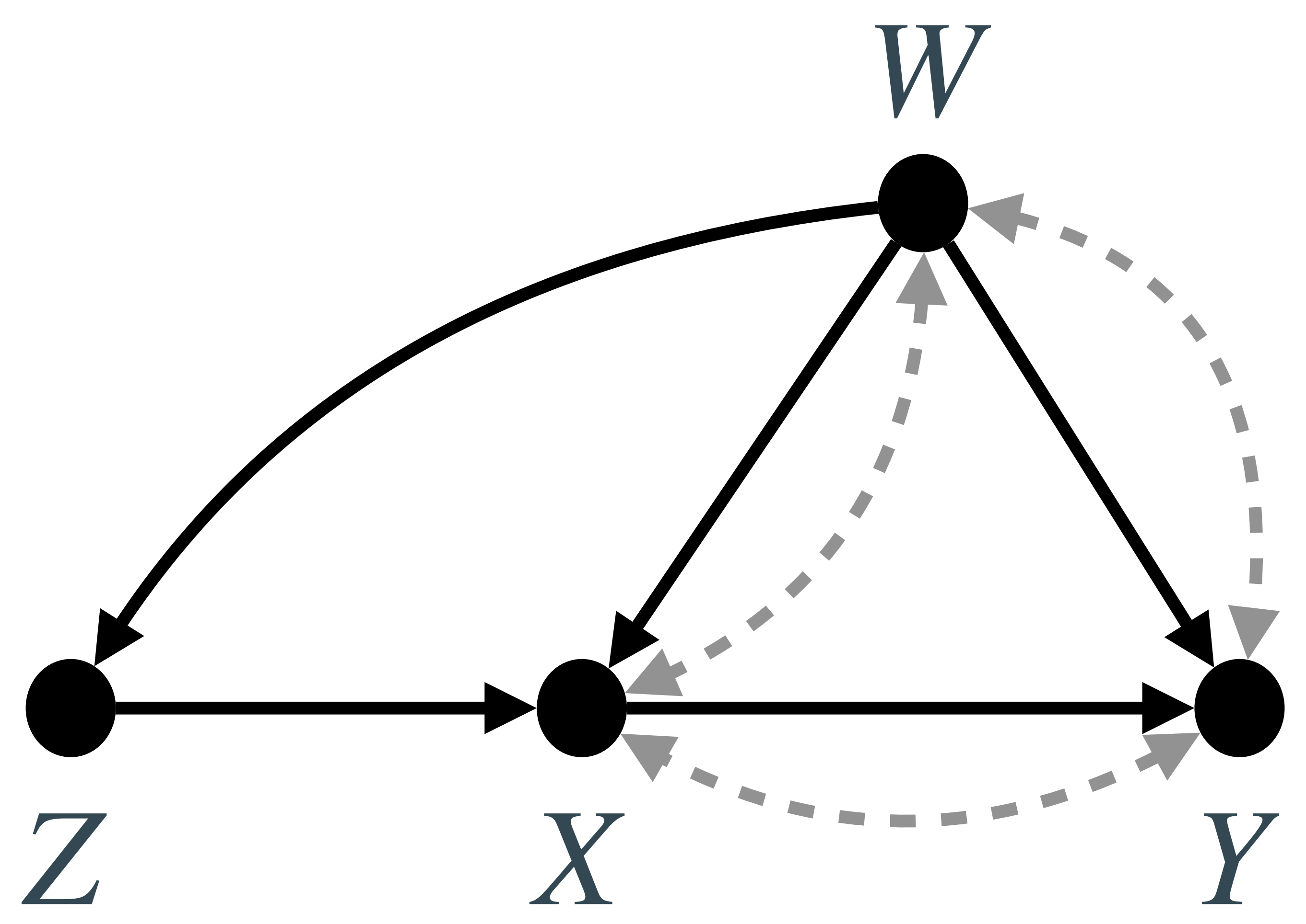
Query: Effect of the participation of 401(k) to the net financial asset



X	401(k) participation	$X = 1$ for participation
Z	401(k) eligibility	$Z = 1$ for being eligible
Y	Net financial asset	
W	Observed covariates (e.g., income, gender, family size, etc.)	
$\leftrightarrow$	Presence of unmeasured confounders	

Rewritten query: A causal effect of $X = x$ to $Y = y$; i.e., $P(y \mid do(x))$.

Unidentifiability of Causal Effects

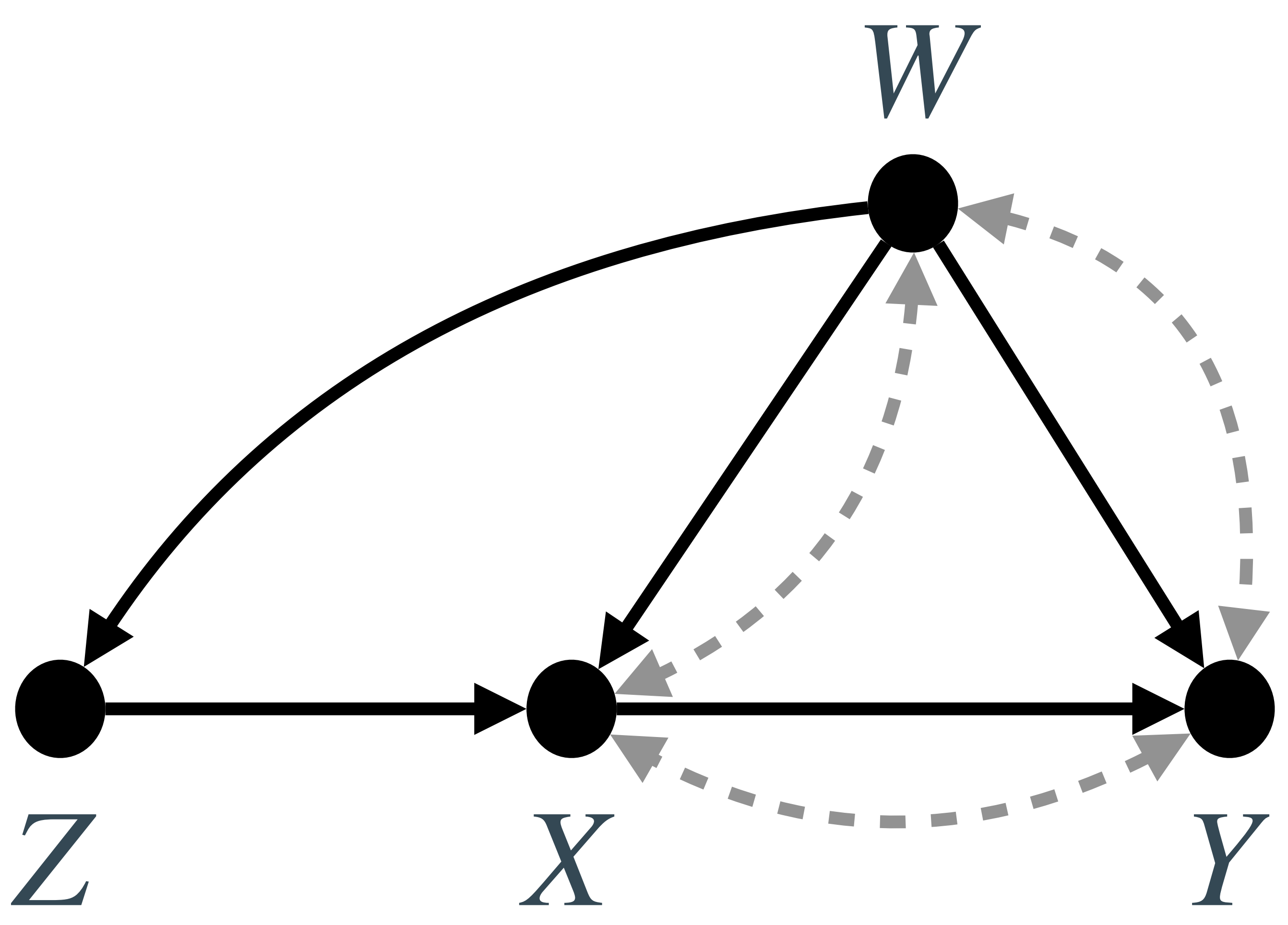


- $P(y \mid do(x))$ is unidentifiable

  (i.e., no unique representation of $P(y \mid do(x))$ w.r.t. the observational distribution P exists)

- Therefore, $P(y \mid do(x))$ is not computable from the observational data.

Exclusion of Defiers



- If a sample is eligible, then the sample either participates or not.

- No eligibility ⇒ No participation; i.e., $Z = 0 \nRightarrow X = 1$.

- This excludes a portion of samples called ‘*defiers*’.

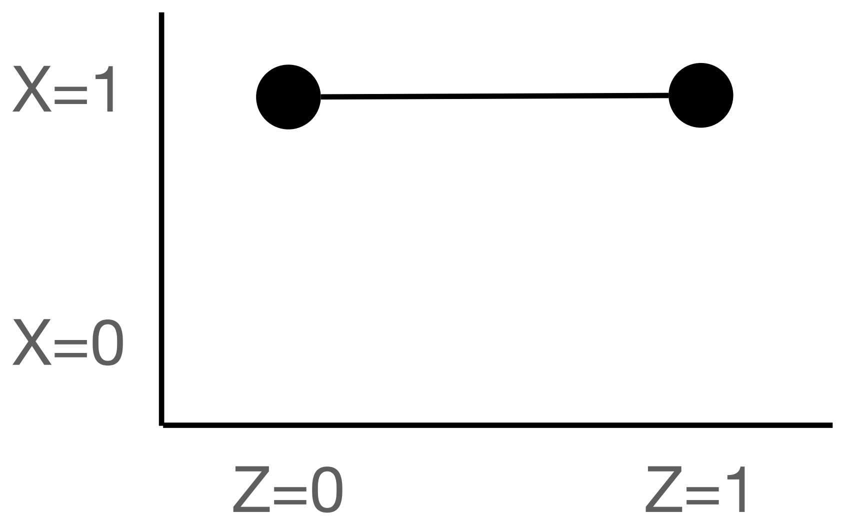
Four types of samples

Samples behaviors on $\{Z, X\}$ are always falling into one of the four types:

Always-taker

$$Z = 0 \Rightarrow X = 1$$

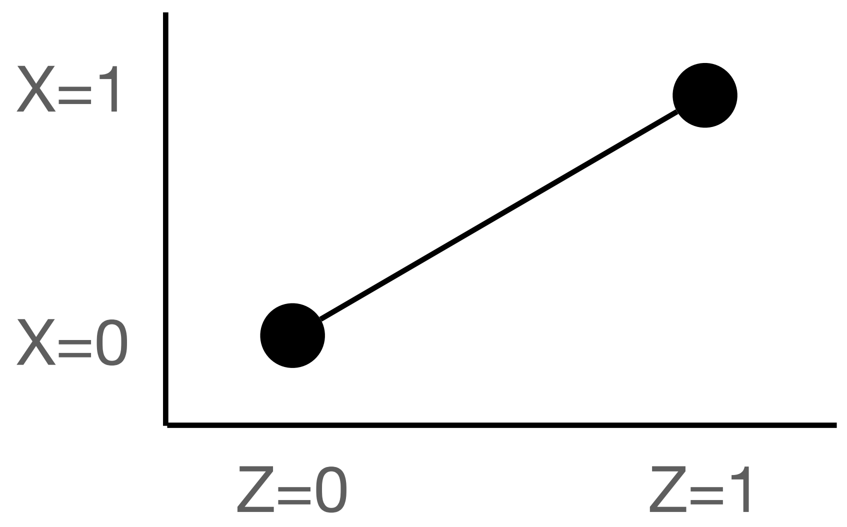
$$Z = 1 \Rightarrow X = 1$$



Compliers

$$Z = 0 \Rightarrow X = 0$$

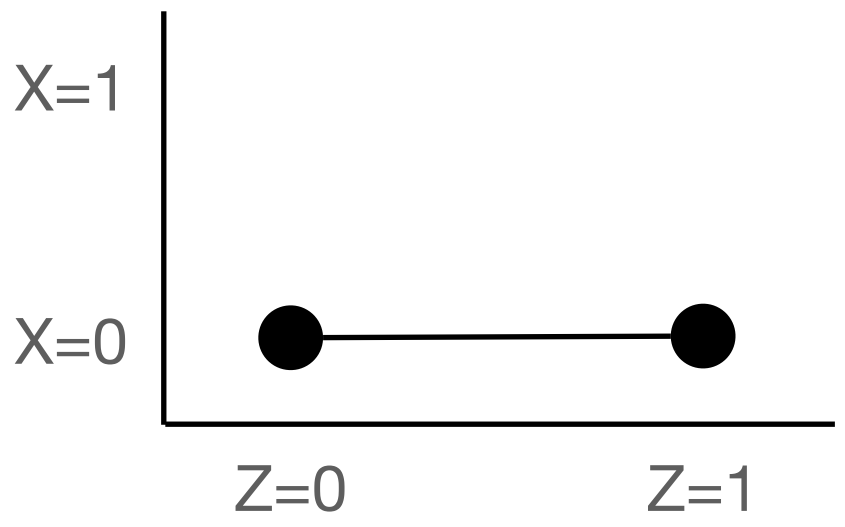
$$Z = 1 \Rightarrow X = 1$$



Never-taker

$$Z = 0 \Rightarrow X = 0$$

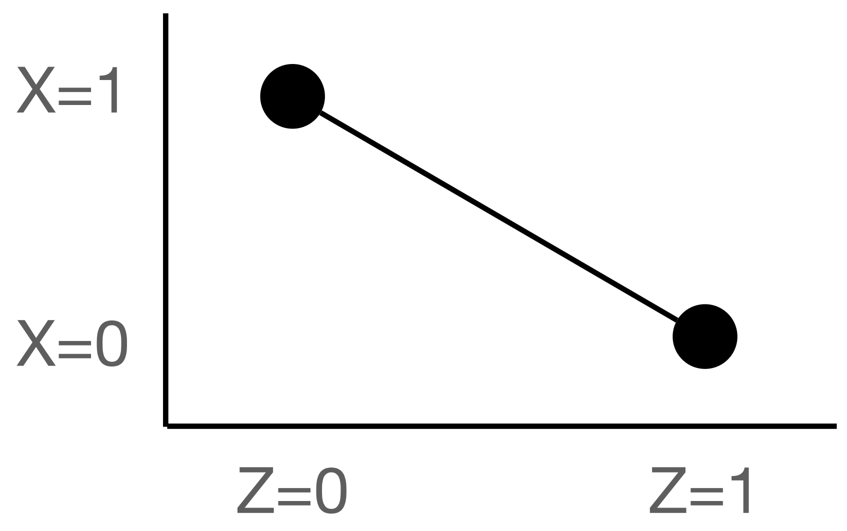
$$Z = 1 \Rightarrow X = 0$$



Defiers

$$Z = 0 \Rightarrow X = 1$$

$$Z = 1 \Rightarrow X = 0$$



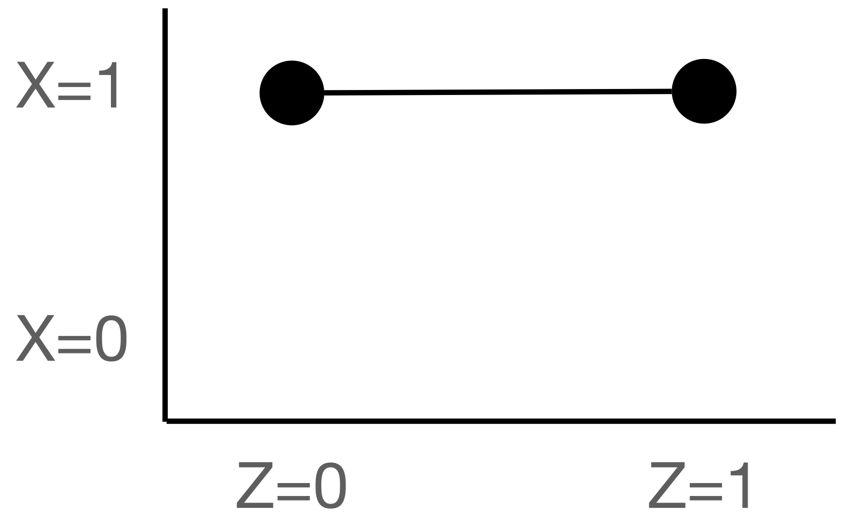
Monotonicity: No defiers

Suppose no defiers.

Always-taker

$$Z = 0 \Rightarrow X = 1$$

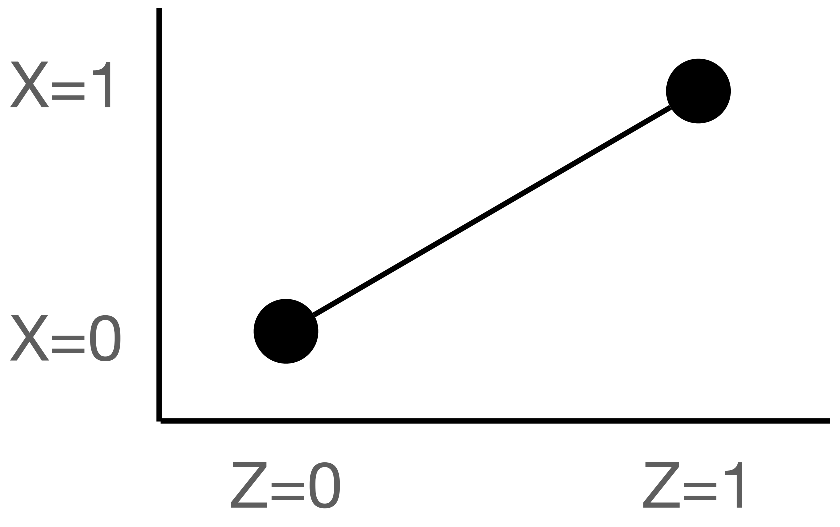
$$Z = 1 \Rightarrow X = 1$$



Compliers

$$Z = 0 \Rightarrow X = 0$$

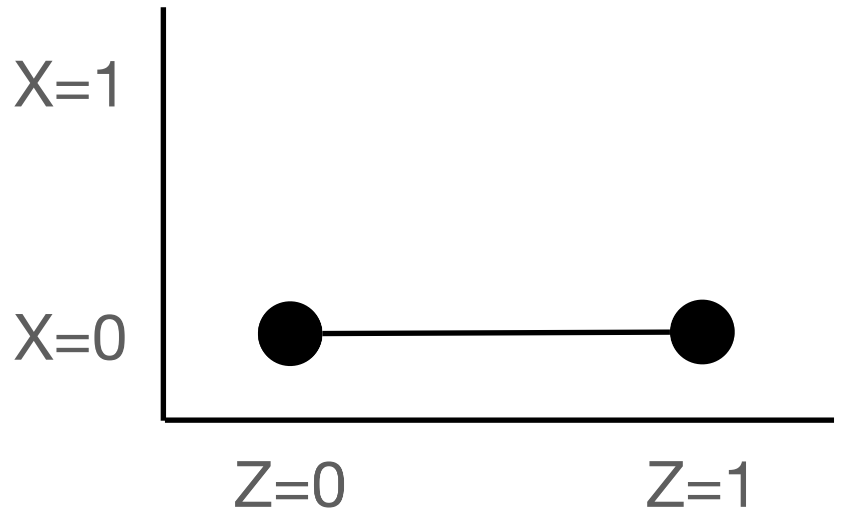
$$Z = 1 \Rightarrow X = 1$$



Never-taker

$$Z = 0 \Rightarrow X = 0$$

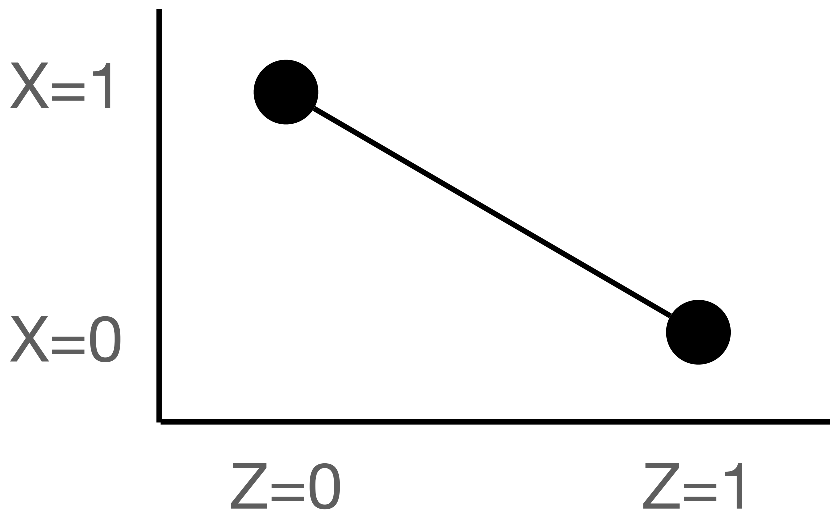
$$Z = 1 \Rightarrow X = 0$$



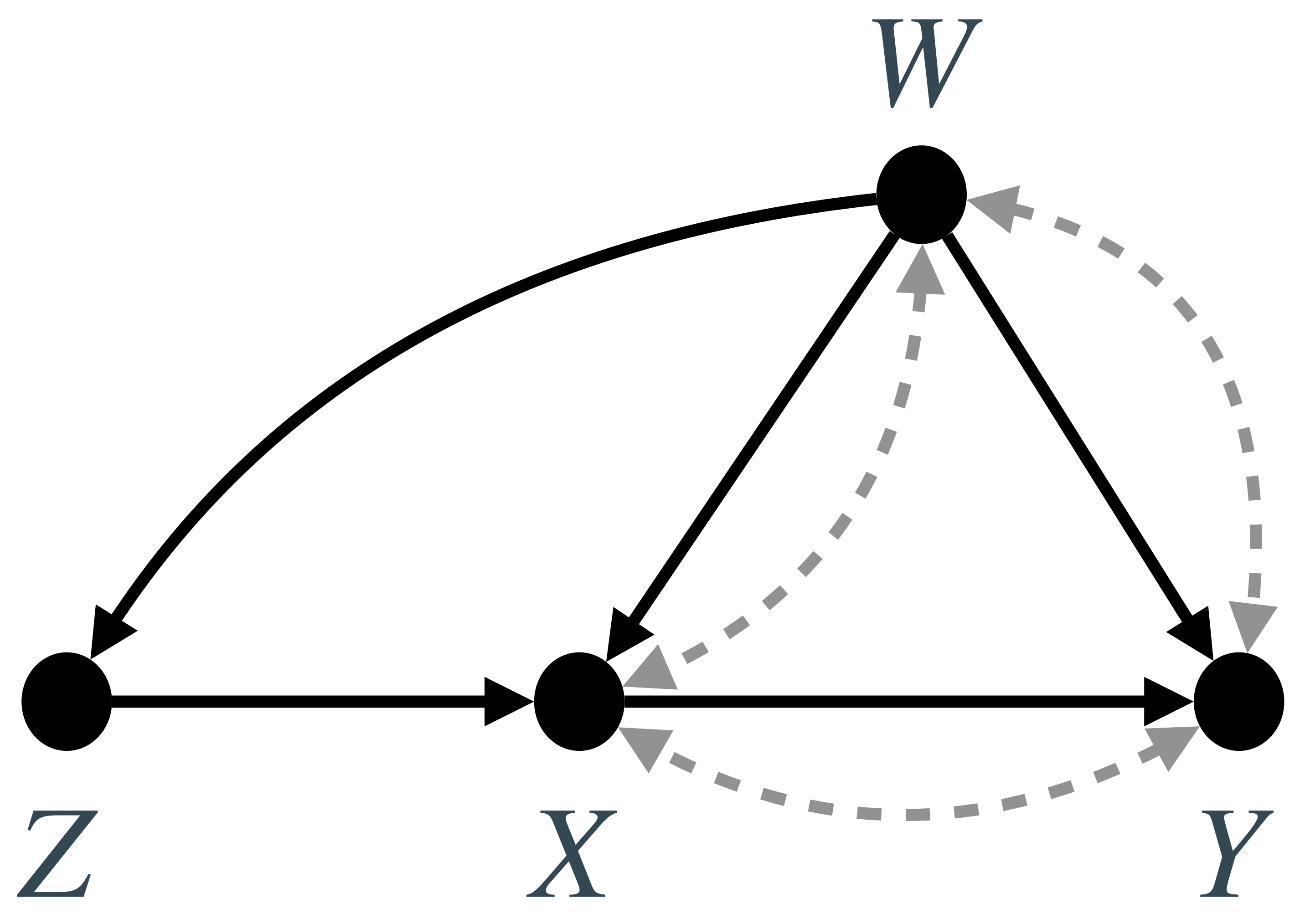
Defiers

$$Z = 0 \Rightarrow X = 1$$

$$Z = 1 \Rightarrow X = 0$$



Local Average Treatment Effect



Let C denote the complier. Under the *monotonicity*, the causal effect for compliers is identifiable and given as follow [🏆Imben and Angrist, 1994; Abadie, 2003]

$$\mathbb{E}[f(Y) \mid do(x), C] = \frac{\mathbb{E}[\mathbb{E}[f(Y)I_x(X) \mid Z = x, W] - \mathbb{E}[f(Y)I_x(X) \mid Z = 1 - x, W]]}{\mathbb{E}[P(X = 1 \mid Z = 1, W) - P(X = 1 \mid Z = 0, W)]}$$

where f is a function of Y.

Insufficiency of LATE

- Most work considered $\mathbb{E}[Y \mid do(x), C]$ (with $f(Y) = Y$) called 'local average treatment effect (LATE)'.

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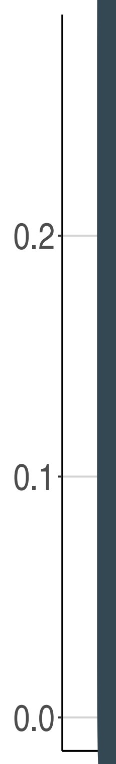


We need the density estimator!

LATE is insufficient to understand the treatment effect.

Challenges in estimating the density

$$\mathbb{E}[f(Y) \mid do(x), C] = \frac{\mathbb{E}[\mathbb{E}[f(Y)I_x(X) \mid Z = x, W] - \mathbb{E}[f(Y)I_x(X) \mid Z = 1 - x, W]]}{\mathbb{E}[P(X = 1 \mid Z = 1, W) - P(X = 1 \mid Z = 0, W)]}$$

😊 We can leverage the existing method for estimating LATE.

😔 The existing methods cannot be directly applied for estimating the density (non-regular estimand).

Toward estimating the density

We propose two methods for estimating the density $p(y_x \mid C)$.

1. **Kernel-smoothing based approach**

   In $\mathbb{E}[f(Y) \mid do(x), C]$, *choose f such that approximates* $\mathbb{E}[f(Y) \mid do(x), C] \approx p(y \mid do(x), C)$.

   Then, we can use the existing methods designed for estimating the LATE $\mathbb{E}[Y \mid do(x), C]$.

2. **Model-based approach**

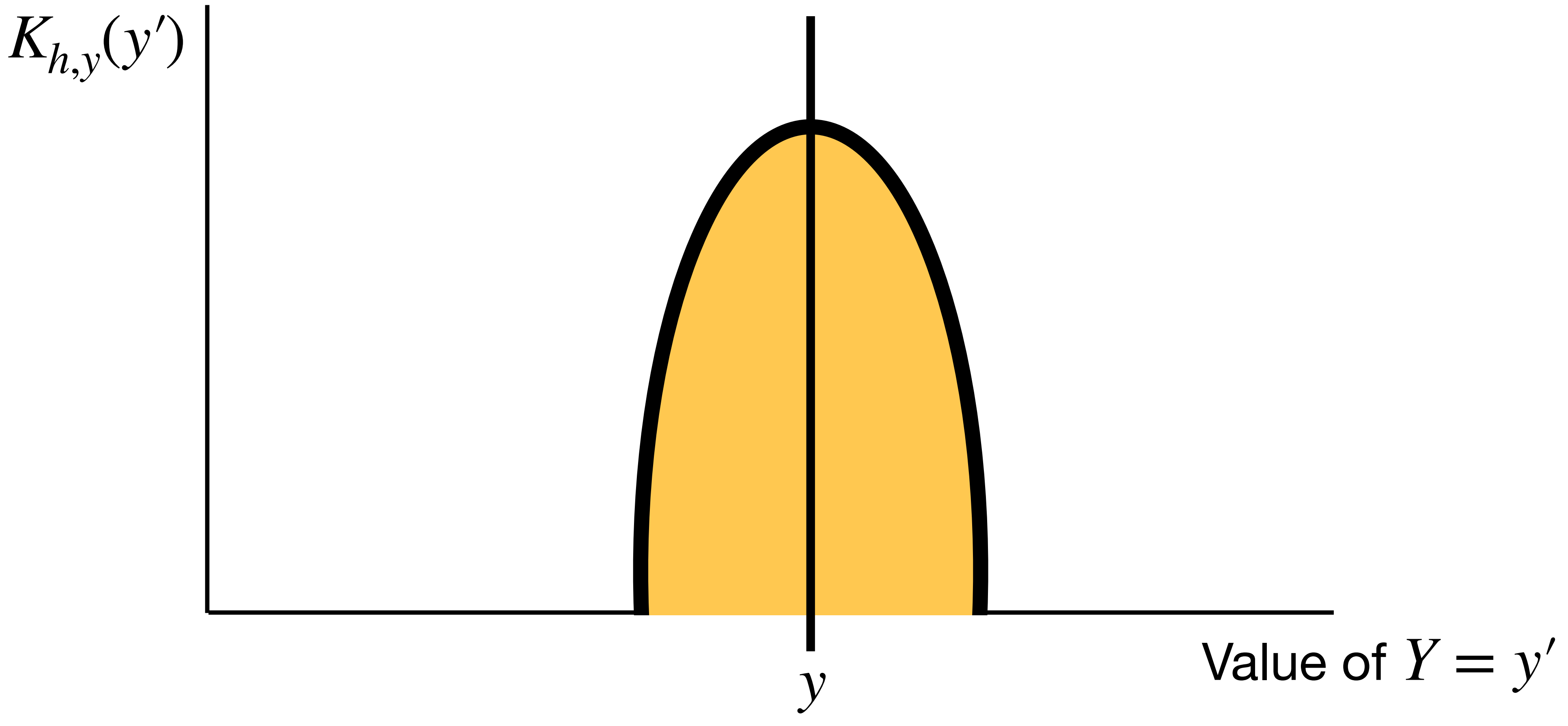
   Assume that the density can be modeled by a parametric model $g(y; \beta)$ (e.g., $p(y \mid do(x), C)$ can be modeled as a normal distribution $g(y; \beta = \{\mu, \sigma\})$).

   Then, we estimate β.

Approach 1 — Kernel-smooth based

In $\mathbb{E}[f(Y) \mid do(x), C]$, choose f for approximating $\mathbb{E}[f(Y) \mid do(x), C] \approx p(y \mid do(x), C)$.

We used $f(Y)$ as a Kernel $K_{h,y}(\cdot)$, which is a smooth approximation for denoting $Y = y$.



For example, Gaussian kernel is given as

$$K_{h,y}(y') \equiv \frac{1}{h\sqrt{2\pi}} \exp\left(-\frac{(y'-y)^2}{2h^2}\right)$$

Approach 1 — Kernel-smooth based

We propose a *double/debiased machine learning* (DML, Chernozhukov et al., 2018) based estimator which is a function of the followings; $\widehat{\psi}_h(y) \equiv f(\widehat{\pi}_z, \widehat{\xi}_x, \widehat{\theta}[K_{h,y}(Y)])$.

$$\pi_z(w) \equiv P(z \mid w); \ \xi_x(z,w) \equiv P(x \mid z, w); \ \theta(x,z,w)[f(Y)] \equiv \mathbb{E}[f(Y)I_x(X) \mid x, w].$$

The proposed estimator exhibits robustness property of DML:

Thm. 2. Debiasedness property of $\widehat{\psi}_h$

$\widehat{\psi}_h$ converges to $p(y \mid do(x), C)$ in $\sqrt{nh}$-rate if

- $\widehat{\pi}_z, \widehat{\xi}_x, \widehat{\theta}$ converges in much slower $n^{-1/4}$ rate; or
- $\widehat{\pi}_z$ or $\{\widehat{\xi}_x, \widehat{\theta}\}$ are correctly specified.

Approach 2 — Model-based

Goal — Learn β that minimizes the divergence between $p(y \mid do(x), C)$ and $g(y; \beta)$.

We propose a *DML* based estimator using the following parameters:

$$\pi_z(w) \equiv P(z \mid w); \; \xi_x(z, w) \equiv P(x \mid z, w); \; \theta(x, z, w)[f(Y)] \equiv \mathbb{E}[f(Y) I_x(X) \mid x, w].$$

For example, suppose $g(y; \beta = \{\mu, \sigma\})$ is a normal distribution. Then, the DML estimates of μ, σ are given as a function $\widehat{\mu} = g_{\mu}(\widehat{\pi}_z, \widehat{\xi}_x, \widehat{\theta})$ and $\widehat{\sigma} = g_{\sigma}(\widehat{\pi}_z, \widehat{\xi}_x, \widehat{\theta})$.

Approach 2 — model-smooth based

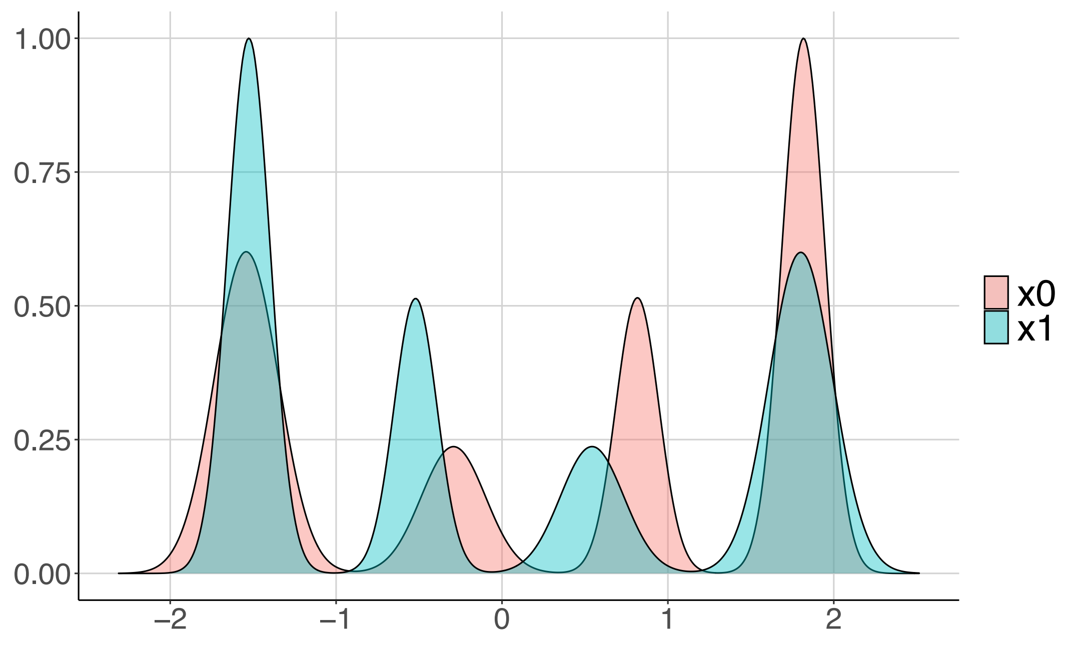
The proposed model-based DML estimators estimates (e.g., $\widehat{\mu} = g_\mu(\widehat{\pi}_z, \widehat{\xi}_x, \widehat{\theta})$ and $\widehat{\sigma} = g_\sigma(\widehat{\pi}_z, \widehat{\xi}_x, \widehat{\theta})$) exhibit robustness property of DML:

Thm. 2. Debiasedness of the model-based approach

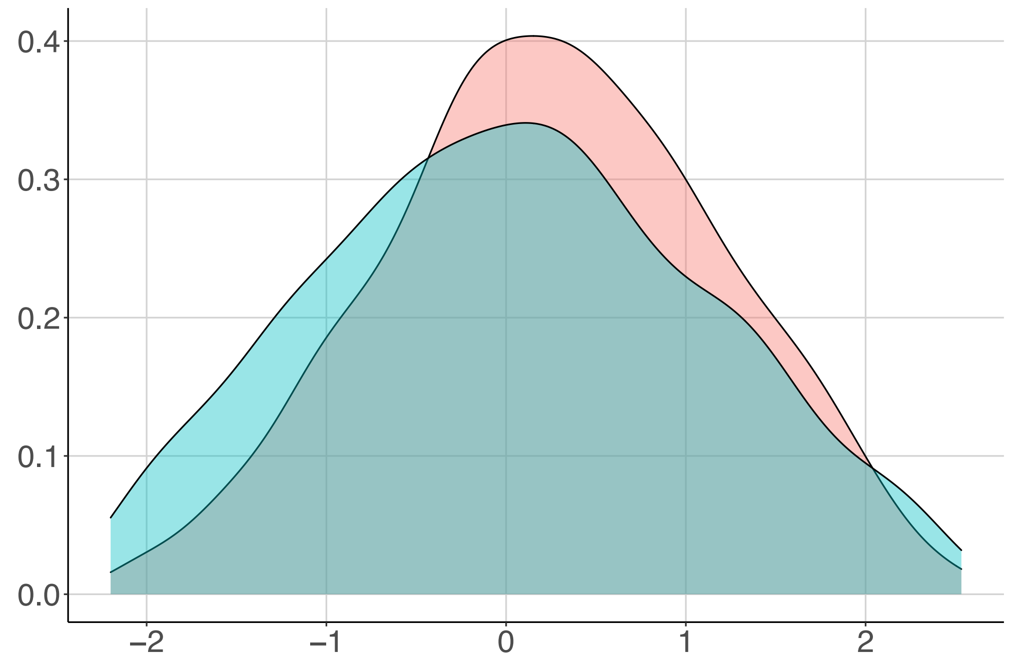
The proposed model-based DML estimator estimates $\widehat{\beta}$ (e.g., $\widehat{\beta} = \{\widehat{\mu}, \widehat{\sigma}\}$) in root-$n$ rate if

- $\widehat{\pi}_z, \widehat{\xi}_x, \widehat{\theta}$ converges in much slower $n^{-1/4}$ rate; or
- $\widehat{\pi}_z$ or $\{\widehat{\xi}_x, \widehat{\theta}\}$ are correctly specified.

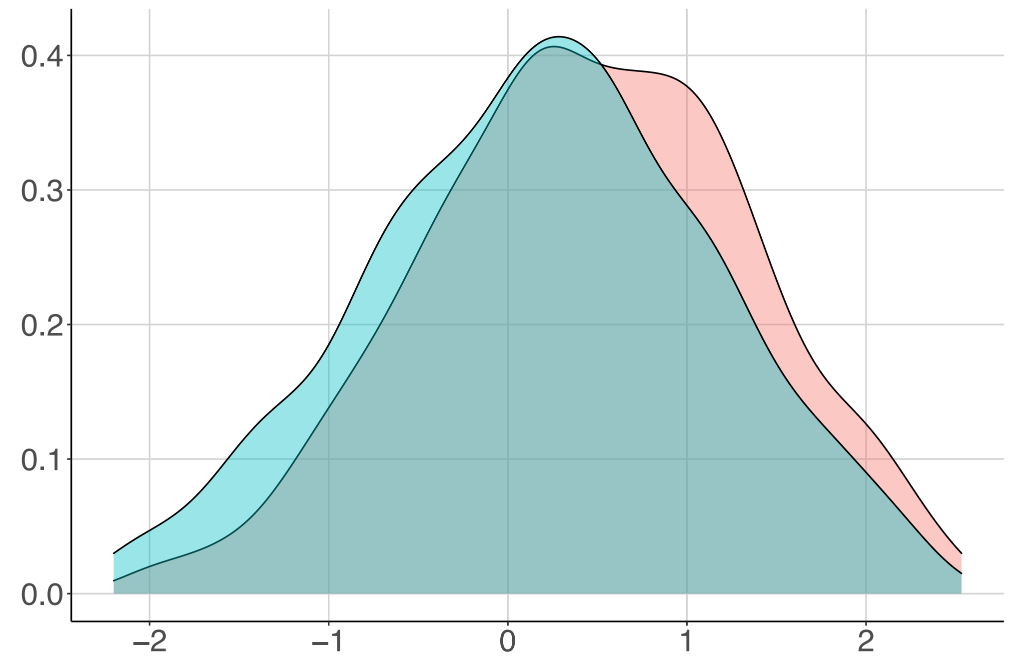
Results: Synthetic dataset



Ground-truth density.

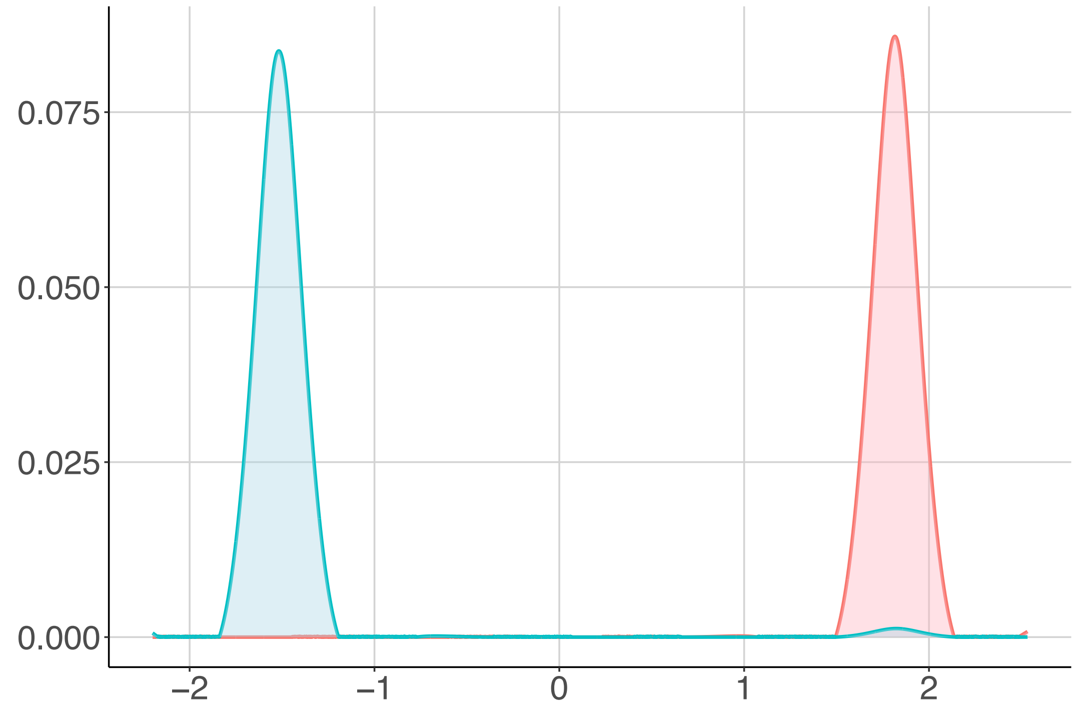


Model-based.
Non-DML

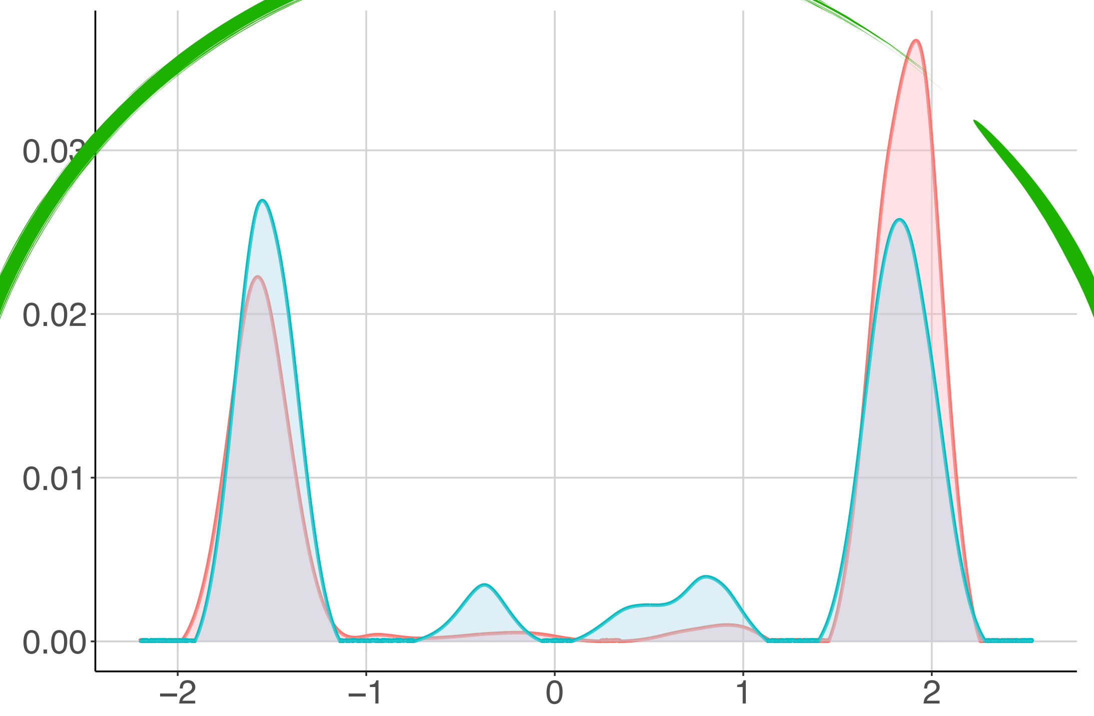


Model-based.
DML

Inaccurate result due to inconsistent assumption (Normal dist.) for the model



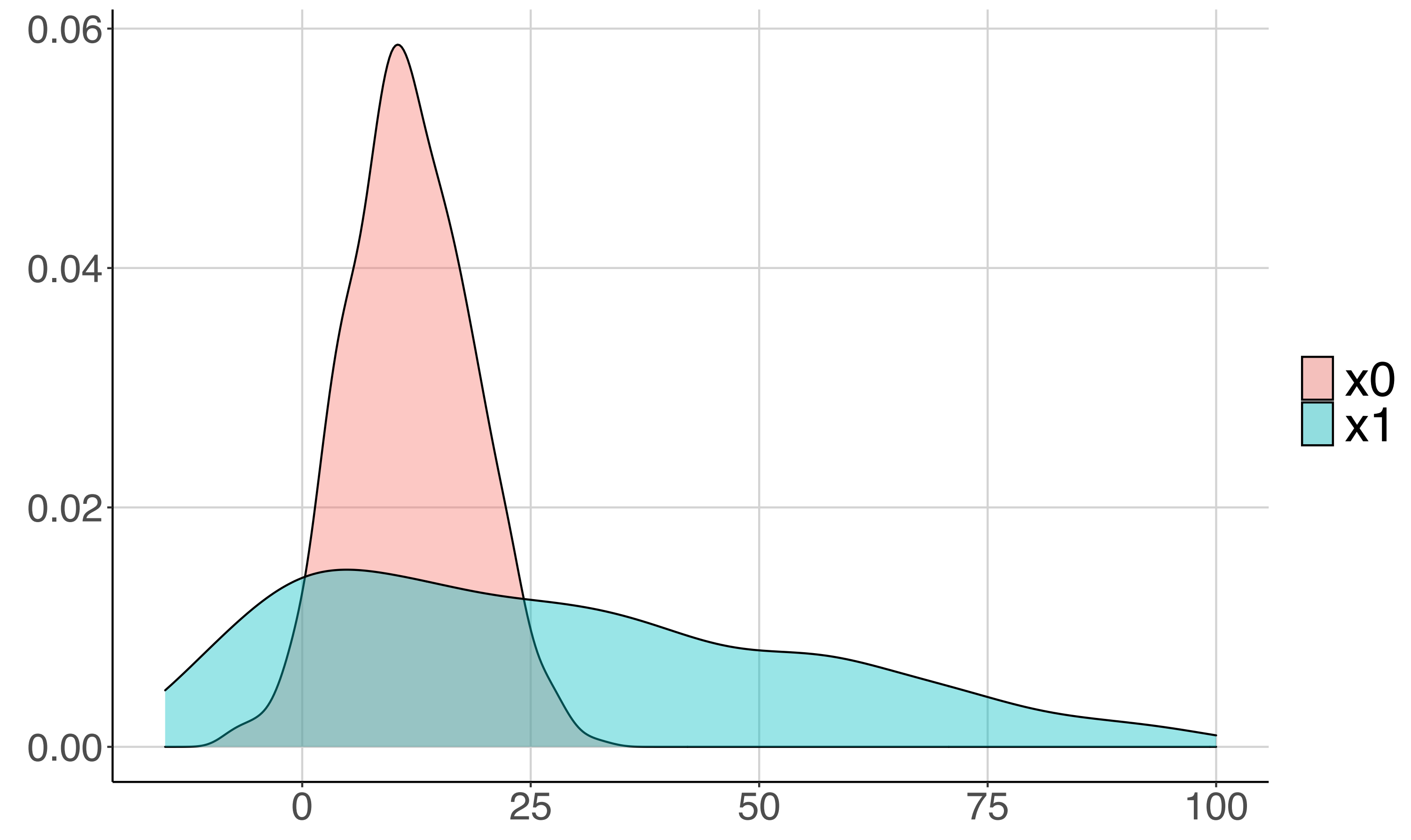
Kernel-based.
Non-DML



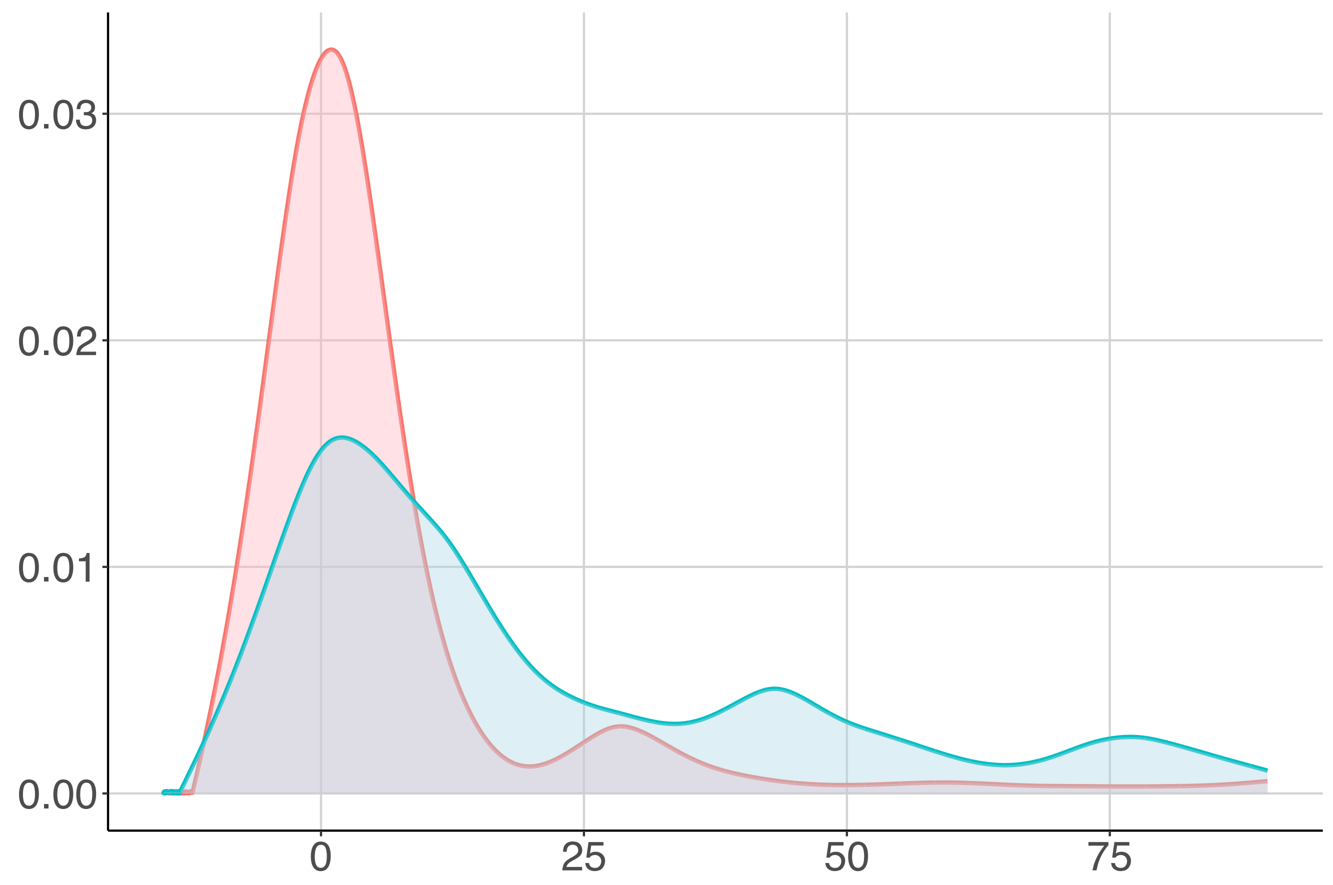
Kernel-based.
DML

Kernel-based DML estimator results in the most accurate result, capturing all modes.

Results: 401(k)



Model-based, DML

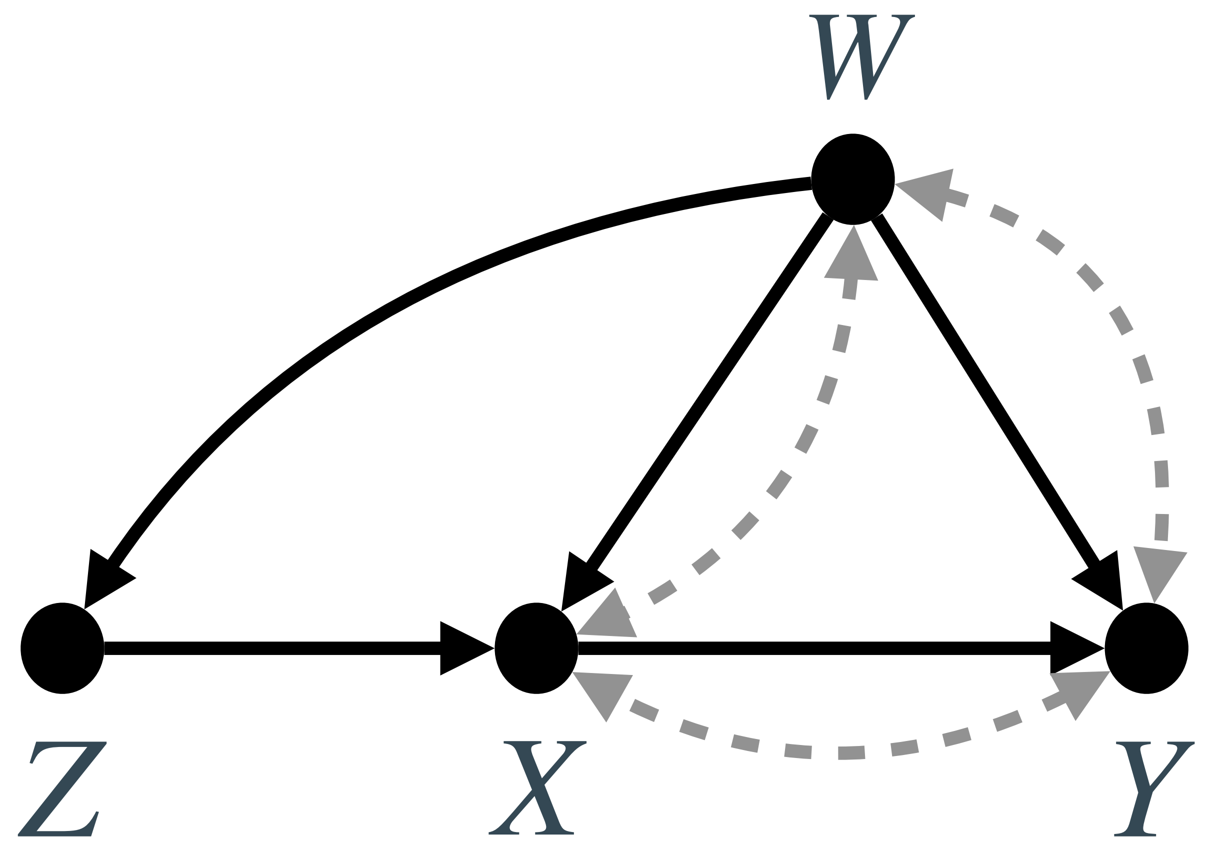


Kernel-based, DML

- Since this is a real-dataset, there is no ground-truth density.
- The implication $-$ $X = 1$ has a positive causal effect on $Y = y$ $-$ is well-known.
- For both of methods, our result is consistent with the known implication.

Conclusion

For a well-known instrumental variable setting,



- We provide the (1) Kernel-smoothing; and (2) model-based on the DML which exhibits robustness properties against the slow convergence of parameter estimates.

- We illustrated the proposed method to the synthetic and real-dataset.