Estimating Identifiable Causal Effects through Double Machine Learning

Yonghan Jung
PURDUE UNIVERSITY

Jin Tian
IOWA STATE UNIVERSITY

Elias Bareinboim
COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

Learning causal effect: 2-step procedures

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Step 1.
(Identification, ID)

Represent the causal query **Q**=P(y|do(x)) as a function of P and set of causal assumption **A** (i.e., **Q**=**F**(P, A)).

Learning causal effect: 2-step procedures

Step 1.
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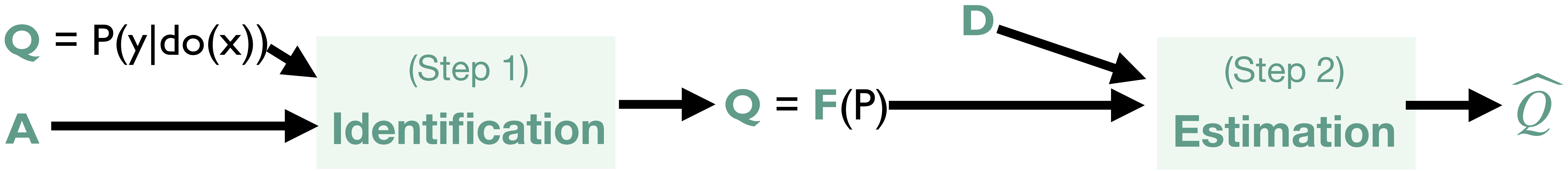
Step 2.
(Estimation)

Estimate the identified estimand **Q**=**F**(P, A) from finite samples **D**.



Current status of causal inference

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Strength

Completeness of Identification algorithm

- There exist ***sound*** and ***complete*** identification algorithms for determining whether a causal query **Q** can be represented as a functional of P (i.e., Q=F(P, A)) from a given causal graph.

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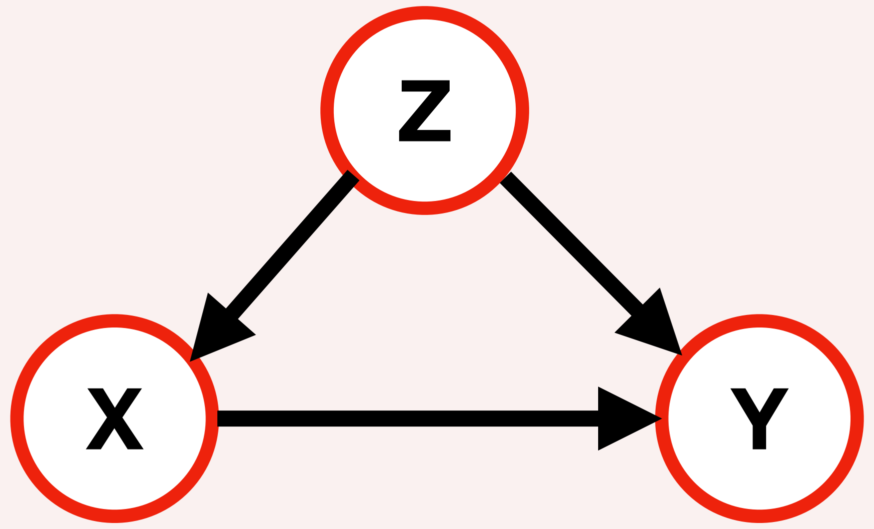

Backdoor graph

Weakness

No sample-efficient estimator

- Estimation has been mainly done on backdoor/ignorability assumption.

Q = P(y|do(x))

G (Causal graph) → **ID algorithm** → **Q** = **F**(P)

D → **Estimation** e.g., IPW,... → $\widehat{Q}$

Under backdoor assumption,

$$Q = \sum P(y \mid x, z) P(z)$$

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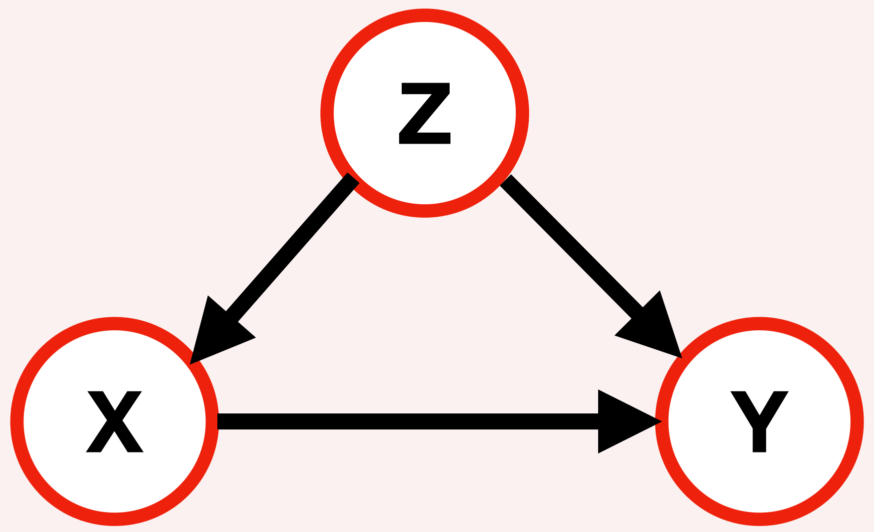

Backdoor graph

Weakness

No sample-efficient estimator

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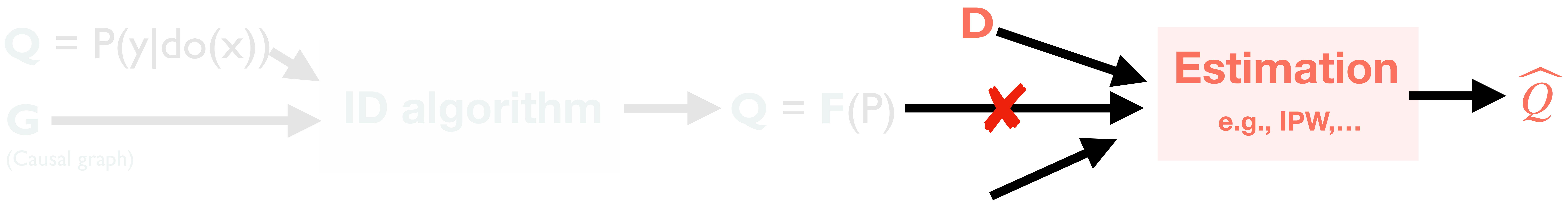
- For general identifiable estimands, it's not known (neither obvious) how to estimate the causal effect sample-efficiently.



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Connecting Identification and Estimation

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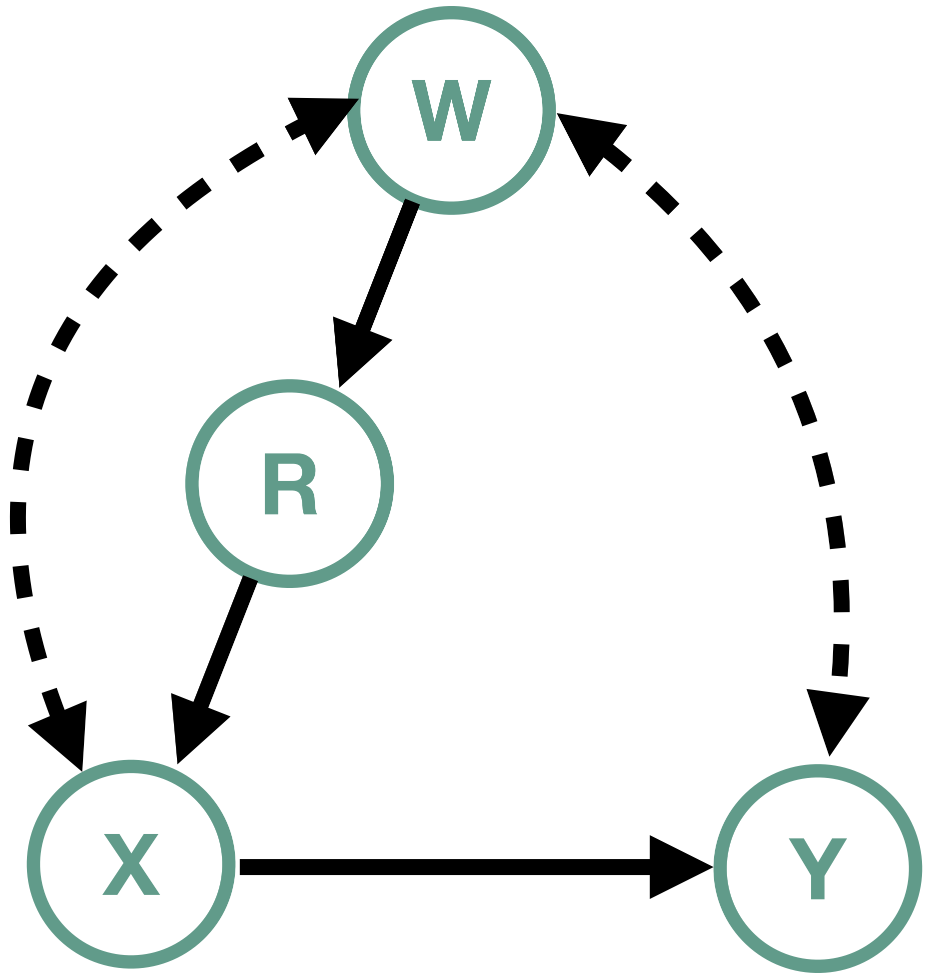

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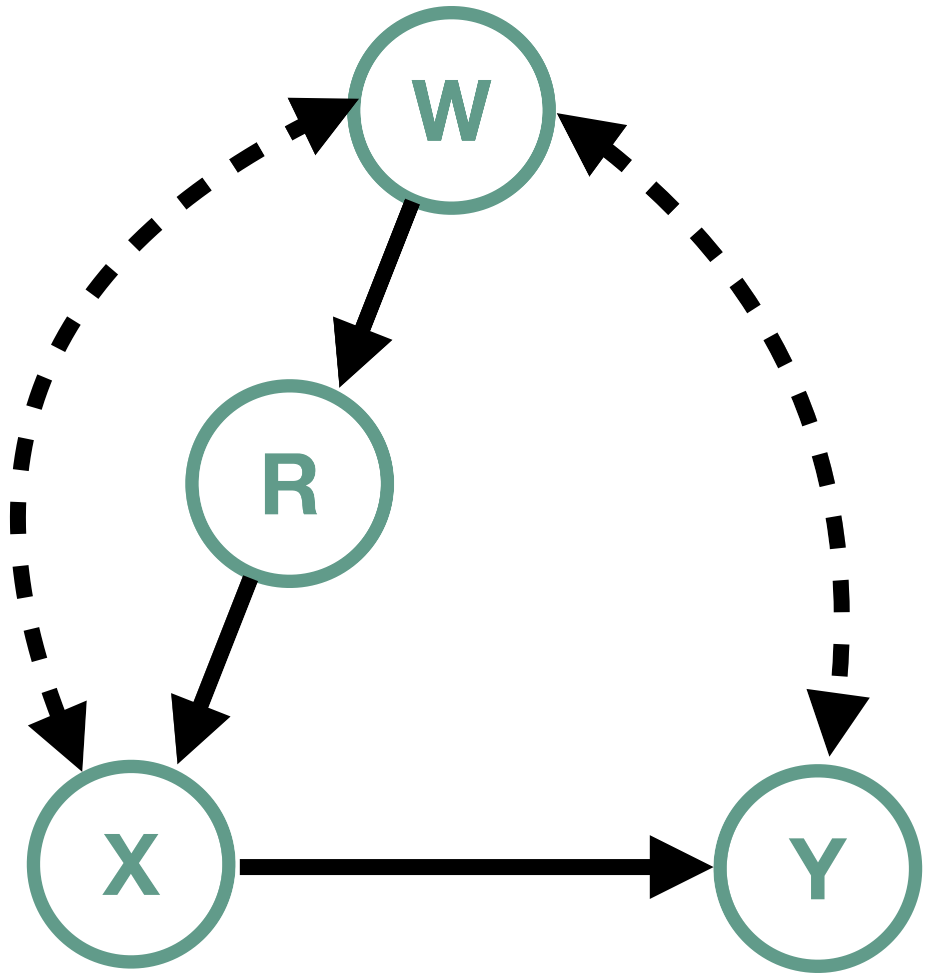
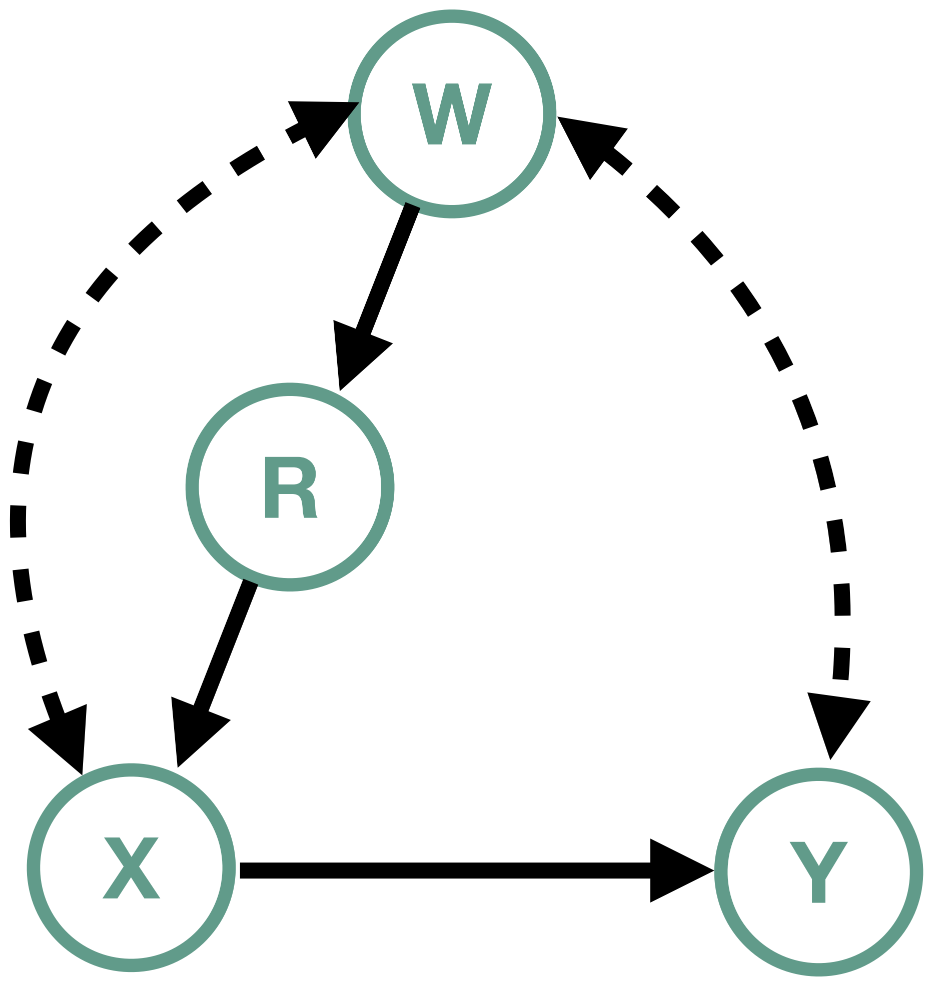


$$P(y \mid do(x)) = \frac{\sum_w P(x, y \mid r, w) P(w)}{\sum_w P(x \mid r, w) P(w)}$$

- Causal effect is identifiable and the functional is given as the below equation.

Connecting Identification and Estimation



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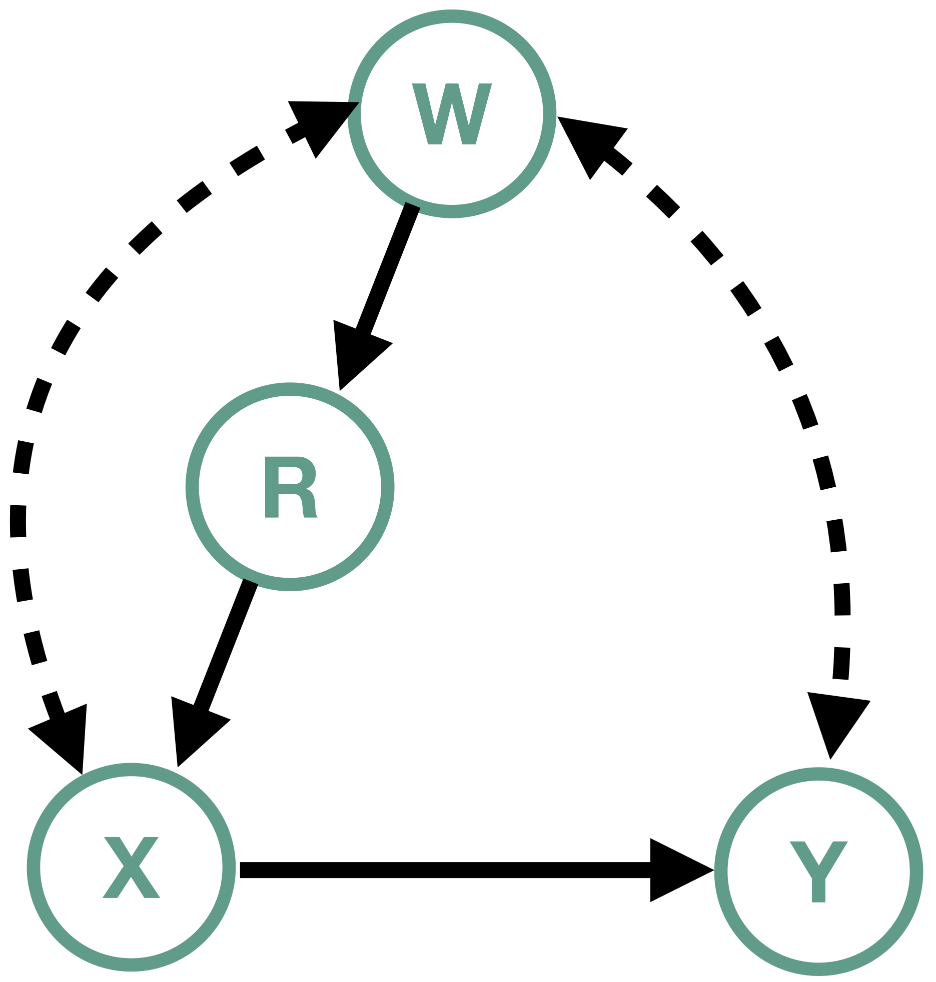

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- **Observation** — Causal functional P is a arithmetic of two adjustments:

$P(x, y \mid r, w)$ adjusting over W=w

$P(x \mid r, w)$ adjusting over W=w

Connecting Identification and Estimation

Intuition

A causal functional is expressible as a function of adjustments. Then, a BD estimator might be applicable in estimating such functional.

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Connecting Identification and Estimation

Intuition

A causal functional is expressible as a function of adjustments. Then, a BD estimator might be applicable in estimating such functional.

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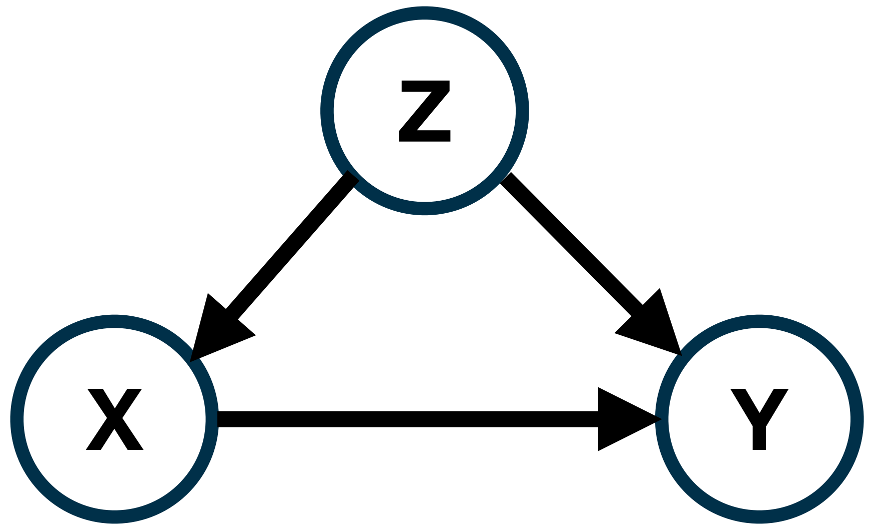
What BD estimators should we leverage?

Recap: Classic BD estimators

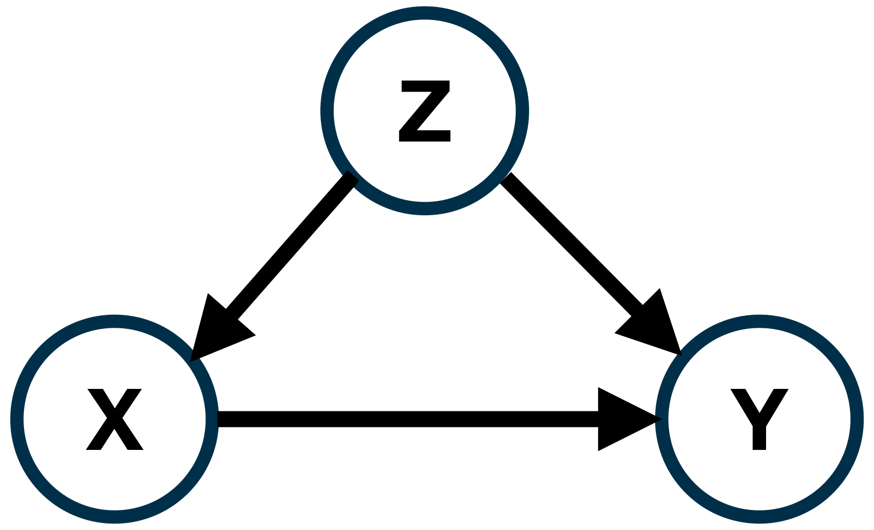


Backdoor graph

$$Q = \sum_z P(y \mid x, z) P(z)$$

Estimand (Q)		
Estimator ($\widehat{Q}$)		
For correct estimation		
For $\sqrt{N}$-consistency		

Recap: Classic BD estimators



Backdoor graph

$$Q = \sum_{z} P(y \mid x, z) P(z)$$

	Inverse Probability Weight	
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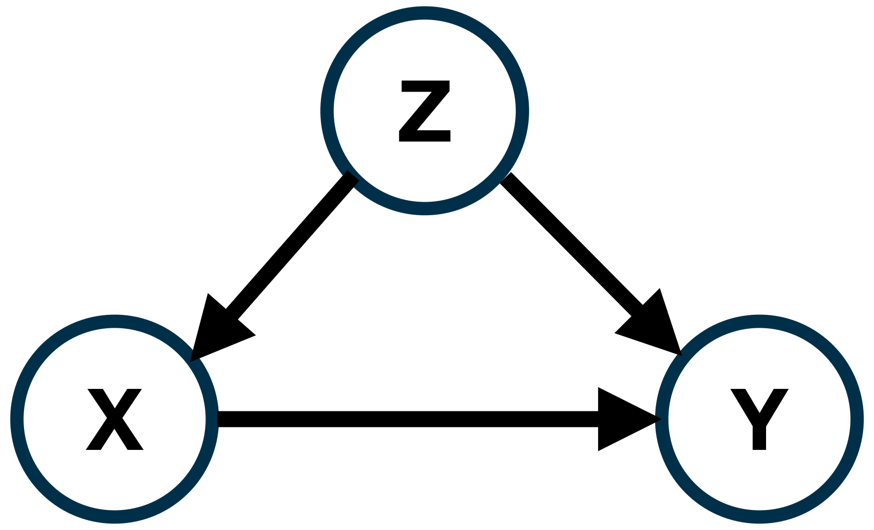

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$$Q = \sum_z P(y \mid x, z) P(z)$$

	Inverse Probability Weight	
Estimand (Q)	$\mathbb{E}\left[\frac{I_x(X)}{P(X \mid Z)} I_y(Y)\right]$	
Estimator ($\widehat{Q}$)		
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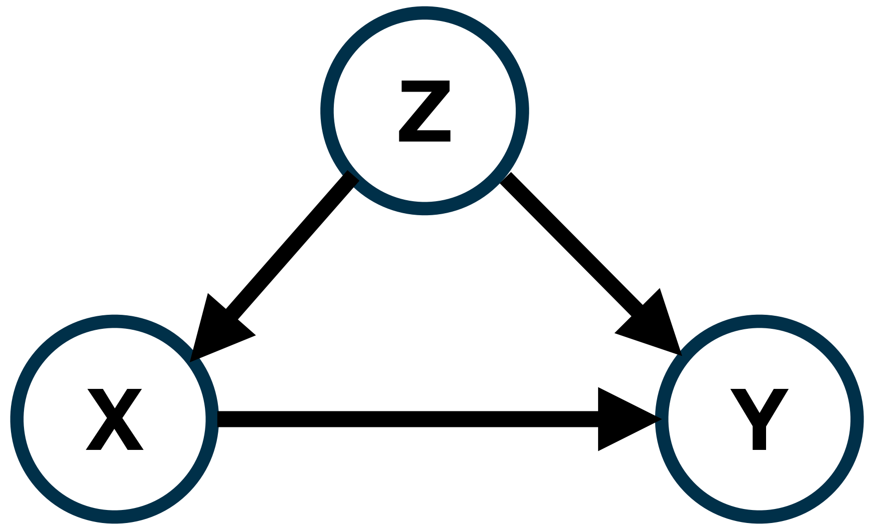
Recap: Classic BD estimators



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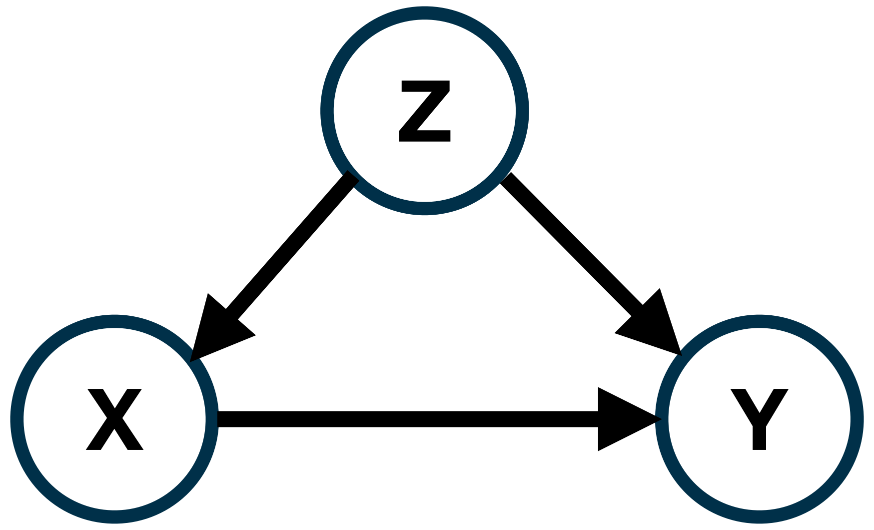

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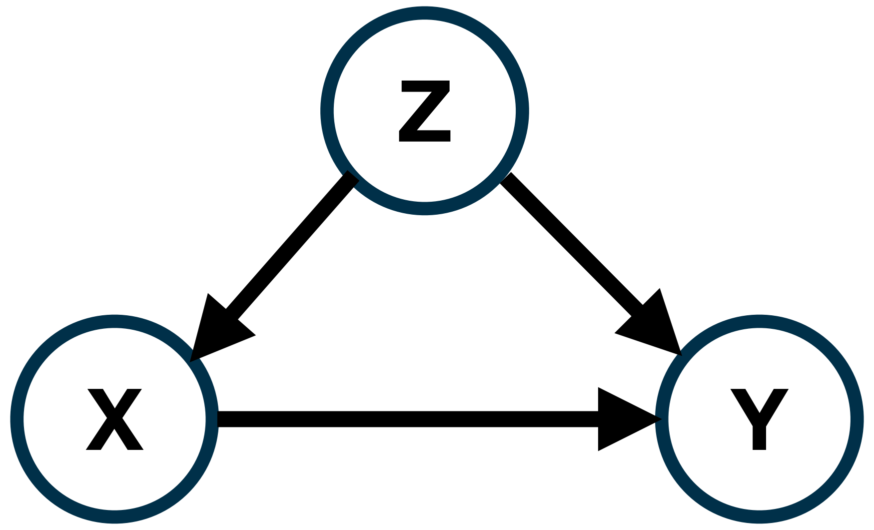

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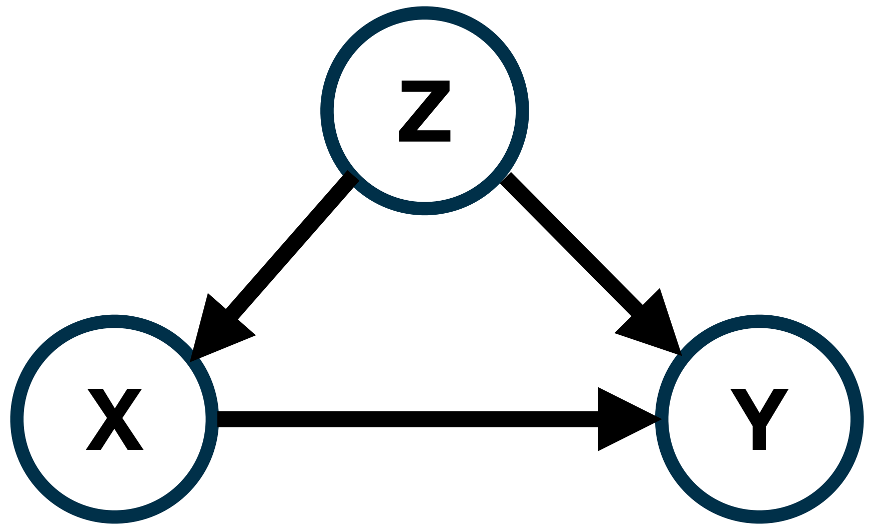

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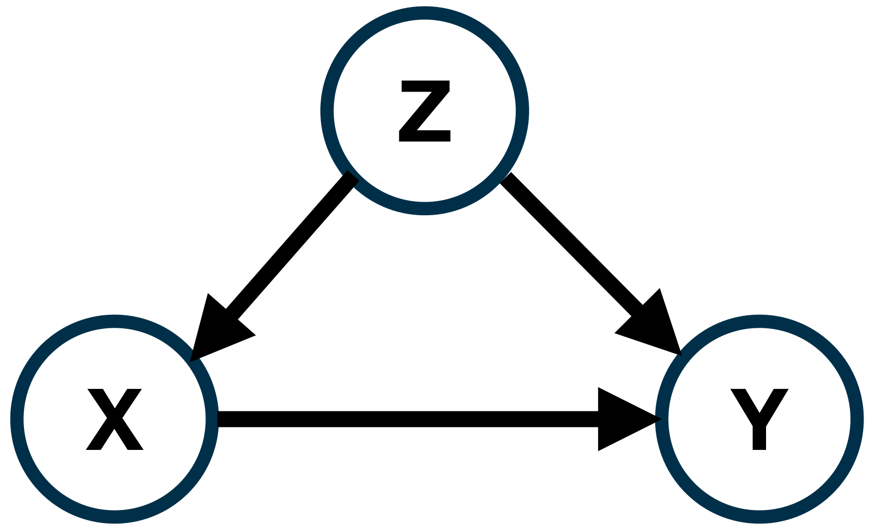

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For correct estimation	Nuisance functional should be correctly estimated .	
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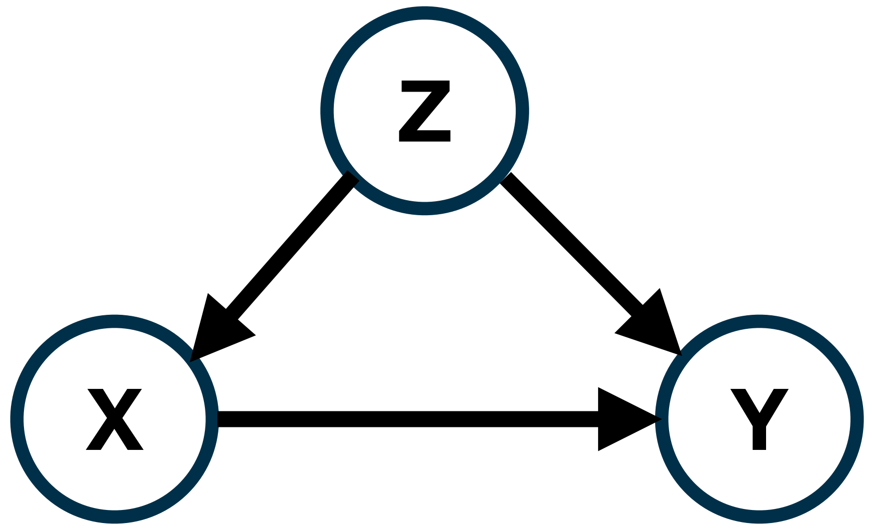

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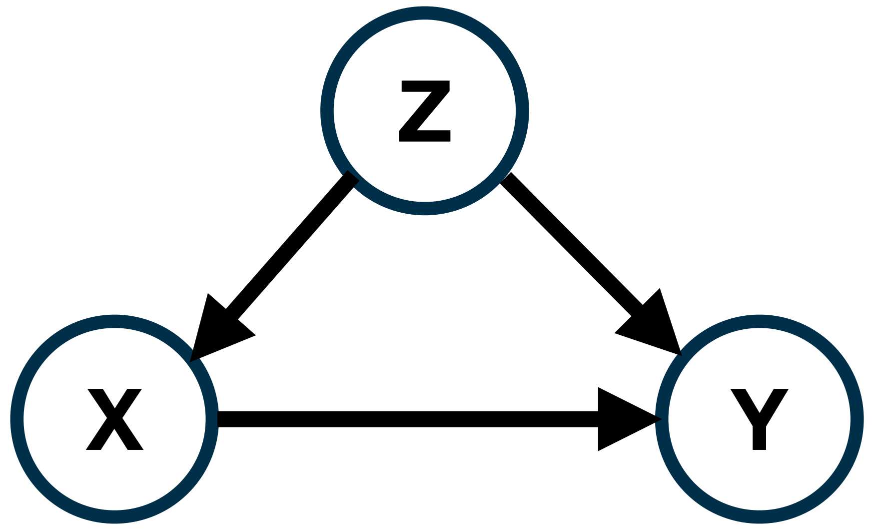

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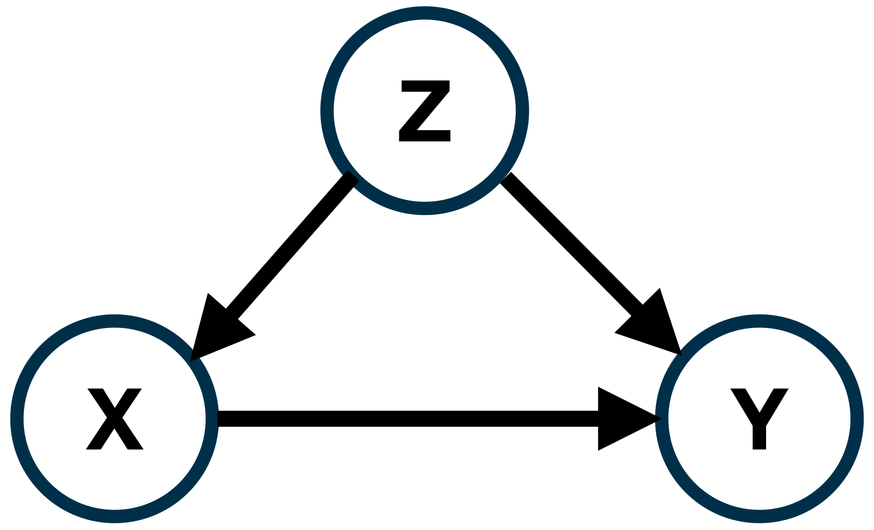
For correct estimation	Incorrectly estimated if estimates of nuisances are misspecified.	
For $\sqrt{N}$-consistency	Estimates for nuisance converges at $o_P(N^{-1/2})$.	

Recap: Classic BD estimators



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For correct estimation	**Incorrectly estimated** if estimates of nuisances are **misspecified.**	
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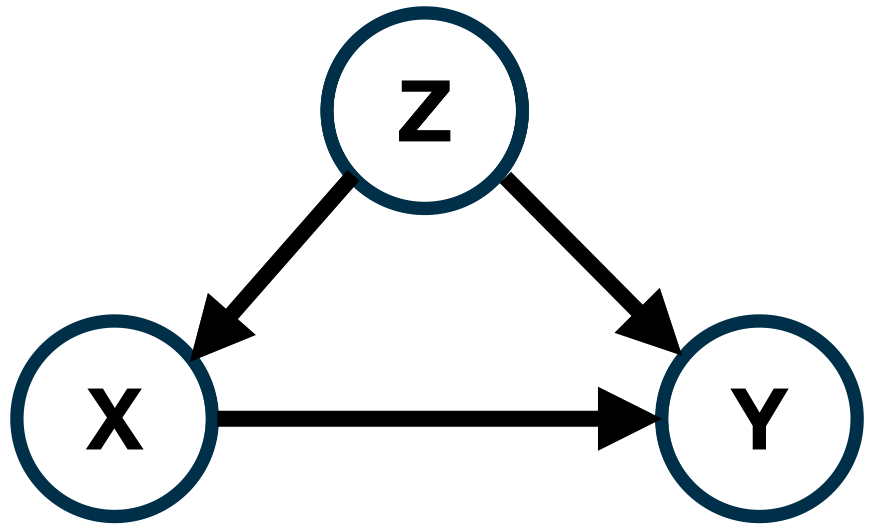

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😈 **Risk of classic estimators**

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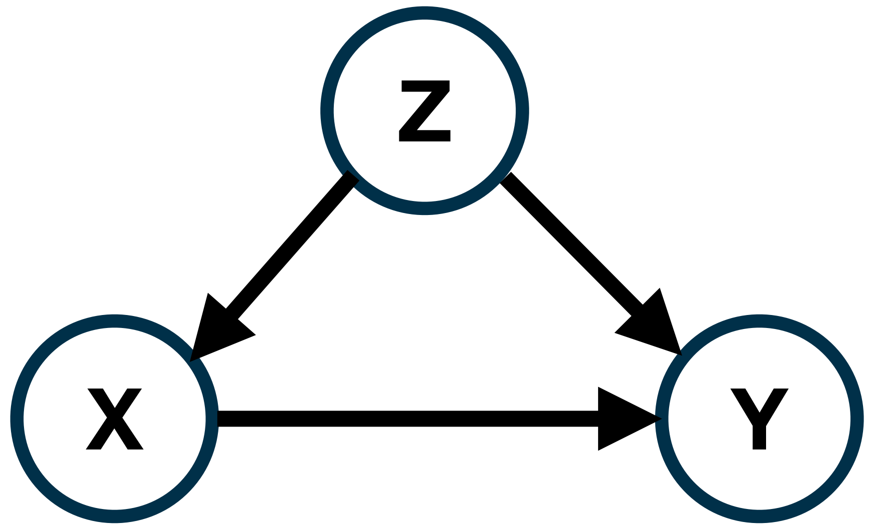

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- **Model misspecification** is common in practice when the data generating process is complicated $\Rightarrow$ Incorrect estimation.

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Estimator ($\widehat{Q}$)

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For $\sqrt{N}$-consistency

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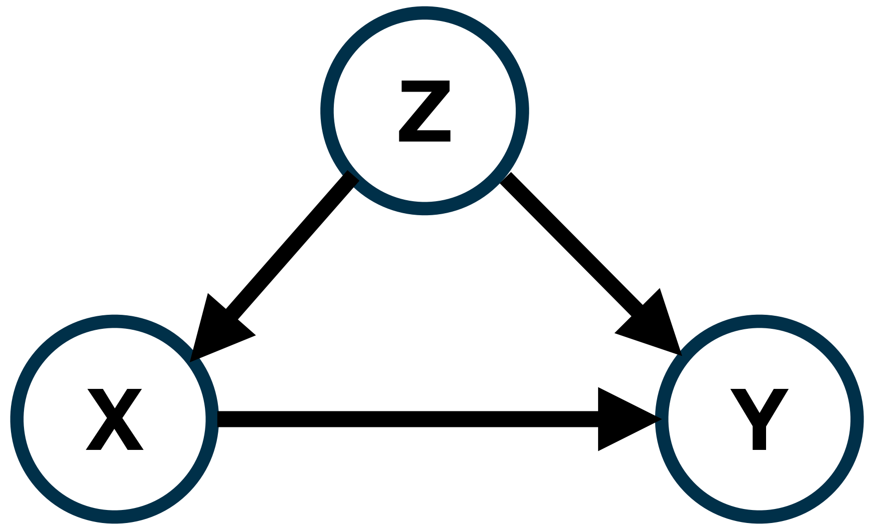
Recap: Classic BD estimators



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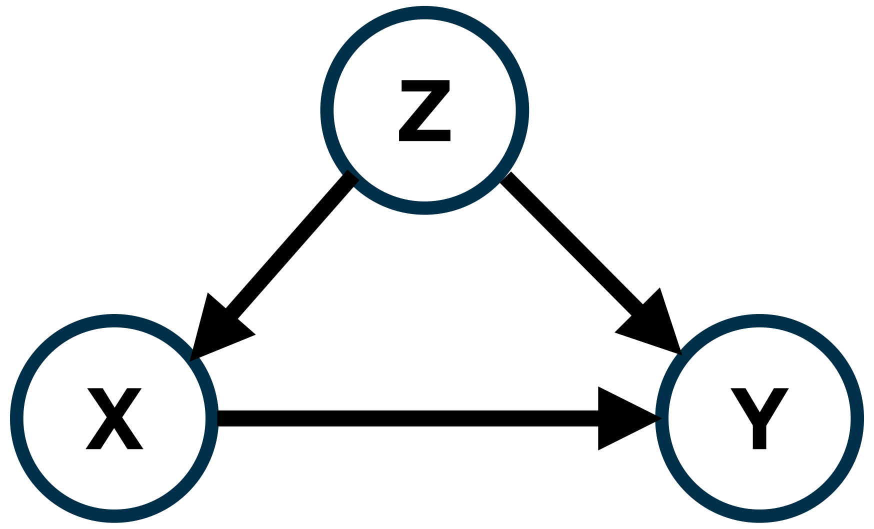
Recent advance: DML estimator



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	Double Machine Learning Estimator
Estimand Representation of Q	
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For correct estimation	
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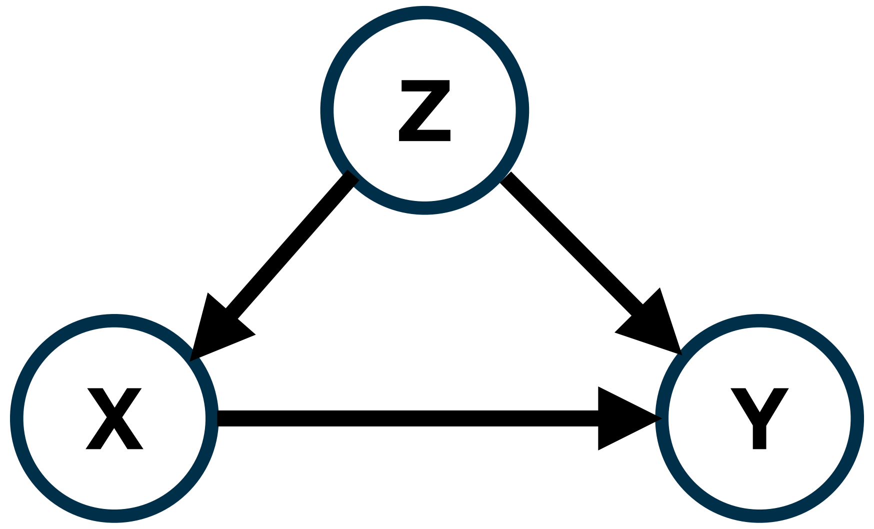

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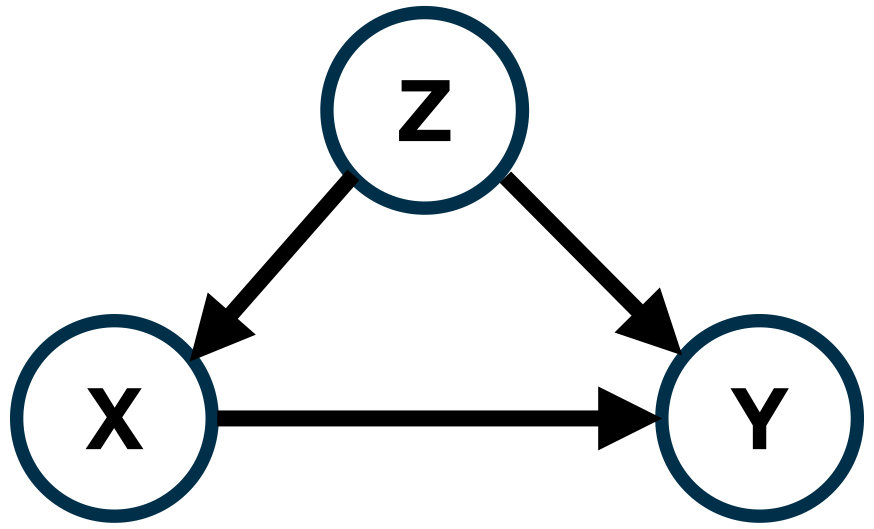
	Double Machine Learning Estimator
Estimand Representation of Q	$\mathbb{E}\left[\frac{I_x(X)}{P(X \mid Z)}\left(I_y(Y) - P(y \mid x, Z)\right) + P(y \mid x, Z)\right]$
Estimators Estimator $\widehat{Q}$	
For correct estimation	
For $\sqrt{N}$-consistency	

Recent advance: DML estimator



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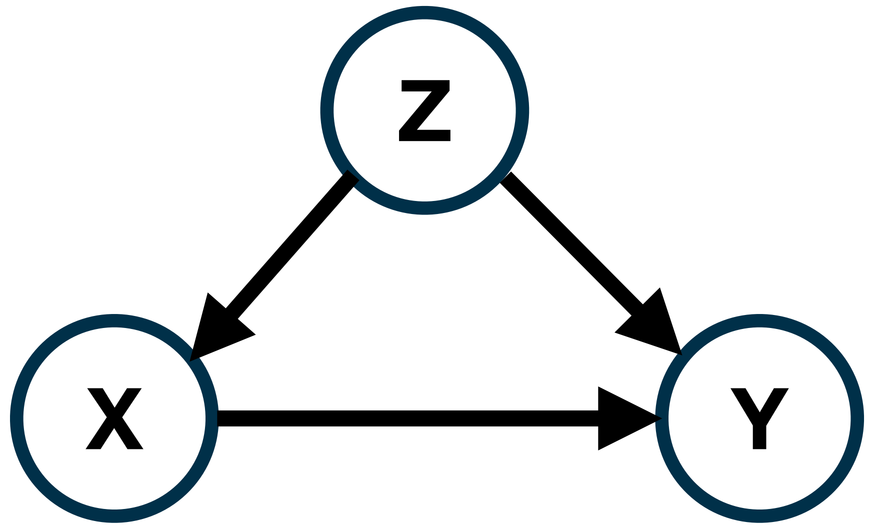

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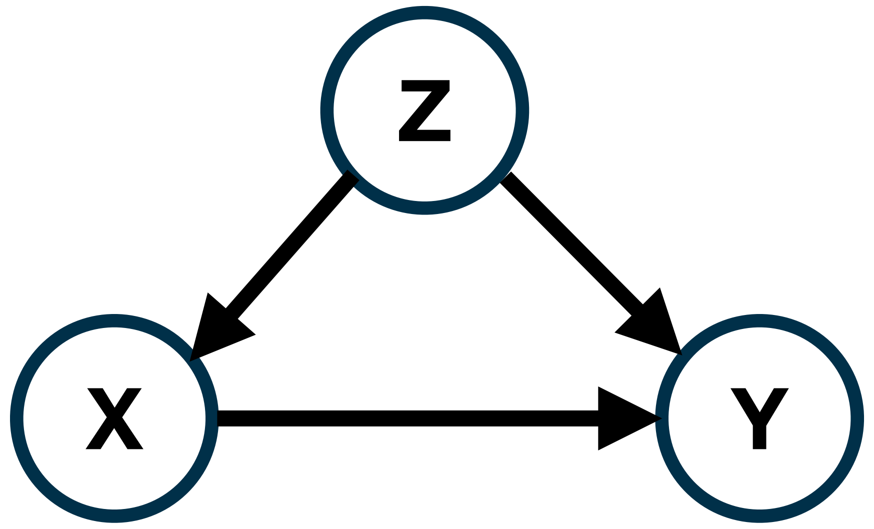
For correct estimation	
For $\sqrt{N}$-consistency	

Recent advance: DML estimator



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Double Machine Learning Estimator

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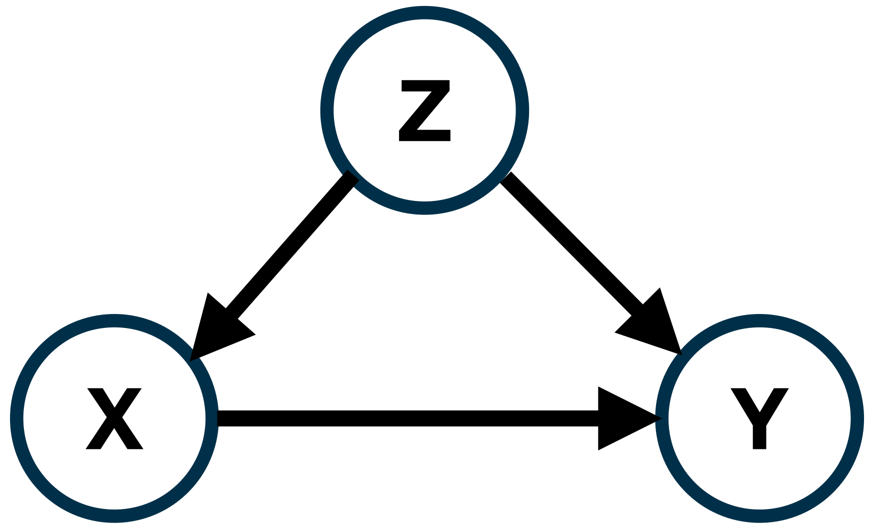
Estimators Estimator $\widehat{Q}$	$\frac{1}{N}\sum_{i=1}^{N}\frac{I_x(X_{(i)})}{\widehat{P}(X_{(i)} \mid Z_{(i)})}\left(I_y(Y) - \widehat{P}(y \mid x, Z)\right) + \widehat{P}(y \mid x, Z)$ where training and evaluation of $\widehat{P}$ are done with two distinct sets of samples ("***Cross fitting***")
For correct estimation	("Doubly robustness") **Either one of two nuisances** should be correctly **estimated**.
For $\sqrt{N}$-consistency	

Recent advance: DML estimator



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Double Machine Learning Estimator

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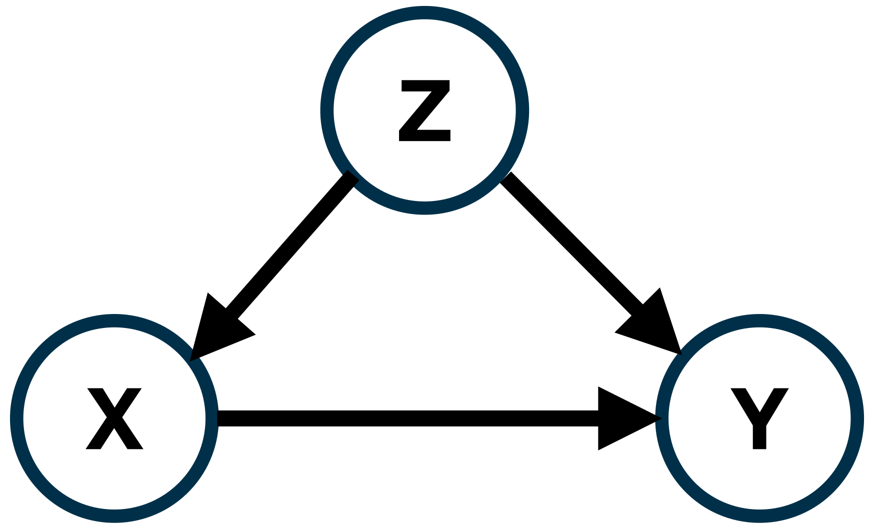
Estimators Estimator $\widehat{Q}$	$\frac{1}{N}\sum_{i=1}^{N} \frac{I_x(X_{(i)})}{\widehat{P}(X_{(i)} \mid Z_{(i)})}\left(I_y(Y) - \widehat{P}(y \mid x, Z)\right) + \widehat{P}(y \mid x, Z)$	where training and evaluation of $\widehat{P}$ are done with two distinct sets of samples ("***Cross fitting***")
For correct estimation	("Doubly robustness")	**Either one of two nuisances** should be correctly **estimated**.
For $\sqrt{N}$-consistency	("Debiasedness")	**Estimates** for **nuisance** converges at $o_P(N^{-1/4})$.

Recent advance: DML estimator

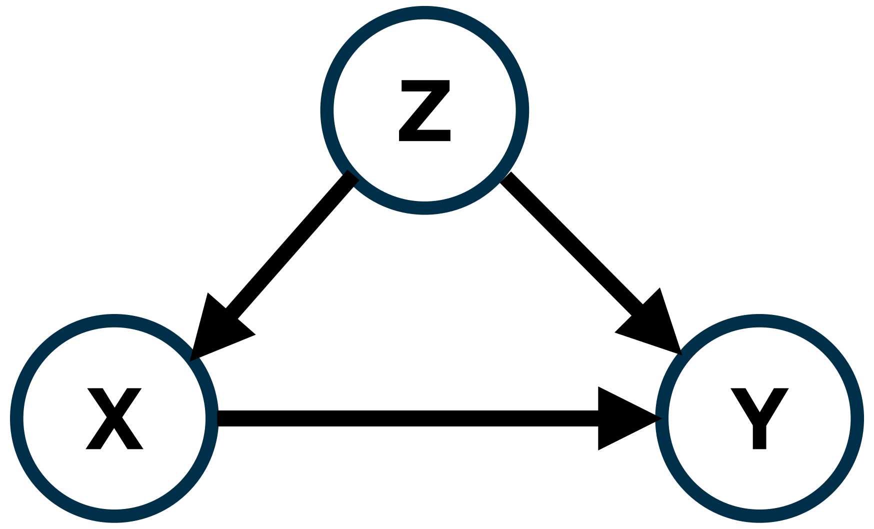


$$Q = \sum_z P(y \mid x, z) P(z)$$

DML estimator is robust!

Estimand Representation of Q		
Estimators Estimator $\widehat{Q}$		
For correct estimation	("Doubly robustness")	Either one of two nuisances should be correctly estimated.
For $\sqrt{N}$-consistency	("Debiasedness")	Estimates for nuisance converges at $o_P(N^{-1/4})$.

Recent advance: DML estimator



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Representation of Q

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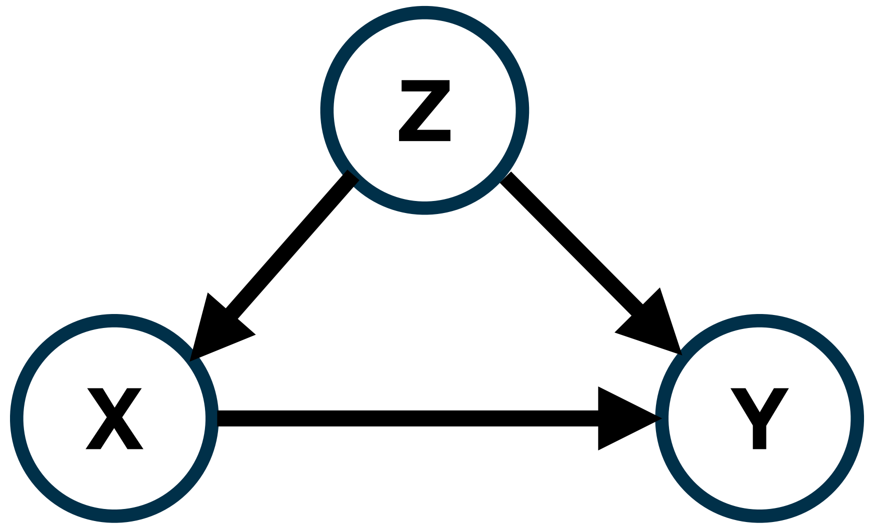

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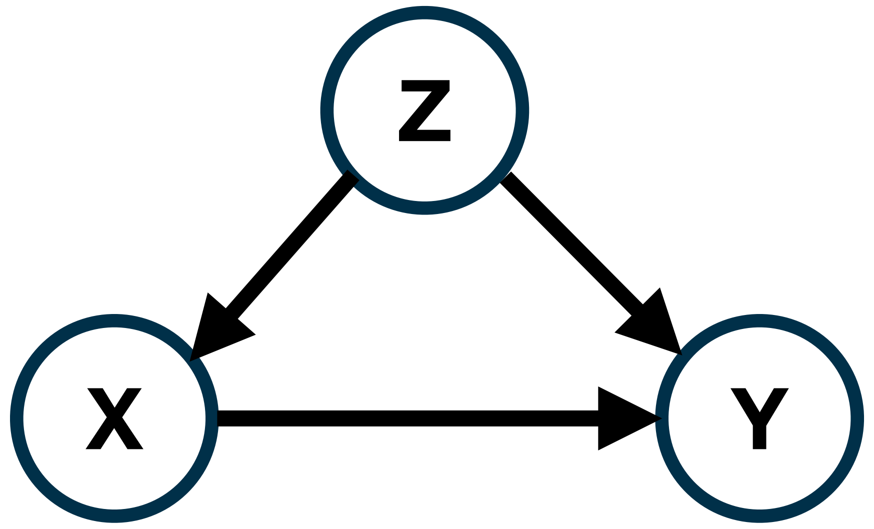

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Is DML estimators applicable for estimating any identifiable causal functional?

Preliminary — DML estimator (Chernozhukov, 2016)

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	Classic	DML
Model class	Donsker	Arbitrary
Permitted class	• Linear regression, • Logistic regression	• Any regressors in Donsker • Neural networks
Forbidden class	Random forest, Lasso, deep neural networks.	x

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💪 Power of DML

One can employ *any* complicated ML model and enjoy debiasedness

Problem Setup

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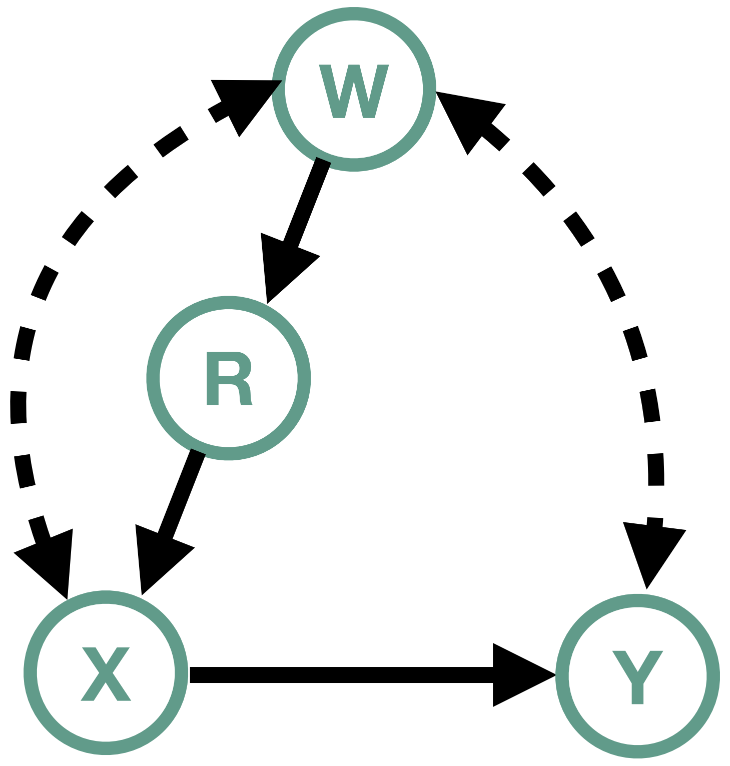

Graph G

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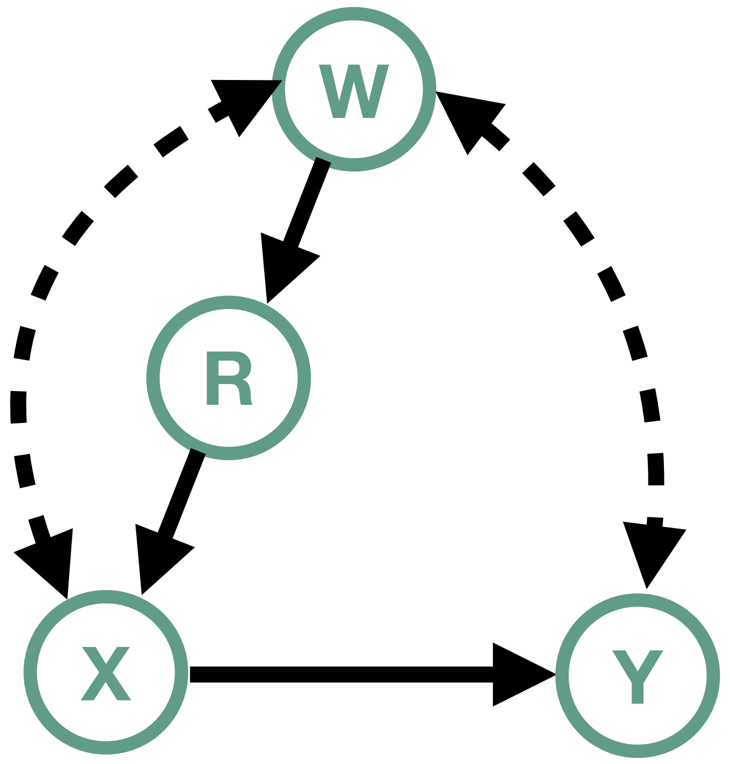

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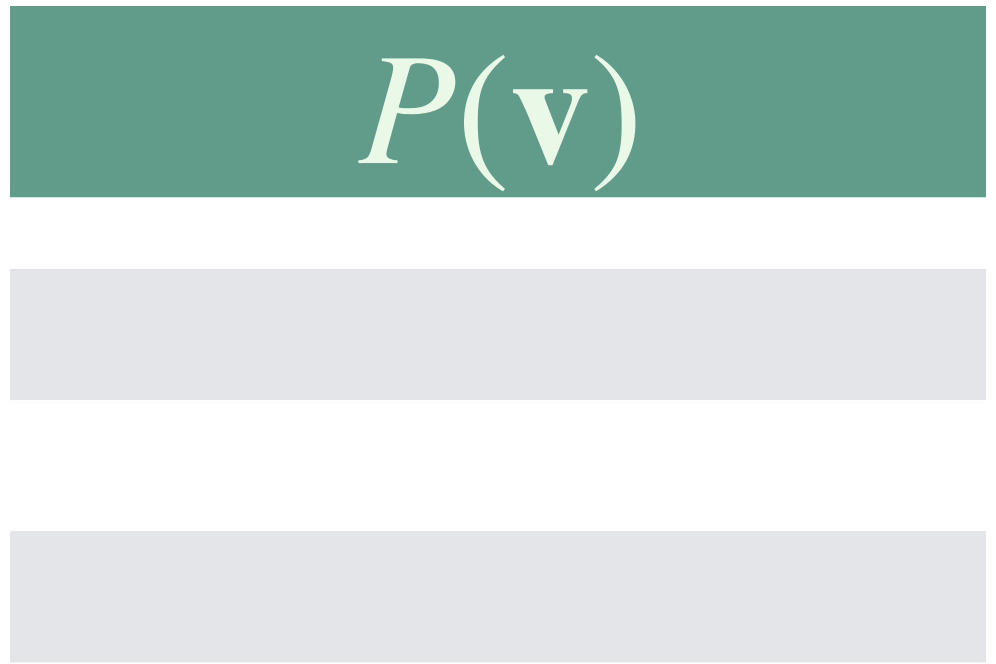

Samples D from joint distribution $P(\mathbf{v})$

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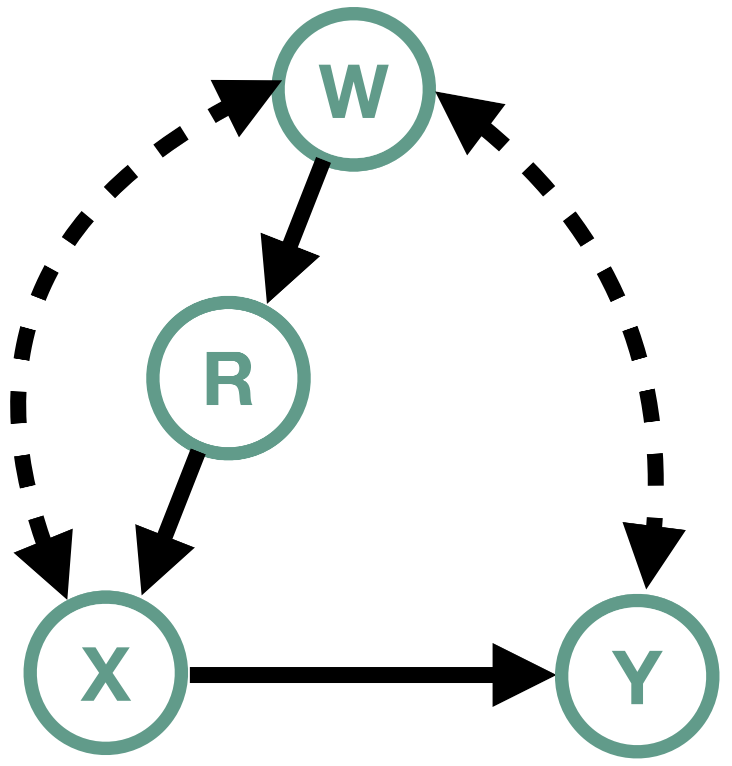

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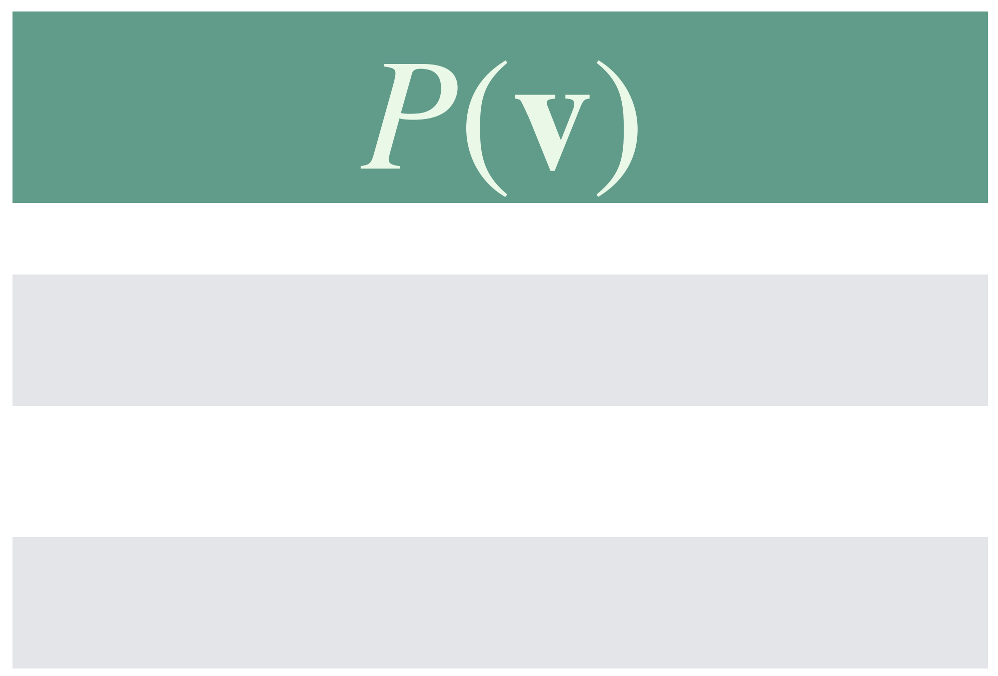

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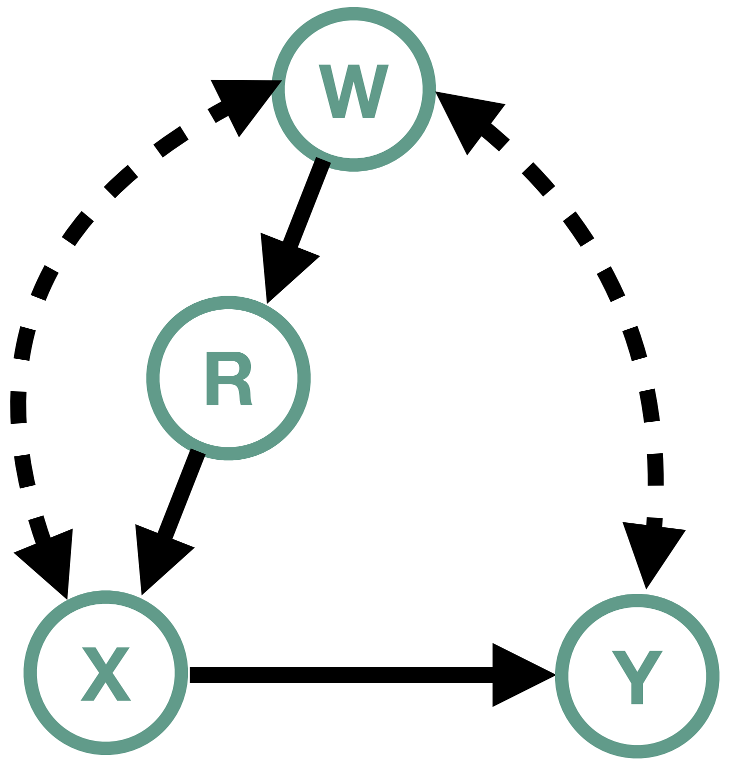

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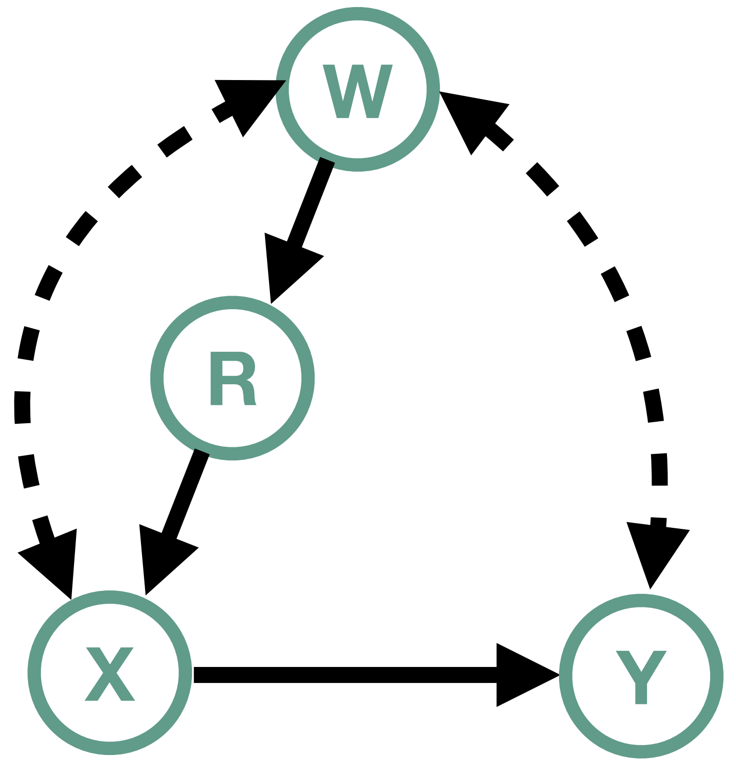

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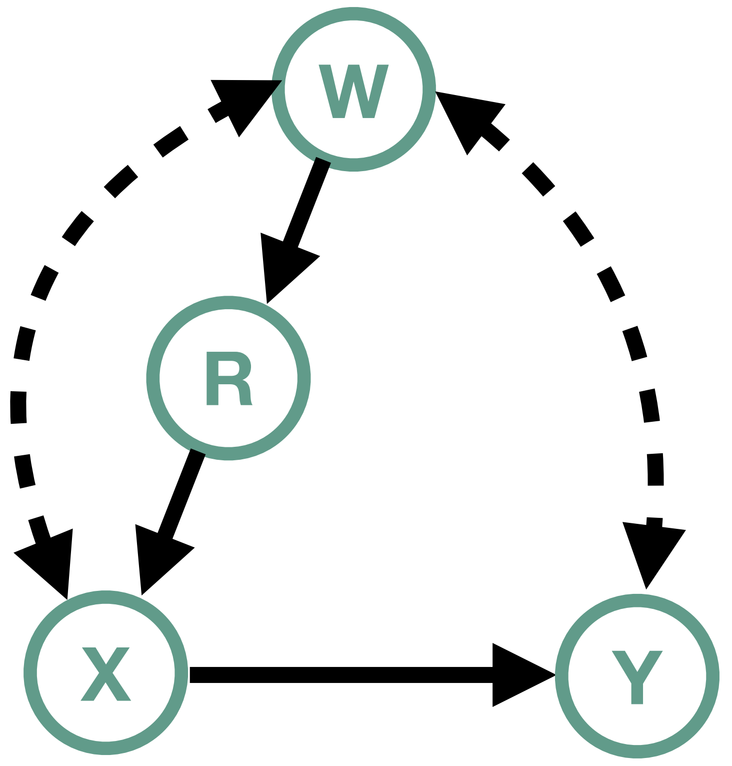

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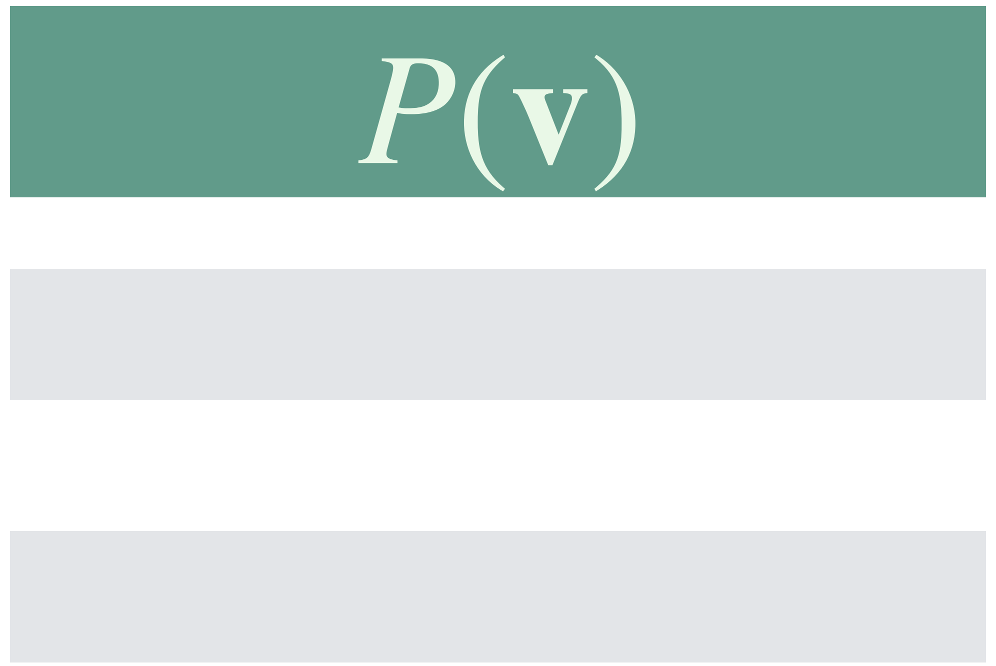

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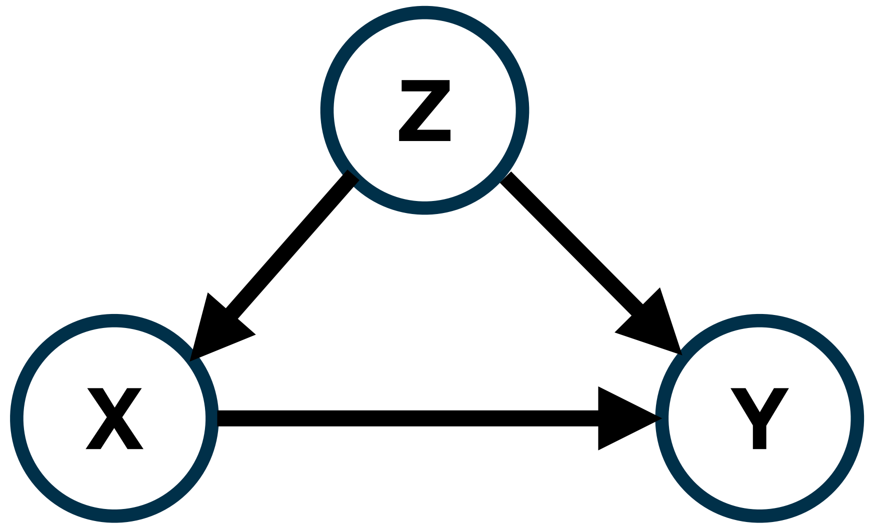

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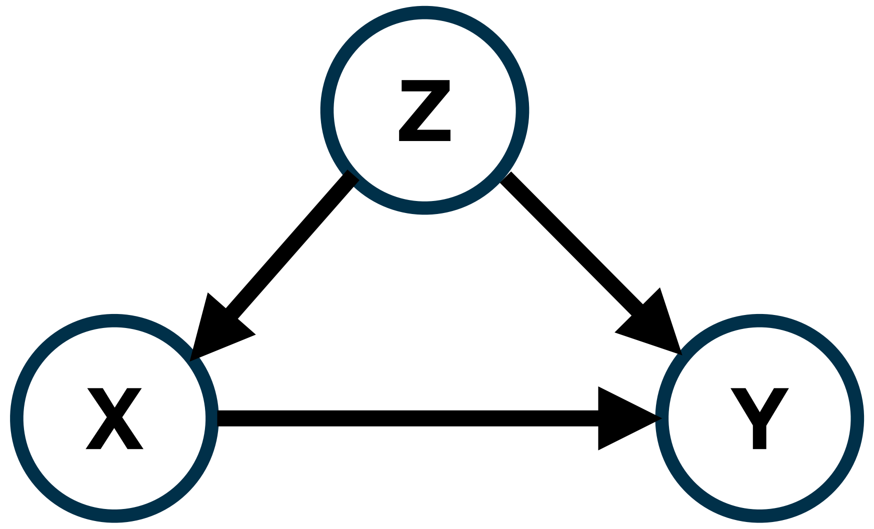

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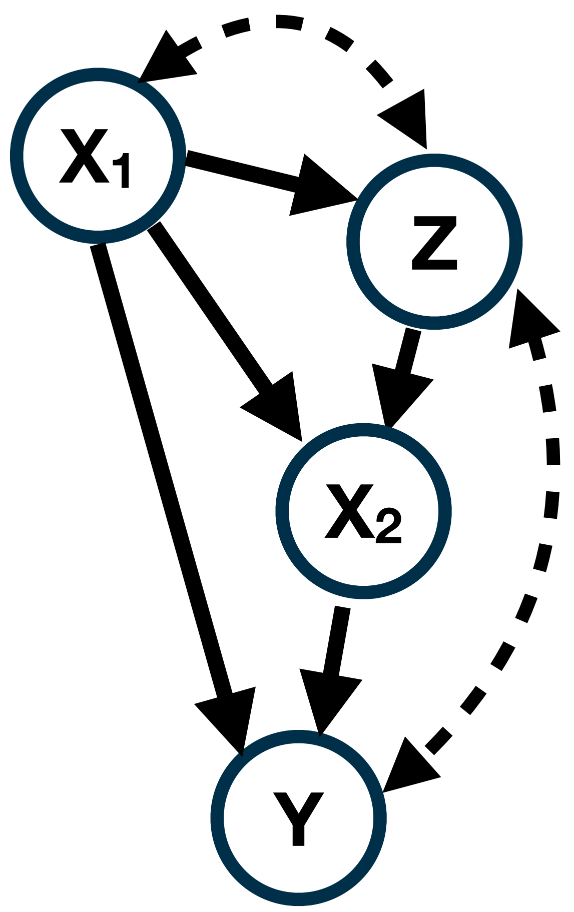
$$P(\mathbf{y} \mid do(\mathbf{x})) = \sum_{\mathbf{z}} \prod_{V_i \in \{\mathbf{Y}, \mathbf{Z}\}} P(\mathbf{v}_i \mid \mathbf{v}^{(i-1)}),$$

- for $\mathbf{V}_i = \mathbf{Y}_i, \mathbf{V}^{(i-1)} = \{\mathbf{X}^{(i)}, \mathbf{Z}^{(i)}, \mathbf{Y}^{(i-1)}\}$; and
- for $\mathbf{V}_i = \mathbf{Z}_i, \mathbf{V}^{(i-1)} = \{\mathbf{Y}^{(i-1)}, \mathbf{X}^{(i-1)}, \mathbf{Z}^{(i-1)}\}$.



$$P(y \mid do(x)) = \sum_z P(y \mid x_2, z, x_1) P(z \mid x_1)$$

Result 0. mSBD — Neyman Orthogonal score & DML estimator

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Neyman orthogonal score for mSBD

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Our Approach for marrying DML + Identification (DML-ID)

Result 0. (mSBD)

- Derive the Neyman orthogonal score when there are no unmeasured confounders b/w (**X**, **Y**) (called *multi-outcome sequential back-door (mSBD)*).

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- **C-component $\mathbf{C}$**: A set of variables s.t. connected by bidirected path.
- **C-factor $Q[\mathbf{C}]$**: A interventional distribution of $\mathbf{C}$ under the intervention $\mathbf{V} \backslash \mathbf{C}$; i.e., $Q[\mathbf{C}] \equiv P(\mathbf{C} \mid do(\mathbf{V} \backslash \mathbf{C}))$

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- A causal functional $P(\mathbf{y} \mid do(\mathbf{x}))$ is identifiable if and only if it is represented as an arithmetic of C-factors of C-component $\mathbf{C}_i$ in G (Tian and Pearl, 2003, Huang and Valtorta 2006).

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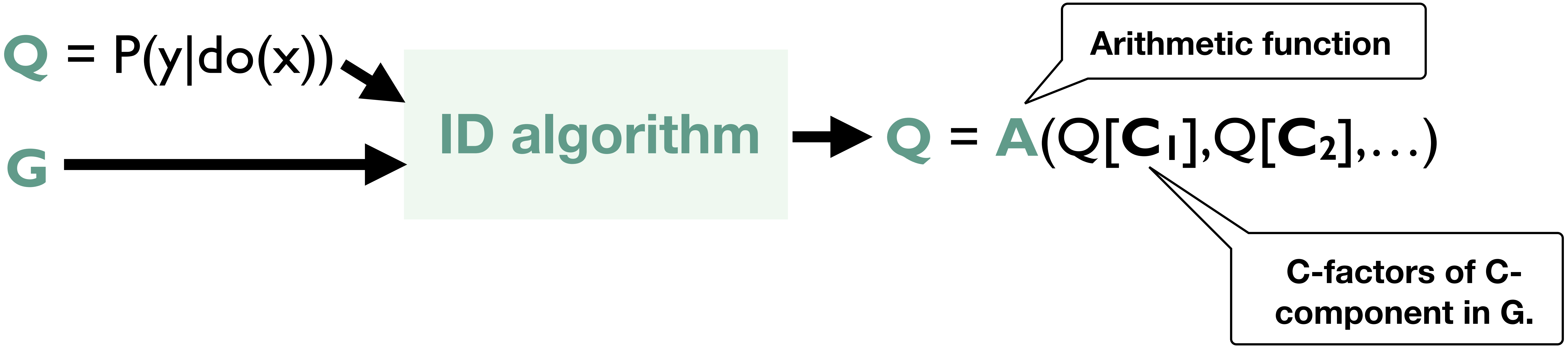

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$Q = A(Q[C_1], Q[C_2], \ldots)$

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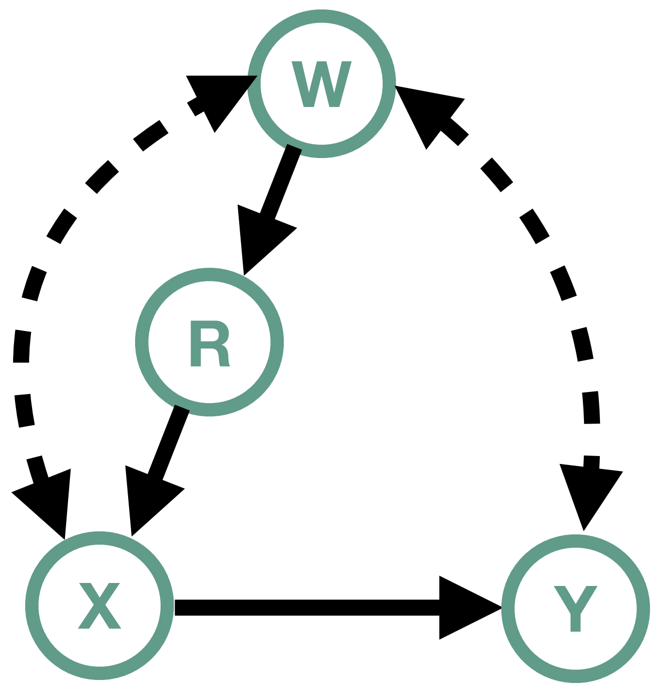

$P(y \mid do(x))$

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$$\mathbf{Q} = \frac{\sum_w Q[W, X, Y]}{\sum_{w,y} Q[W, X, Y]} = \frac{\sum_w P(x, y \mid r, w) P(w)}{\sum_w P(x \mid r, w) P(w)}$$

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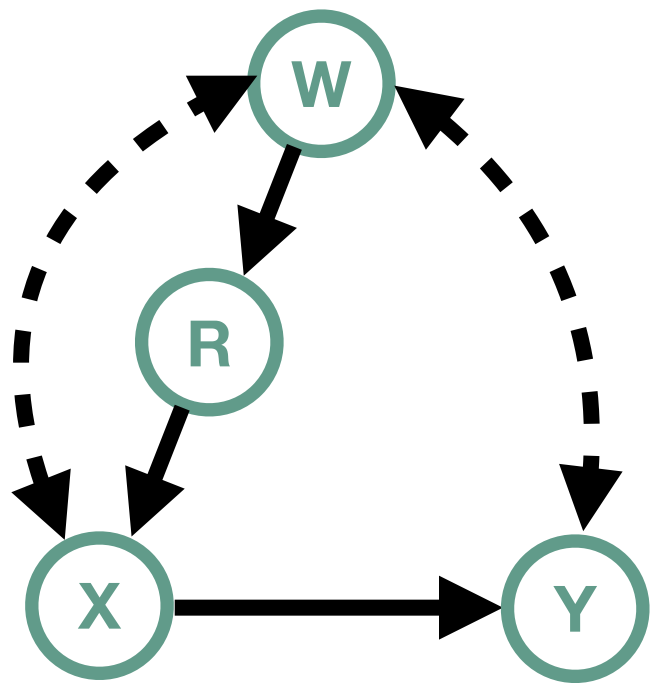

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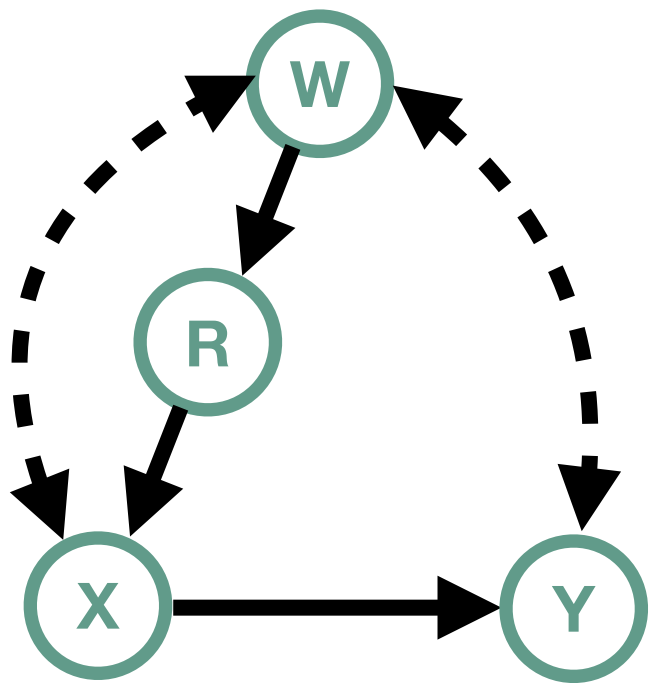

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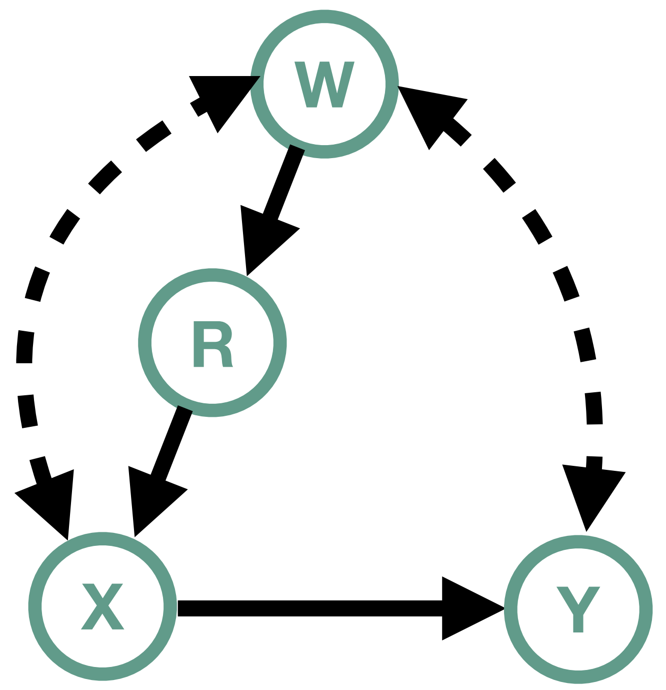

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→ **Q** $= \dfrac{M[(x,y) \mid r; w]}{M[x \mid r; w]} = \dfrac{\sum_w P(x,y \mid r,w)P(w)}{\sum_w P(x \mid r,w)P(w)}$

mSBD adjustment of $P(x,y \mid r,w)$ **over W**

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Neyman orthogonal score

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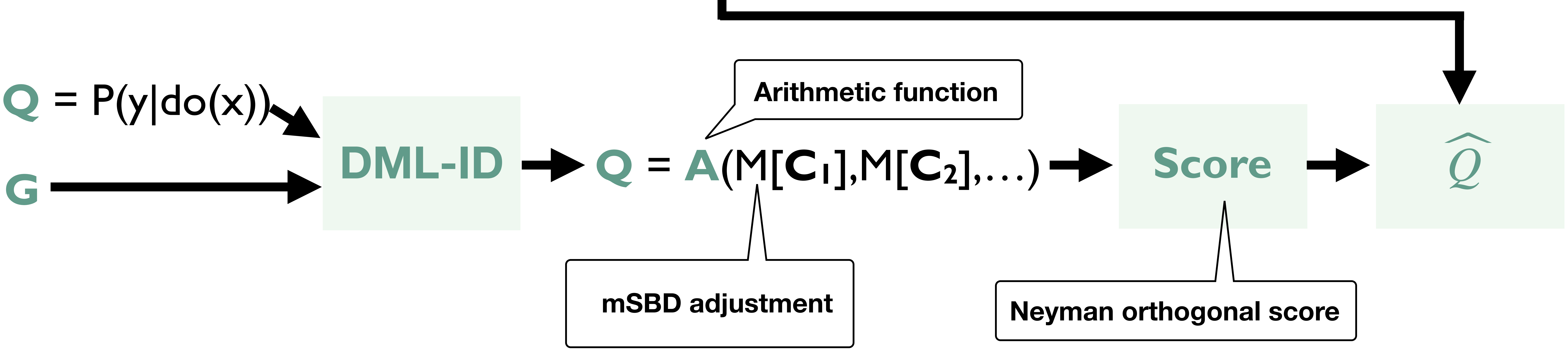

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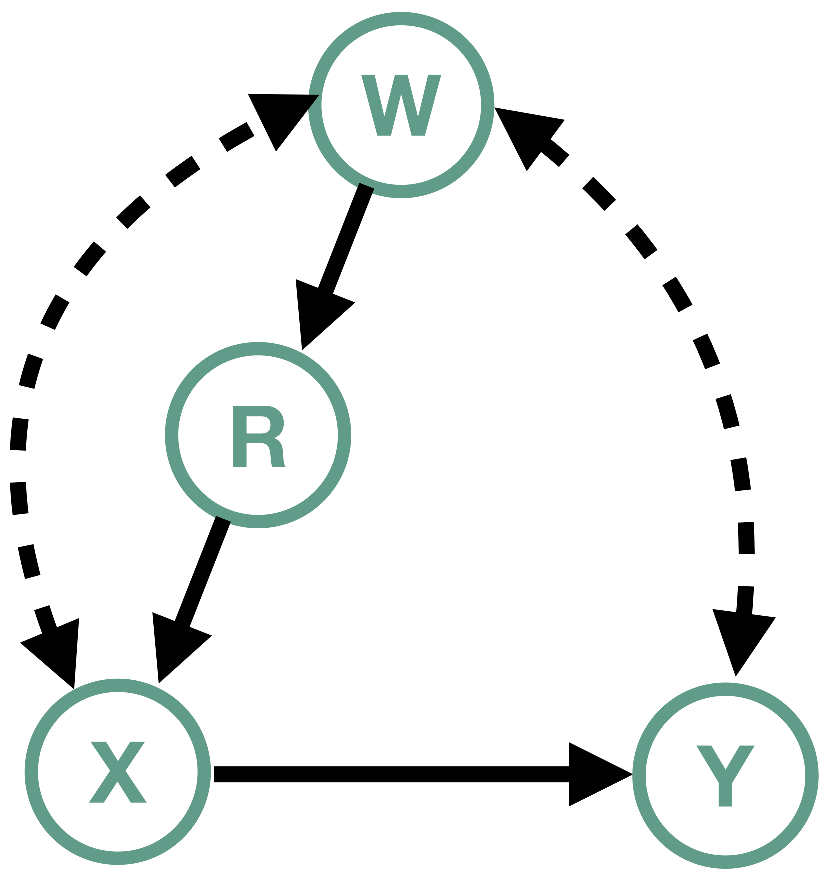

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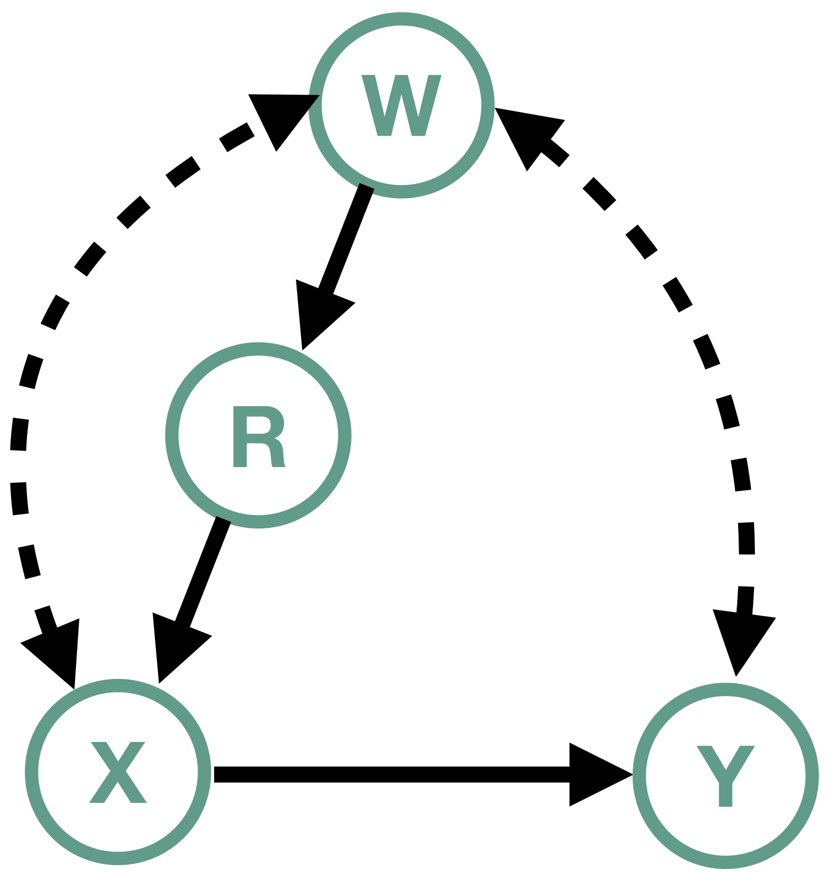

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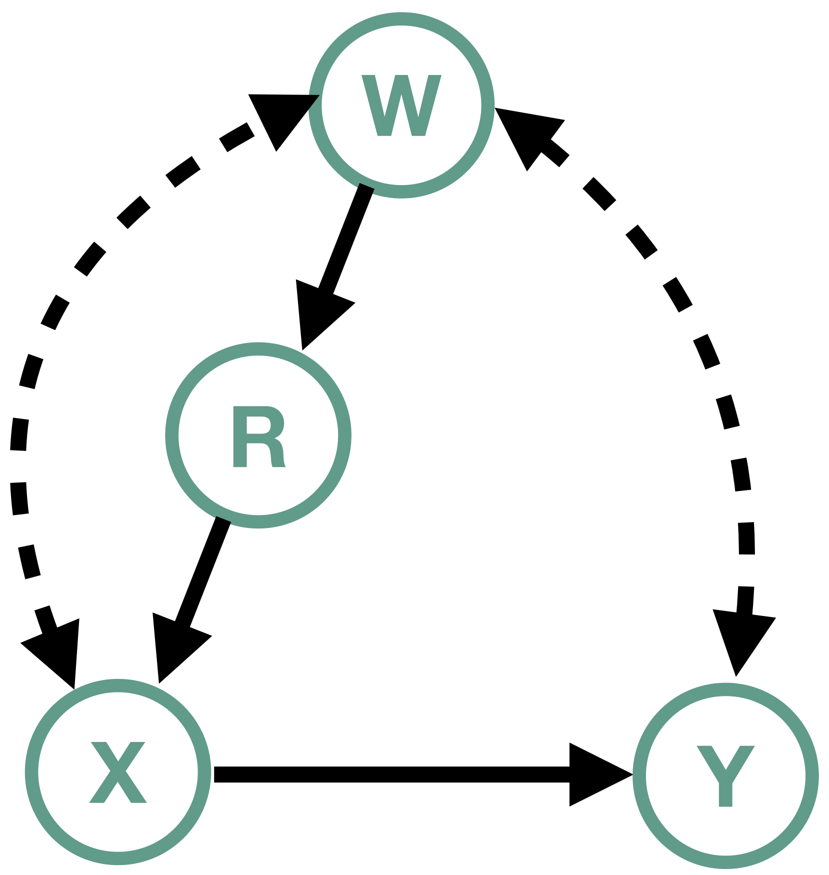

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- A reason that we compare with the plug-in estimator is because this is only viable estimator that is working for arbitrary causal functional.

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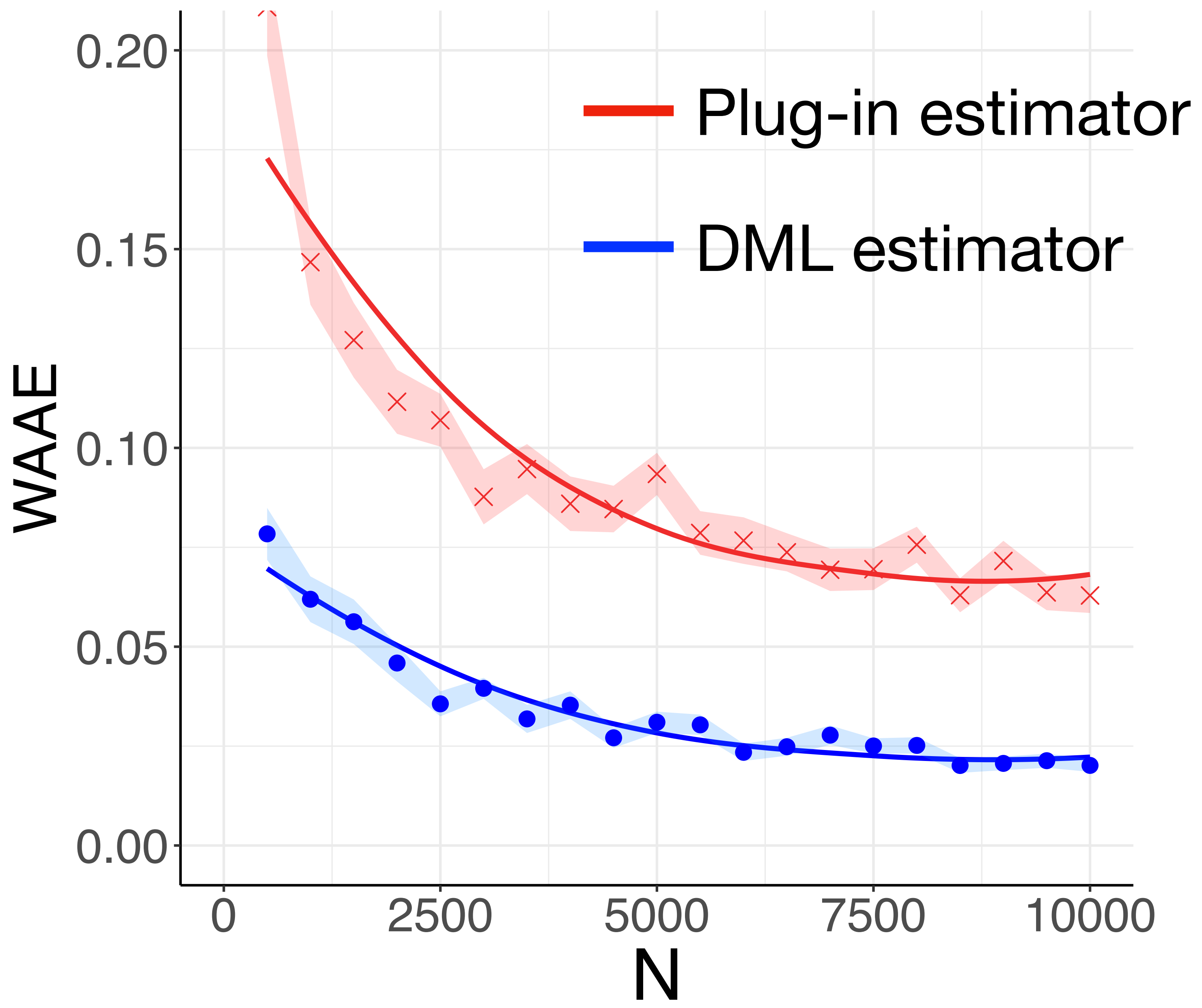		

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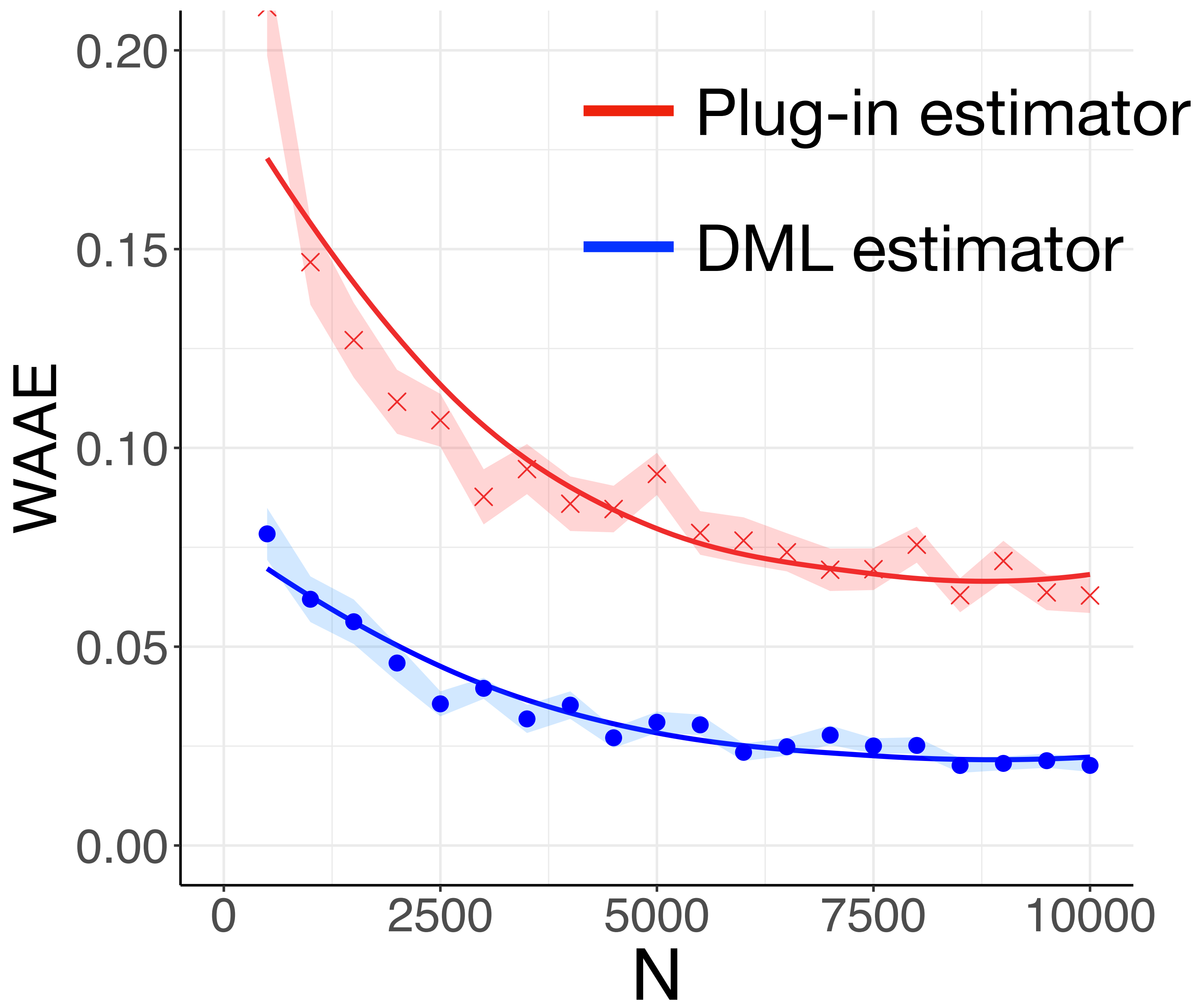		

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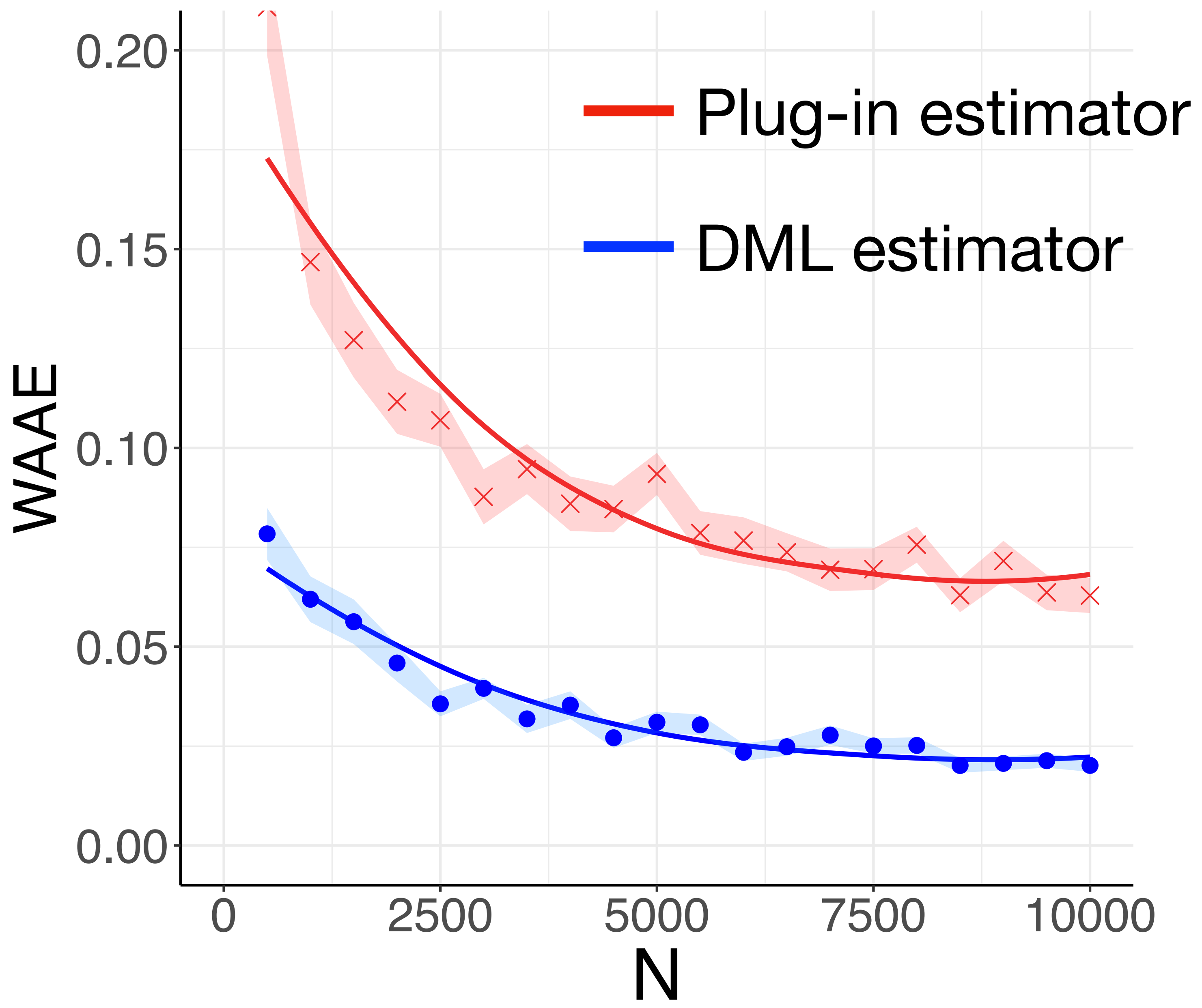

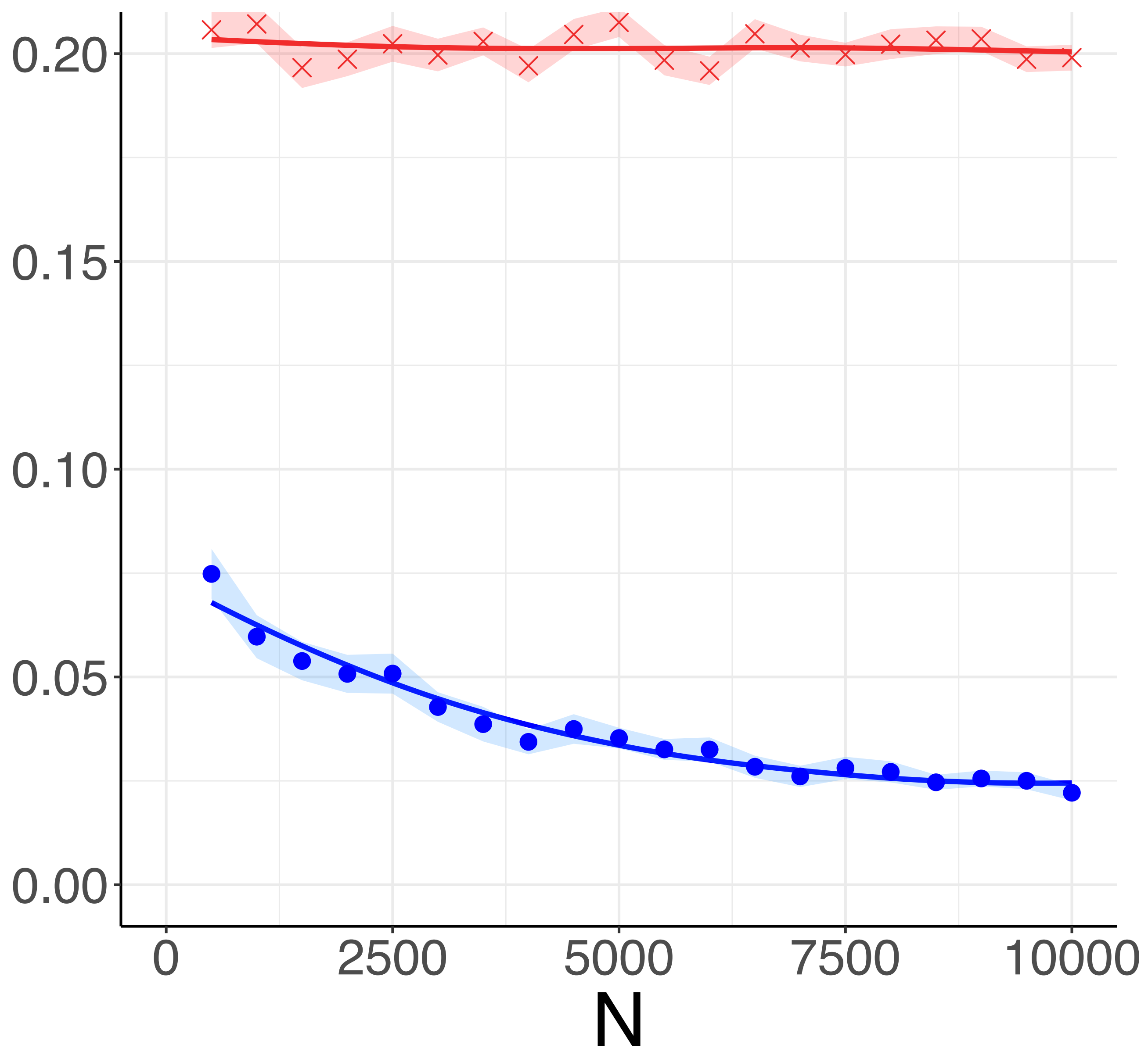

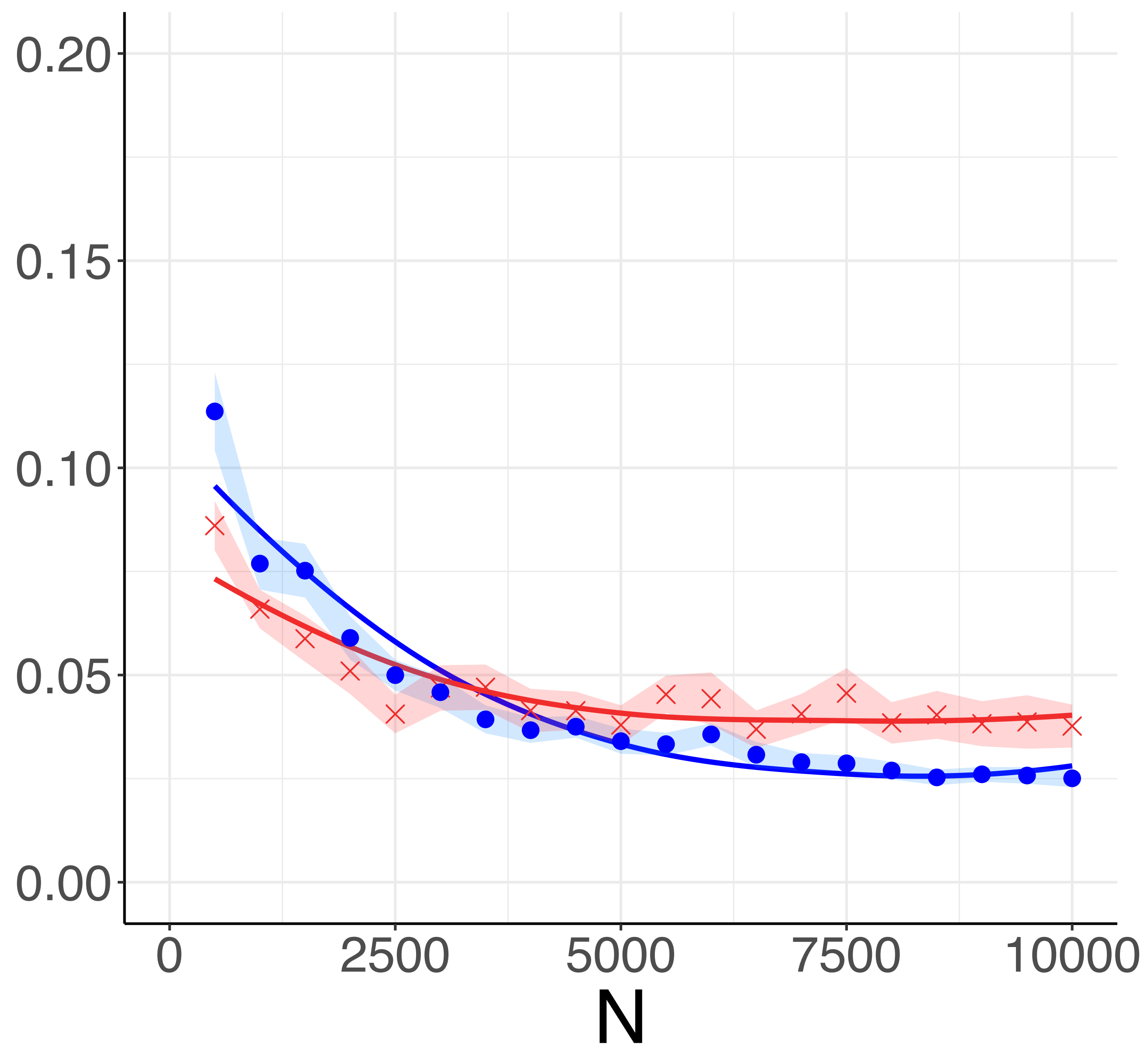

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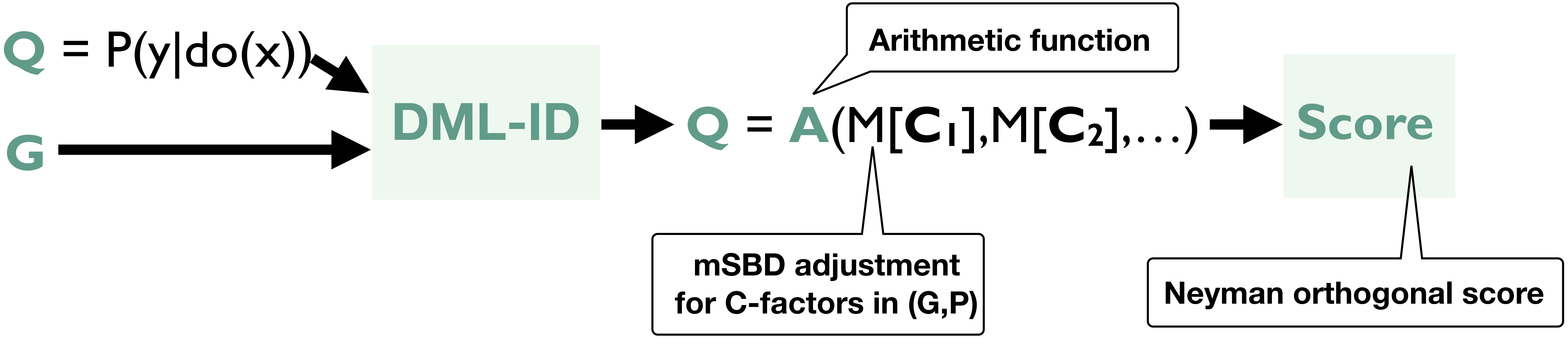
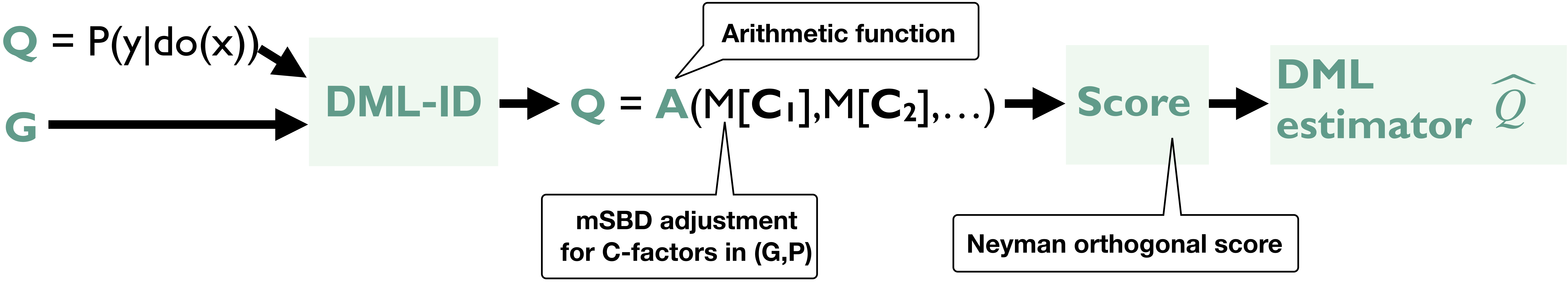


Conclusion

- **Result 1** — We develop a systematic procedure for deriving Neyman orthogonal score for estimands of any identifiable causal effects.

- **Result 2** — We develop DML estimators for any identifiable causal effect, which enjoy debiasedness and doubly robustness against model misspecification and bias.



Thank you for listening!