Estimating Joint Treatment Effects by Combining Multiple Experiments

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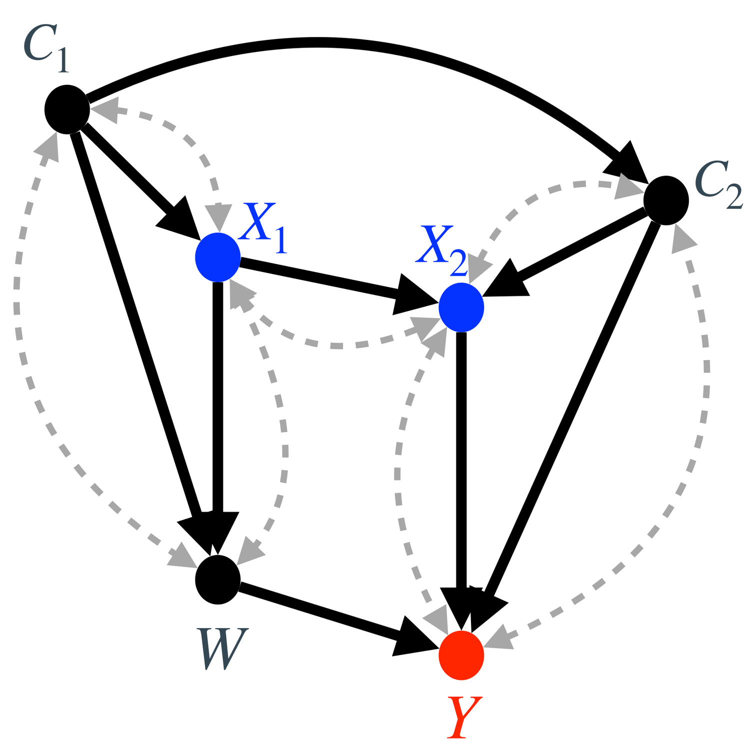
Summary

This paper provides identification conditions for joint treatment effects from multiple marginal experiments and develops estimators using data from multiple experiments.

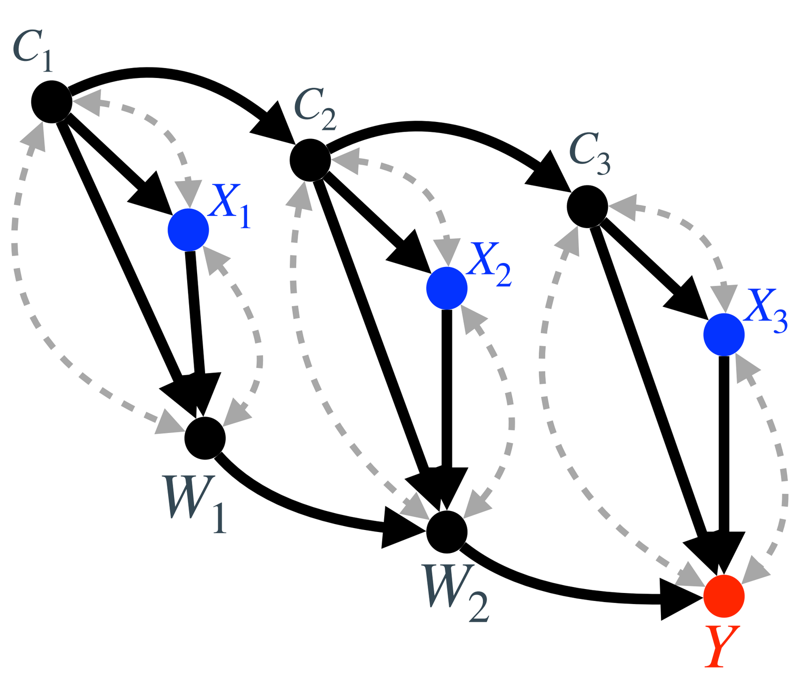
Example 1: Treatment-Treatment Interaction



C_1: Covariate for the 1st experiment
X_1: Treatment in the 1st experiment.
W: Outcome in the 1st experiment.
C_2: Covariate for the 2nd experiment.
X_2: Treatment in the 2nd experiment.
Y: Outcome in the 2nd experiment.
$\mathbf{V} := \{C_1, X_1, W, C_2, X_2, Y\}$.

Input: Two samples $\{D_1, D_2\}$ where $D_1 \sim P(\mathbf{V} \mid do(X_1 = x_1))$ and $D_2 \sim P(\mathbf{V} \mid do(X_2 = x_2))$.
Output: Estimate the effect $\mathbb{E}[Y \mid do(x_1, x_2)]$.

Example 2: Multiple Treatments Interaction



W_2: Outcome for the 2nd experiment
C_3: Covariate for the 3rd experiment.
X_3: Treatment in the 3rd experiment.
Y: Outcome in the 2nd experiment.
$\mathbf{V} := \{C_1, X_1, W, C_2, X_2, W_2, C_3, X_3, Y\}$

Input: Multiple samples $\{D_1, \cdots, D_m\}$ where $D_i \sim P(\mathbf{V} \mid do(X_i = x_i))$ for $i = 1, \cdots, m$.
Output: Estimate the effect $\mathbb{E}[Y \mid do(x_1, \cdots, x_m)]$.

Treatment-Treatment Interaction (TTI)

Identification through AC-TTI

1. The 2nd experiment's treatment X_2 doesn't have a direct effect on the previous experiment (C_1, W); and
2. There are no unmeasured confounders between the treatment X_1 in the previous experiments and the outcome of interest Y.

Then,

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}_{P_{do(x_1)}}[\mathbb{E}_{P_{do(x_2)}}[Y \mid C_1, W, x_1]],$$

where the r.h.s. can be estimated from the samples from $P(\mathbf{V} \mid do(x_2))$ and $P(\mathbf{V} \mid do(x_1))$, respectively.

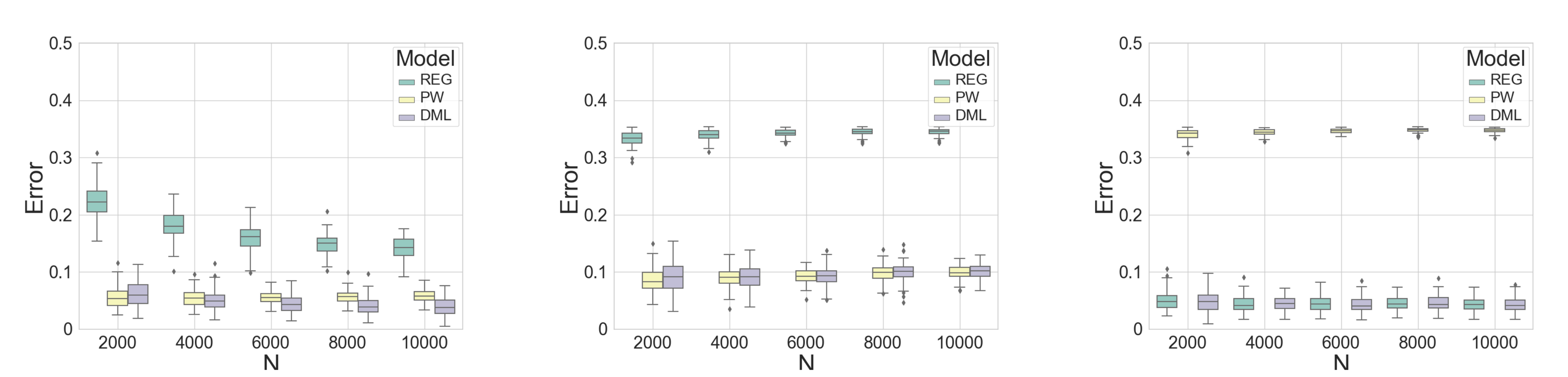
Doubly Robust AC-TTI Estimator

- The nuisances are $\mu_0(C_1, X_1, W) := \mathbb{E}_{P_{do(x_2)}}[Y \mid C_1, X_1, W]$; and $\pi_0(C_1, X_1, W) := \dfrac{P(W \mid C_1, do(x_1))}{P(W, X_1 \mid C_1, do(x_2))}$.
- The doubly robust estimator T^{tti} for the AC-TTI is the following:

$$\mathbb{E}_{D_2}[\pi(C_1, X_1, W) I_{x_1}(X_1)\{Y - \mu(C_1, X_1, W)\}] + \mathbb{E}_{D_1}[\mu(C_1, X_1, W)\}]$$

where $\mathbb{E}_{D_2}$ is an empirical average, and π, μ are estimated nuisance after sample-splitting.

Then, T^{tti} is doubly robust w.r.t. $\{\mu, \pi\}$.



(a) Fast Convergence (b) Doubly Robustness (μ) (c) Doubly Robustness (π)

- We compared DR-AC-TTI estimator with competing methods (regression, inverse-probability-weighting based).
- The result showed the doubly robustness and fast convergence of the DR-AC-TTI estimator.

Multiple-Treatment Interaction (TTI)

Identification through AC-MTI

1. The i'th experiment's treatment X_i doesn't have the direct effect on the previous observations $(\mathbf{X}^{(i-1)}, \mathbf{C}^{(i-1)}, \mathbf{W}^{(i-1)})$; and
2. There are no unmeasured confounders between X_i and Y conditioning on previous observations $(\mathbf{X}^{(i-1)}, \mathbf{C}^{(i-1)}, \mathbf{W}^{(i-1)})$.

Then, the causal effect is identifiable as follows: Let $\mu_0^m(\mathbf{X}^{(m-1)}, \mathbf{W}^{(m-1)}, \mathbf{C}^{(m-1)}) := \mathbb{E}_{P_{do(x_m)}}[Y \mid \mathbf{X}^{(m-1)}, \mathbf{W}^{(m-1)}, \mathbf{C}^{(m-1)}]$.
Let $\mu_0^i(\mathbf{X}^{(i-1)}, \mathbf{W}^{(i-1)}, \mathbf{C}^{(i-1)}) := \mathbb{E}_{P_{do(x_i)}}[\overline{\mu}_0^{i+1}(\mathbf{X}^{(i-1)}, \mathbf{W}^{(i-1)}, \mathbf{C}^{(i-1)}]$ where $\overline{\mu}_0^i := \mu_0^{i+1}(x_i, \mathbf{X}^{(i-1)}, \mathbf{W}^{(i)}, \mathbf{C}^{(i)})$. Then,

$$\mathbb{E}[Y \mid do(x_1, \cdots, x_m)] = \mathbb{E}_{P_{do(x_1)}}[\mu_0^2(W_1, C_1, x_1)].,$$

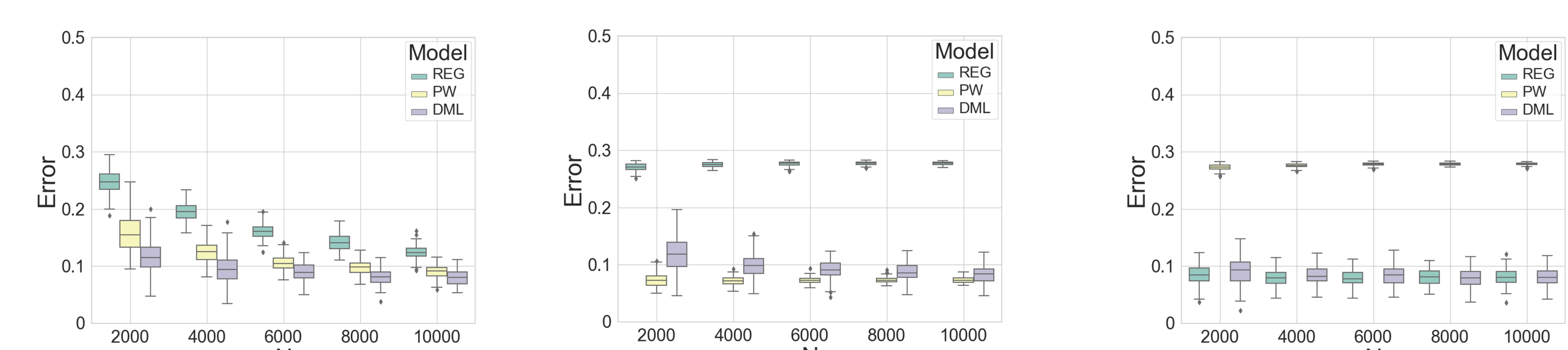
where each nuisances can be estimated from the samples from marginal experiments $P(\mathbf{V} \mid do(x_i))$.

Doubly Robust AC-MTI Estimator

- $\pi_0^{(i)} := \prod_{j=1}^{i} \dfrac{P(W_i \mid C_i, \mathbf{H}_{i-1}, do(x_i))}{P(W_i, X_i \mid C_i, \mathbf{H}_{i-1}, do(x_m))}$ for $\mathbf{H}_i := \{\mathbf{C}^{(i)}, \mathbf{W}^{(i)}, \mathbf{X}^{(i)}\}$.
- The doubly robust estimator T^{mti} for the AC-MTI is the following:

$$T^{mti} := \sum_{j=1}^{m} \mathbb{E}_{D_j}[\pi^{(j)}\{\overline{\mu}^{i+1} - \mu^i\}] + \mathbb{E}_{D_1}[\mu^1\}].$$

Then, T^{mti} is doubly robust w.r.t. $\{\mu^i, \pi^i\}$ for $i = 1, \cdots, m$.



(a) Fast Convergence (b) Doubly Robustness (μ) (c) Doubly Robustness (π)

- The result showed the doubly robustness and fast convergence of the DR AC-MTI estimator.