Unified Covariate Adjustment for Causal Inference

Yonghan Jung[1], Jin Tian[2], Elias Bareinboim[3] [1] Purdue University, [2] MBZUAI, [3] Columbia University

Yonghan is on the academic job market! Visit www.yonghanjung.me

Motivation

Causal effects are often identified as *various forms of covariate adjustments (which is a summation of products of conditional distributions).*

Back-door adjustment	$\mathbb{E}[Y \mid \mathrm{do}(x)] = \sum_{z} \mathbb{E}[Y \mid x, z]P(z)$
Front-door adjustment	$\mathbb{E}[Y \mid \mathrm{do}(x)] = \sum_{x',z,c} \mathbb{E}[Y \mid x', z, c]P(z \mid x, c)P(x', c)$
Verma's equation	$\mathbb{E}[Y \mid \mathrm{do}(x)] = \sum_{a,b,x'} \mathbb{E}[Y \mid a, b, x]P(b \mid a, x')P(a \mid x)P(x')$
Domain Generalization	$\mathbb{E}[Y \mid \mathrm{do}(x), S = 0] = \sum_{z} \mathbb{E}[Y \mid x, z, S = 1]P(z \mid S = 0)$
Policy Intervention	$\mathbb{E}[Y_{\sigma_X}] = \sum_{z,x'} \mathbb{E}[Y \mid x', z]\sigma_X(x' \mid z)P(z)$
Effect of Treatment on the Treated	$\mathbb{E}[Y_{X=1} \mid X = 0] = \sum_{z} \mathbb{E}[Y \mid X = 1, z]P(z \mid X = 0)$

However, no *unified* estimators for those various covariate adjustments with (1) *computational efficiency* and (2) *statistical robustness exists.*

United Covariate Adjustment (UCA)

The UCA is $\sum_{c_1,\cdots,c_m} \mathbb{E}_{P^{m+1}}[Y \mid c^{(m)}, r^{(m)}] \prod_{i=1}^{m} P^i(c_i \mid c^{(i-1)}, r^{(i-1)})\sigma^i_{R_i}(r_i \mid c^{(i)}, r^{(i-1)})$,

where P^i is an arbitrary distribution and $\sigma^i_{R_i}$ is an intervention policy.

Various forms of covariate adjustment, beyond the back-door adjustment, can be represented as UCA.

Example: UCA for expressing the front-door adjustment

The UCA reduces to $\sum_{x',z,c} \mathbb{E}[Y \mid x', z, c]P(z \mid x, c)P(x', c)$, with

$(R_1, R_2) = \varnothing, (C_1, C_2, Y) = (X, Z, Y), P^1(C_1) = P(X), P^2(C_2 \mid C_1) := P(Z \mid x), P^3(Y \mid C_1, C_2) = P(Y \mid Z, X)$.

DML-UCA: Estimator for UCA

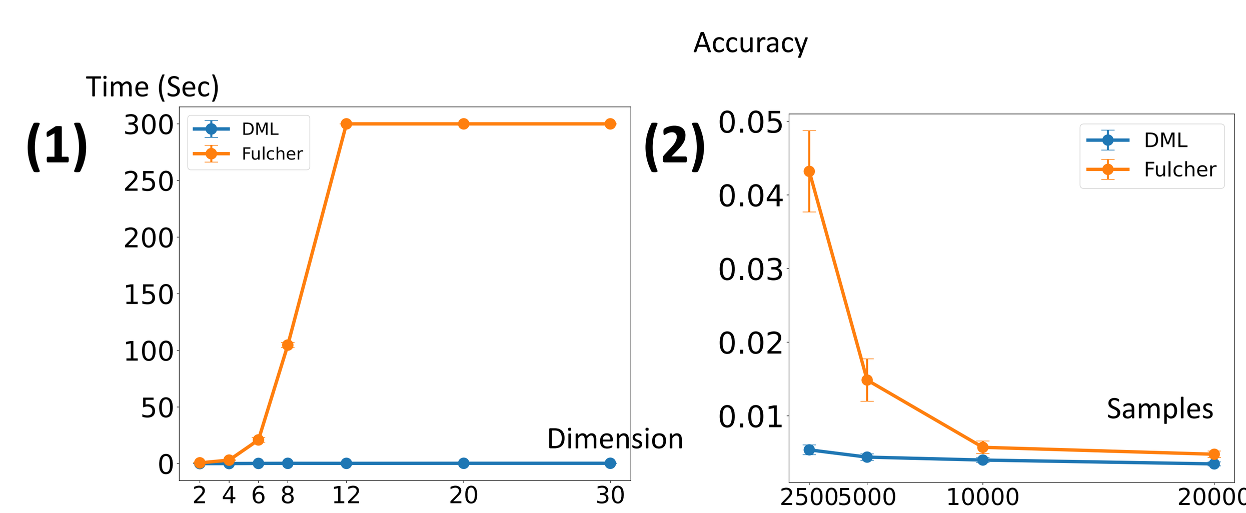
DML-UCA$(\{\hat{\mu}^i, \hat{\pi}^i\}, \{D^i\}) = \sum_{i=1}^{m} \mathbb{E}_{D^{i+1}}[\hat{\pi}^i\{\check{\mu}^{i+1} - \hat{\mu}^i\}] + \mathbb{E}_{D^1}[\check{\mu}^1]$ (where is $\{\hat{\mu}^i, \hat{\pi}^i\}$ are nuisance estimates and $D^i \sim P^i$ are samples) satisfies the following:

1. *Computational Efficiency* (evaluable in polynomial time); and
2. *Doubly Robustness* (If $\hat{\mu}^i$,$\hat{\pi}^i$ converges to μ^i_0, π^i_0 at $n^{-1/4}$ rate, DML-UCA converges at $n^{-1/2}$ rate).

Example: DML-UCA for the front-door adjustment

DML-UCA evaluates $\mathbb{E}_P[\pi^2_0(Z, X, C)\{Y - \mu^2_0(Z, X, C)\} + \mathbb{E}_P[\pi^1(C)\{\mu^2(Z, X', C) - \mu^1(X', C)\} \mid x] + \mathbb{E}_P[\mu^1_0(X, C)]$,

- $\mu^2 = \mathbb{E}[Y \mid Z, X, C]$, $\mu^1_0 = \mathbb{E}[\mu^2_0(Z, X', C) \mid x, C, X']$, and X' is an independent copy of X.
- $\pi^2_0 = P(Z \mid x, C)/P(Z \mid X, C)$, $\pi^1 = P(x)/P(x \mid C)$.



(1) Computational Efficiency: The estimator can be estimable in a polynomial time.

(2) Doubly Robustness: The estimator converges fast even when $\{\pi^i\}$ and $\{\mu^i\}$ converges slow.

Conclusion

1. *Unified Covariate Adjustment (UCA)* unifies various covariate adjustments in a form of a summation of products of conditional distributions.
2. We develop *DML-UCA* that is *computationally efficient* and *doubly robust* and provide its finite sample guarantee.