Estimating Identifiable Causal Effects on Markov Equivalence Class through Double Machine Learning

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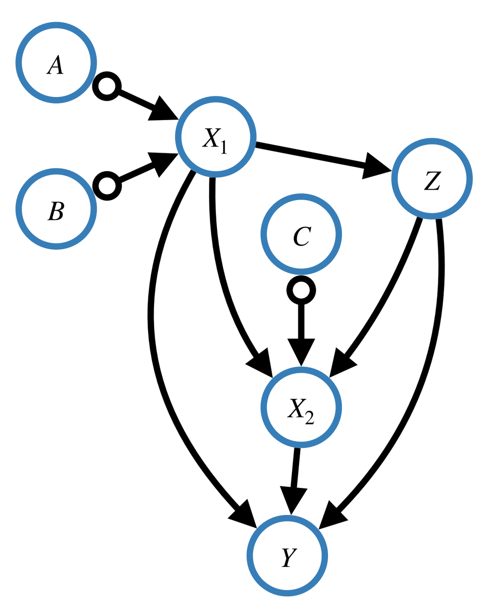
Summary

- Recently, *Double/Debiased Machine Learning (DML)* [1] based robust estimators have been developed for any identifiable causal effects from *a given fully specified graph* [2].

- Given observational data, we can only learn the *Markov equivalence class (MEC)* of the causal graph. A complete result for identifying causal effects from the Markov equivalence class has recently been developed [3].

- In this paper, we developed robust DML estimators for *any identifiable functional from a MEC, which doesn't* require fully specified graphs in prior.

Causal effect identification on MEC



[Figure 1] Example for Markov equivalence class

Markov equivalence class: MEC is a collection of graphs compatible with data (i.e., Two graphs G_1, G_2 are in the same MEC if the same conditional independences are implied by the graphs).

Identification on PAG: A *complete* identification algorithm (i.e., the causal effect $P(y \mid do(x))$ is identifiable if and only if the algorithm returns an output) has been developed [3].

[1] Chernozhukov, Victor, et al. "Double/debiased machine learning for treatment and structural parameters." *The Econometrics Journal* 21.1 (2018): C1-C68.

[2] Jung, Yonghan, Jin Tian, and Elias Bareinboim. "Estimating identifiable causal effects through double machine learning." *Proceedings of the 35th AAAI Conference on Artificial Intelligence*. 2021.

[3] Jaber, Amin, Jiji Zhang, and Elias Bareinboim. "Causal identification under Markov equivalence: Completeness results." *International Conference on Machine Learning*. PMLR, 2019

DML for Canonical Expression

- **[Canonical Expression 1 (CE-1),** Def. 3] is given as

$$\mathcal{Q} = \sum_{\mathbf{a}} \prod_{\mathbf{B}_i \in \mathbf{C}} P(\mathbf{b}_i \mid \mathrm{Pre}(\mathbf{b}_i))$$

where $\mathbf{A}, \mathbf{B}_i, \mathbf{C}$ are a subset of variables.

- An *uncentered influence function* (UIF), a key ingredient in constructing DML estimators, for CE-1 is given as

Lemma 1: UIF for CE-1

$$\mathcal{V}(\mathbf{V}; \eta = \{\theta, \omega\}) \equiv \theta_{0,1} + \sum_k \omega_k(\theta_{k,1} - \theta_{k,2})$$

where $\{\theta_{k,1}, \theta_{k,2}\}$and $\{\omega_k\}$ are parameters represented by the conditional expectations, and can be estimated through regressions.

- The DML estimator can be constructed based on the UIF; For (D_a, D_b) are ***randomly split samples*** and (η_a, η_b) are nuisances trained using data (D_a, D_b),

$$T_N \equiv \frac{2}{N} \sum_{\mathbf{V}_{(i)} \in D_a} \mathcal{V}(\mathbf{V}_{(i)}; \eta_b)) + \frac{2}{N} \sum_{\mathbf{V}_{(i)} \in D_b} \mathcal{V}(\mathbf{V}_{(i)}; \eta_a))$$

1. **Doubly Robust:** T_N converges to ψ whenever $\{\theta\}$ or $\{\omega\}$ are correct.
2. **Debiasedness:** T_N converges at $\sqrt{N}$ rate to ψ even when $\eta = \{\theta, \omega\}$ converges $N^{-1/4}$ rate.

Illustration 1: Derivation of UIF for Figure 1

A causal effect is expressed as CE-1, since

$$P(y \mid do(x_1, x_2)) = \sum_{z,a,b,c} P(y \mid x_2, x_1, z, z, a, b, c) P(z \mid x_1, a, b, c) P(a, b, c)\,.$$

The UIF can be derived by Lemma 1 (Illustration 1 in the paper) as,

$$\mathcal{V} = \theta_{0,1} + \omega_1(\theta_{1,1} - \theta_{1,2}) + \omega_2(\theta_{2,1} - \theta_{2,2}),$$

where $\{\theta_{\cdot,\cdot}, \omega_{\cdot}\}$ are specified in the Illustration 1 in the paper.

DML for *any* identifiable causal estimands

1. (**DML-IDP** in Algo. 1) represents a causal effect as a function of CE-1 & derives/expresses a UIF for any identifiable causal effects as a function of UIFs of CE-1 in Lemma 1.

Theorem 1: Expressibility

An UIF for any identifiable causal effects can be expressed as a function of UIFs of CE-1 (in Lemma 1), through Algo. 1.

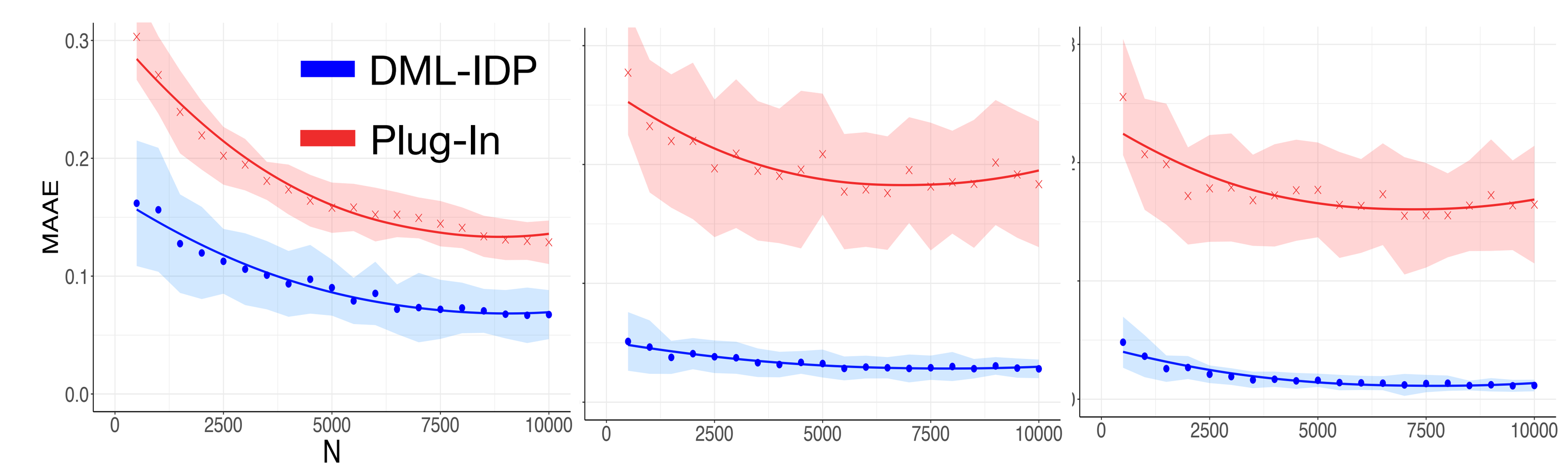
2. (**DML estimator** in Def. 5) constructs T_N based on the UIF.

Theorem 2: DML Properties

A DML estimator T_N (named DML-IDP) achieves doubly robustness and debiasedness, with respect to $\{\theta\}$ and $\{\omega\}$.

Simulation for Figure 1

- DML-IDP is compared with the plug-in estimator, the only viable estimator working for identifiable causal functional for MEC.



- **(Debiasedness; Left)** DML converges (i.e., the error 'MAAE' decreases) faster even when nuisances converge slower rate ($N^{-1/4}$).
- **(Doubly Robustness; (Center, Right))** DML converges even when models for either $\{\theta\}$ (center) or $\{\omega\}$ (right) is misspecified.

Conclusions

- (**DML-IDP**) We developed algorithms to derive corresponding UIFs for any causal effects identifiable from the MEC.
- (**DML estimator**) We introduced a general purpose causal estimators achieving ***doubly robustness*** and ***debiasedness*** properties based on the derived UIF.