Estimating Joint Treatment Effect from Marginal Experiments

based on:
Estimating Joint Treatment Effects by Combining Multiple Experiments, ICML-23

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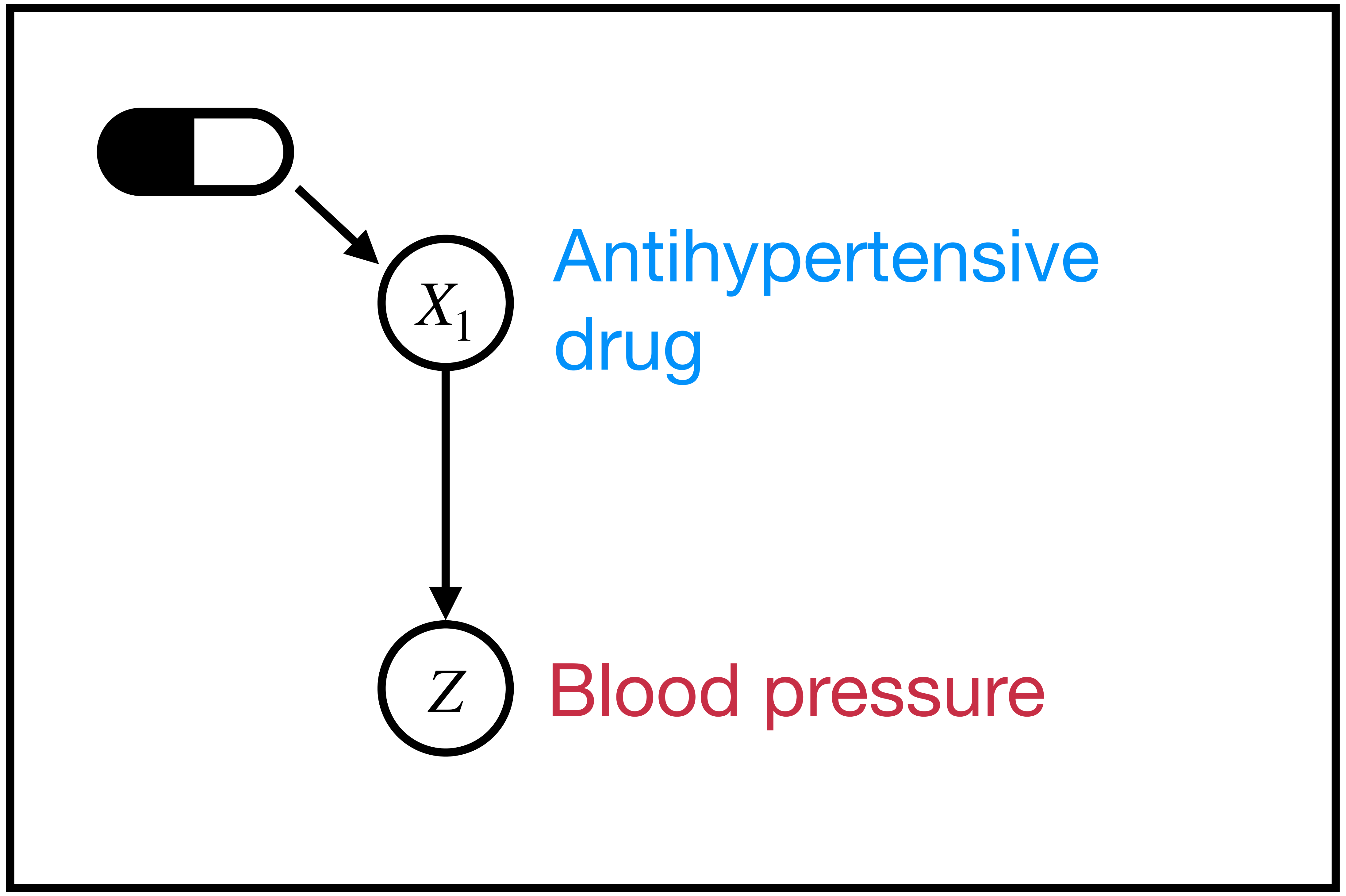
2023 Fall QM Seminar

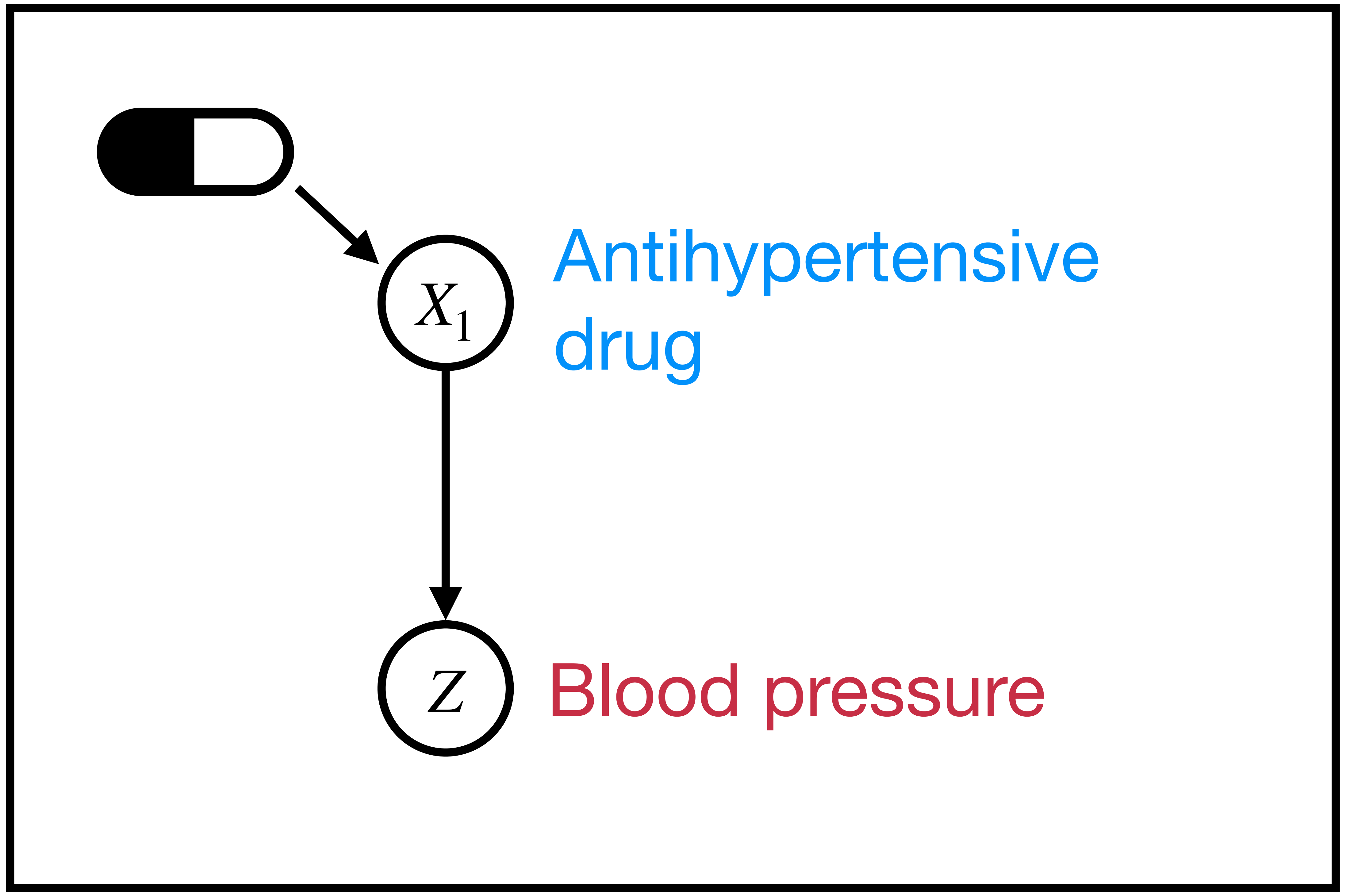
Treatment-Treatment interaction

Treatment-Treatment interaction



Experiment on X_1

Treatment-Treatment interaction



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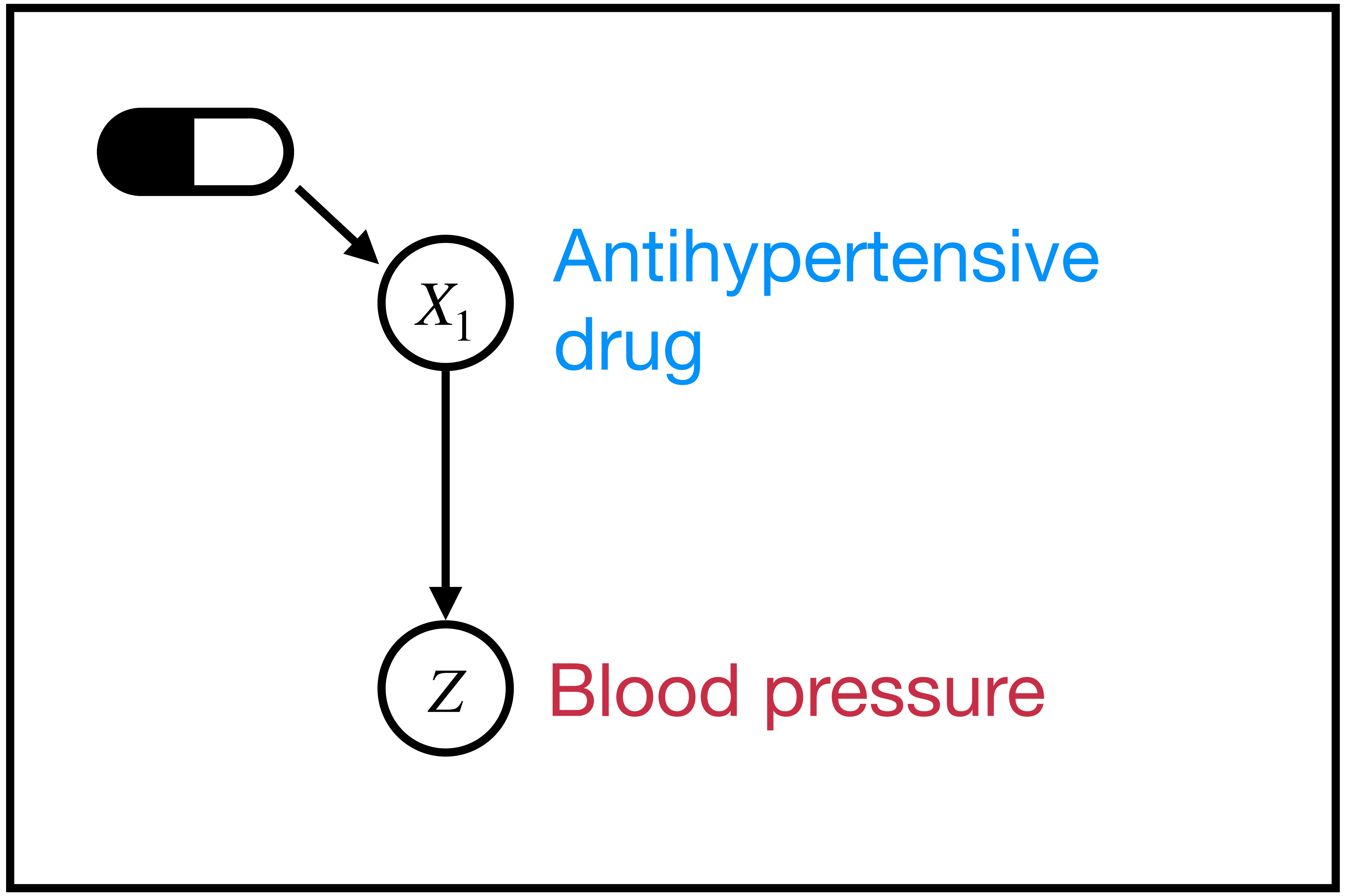

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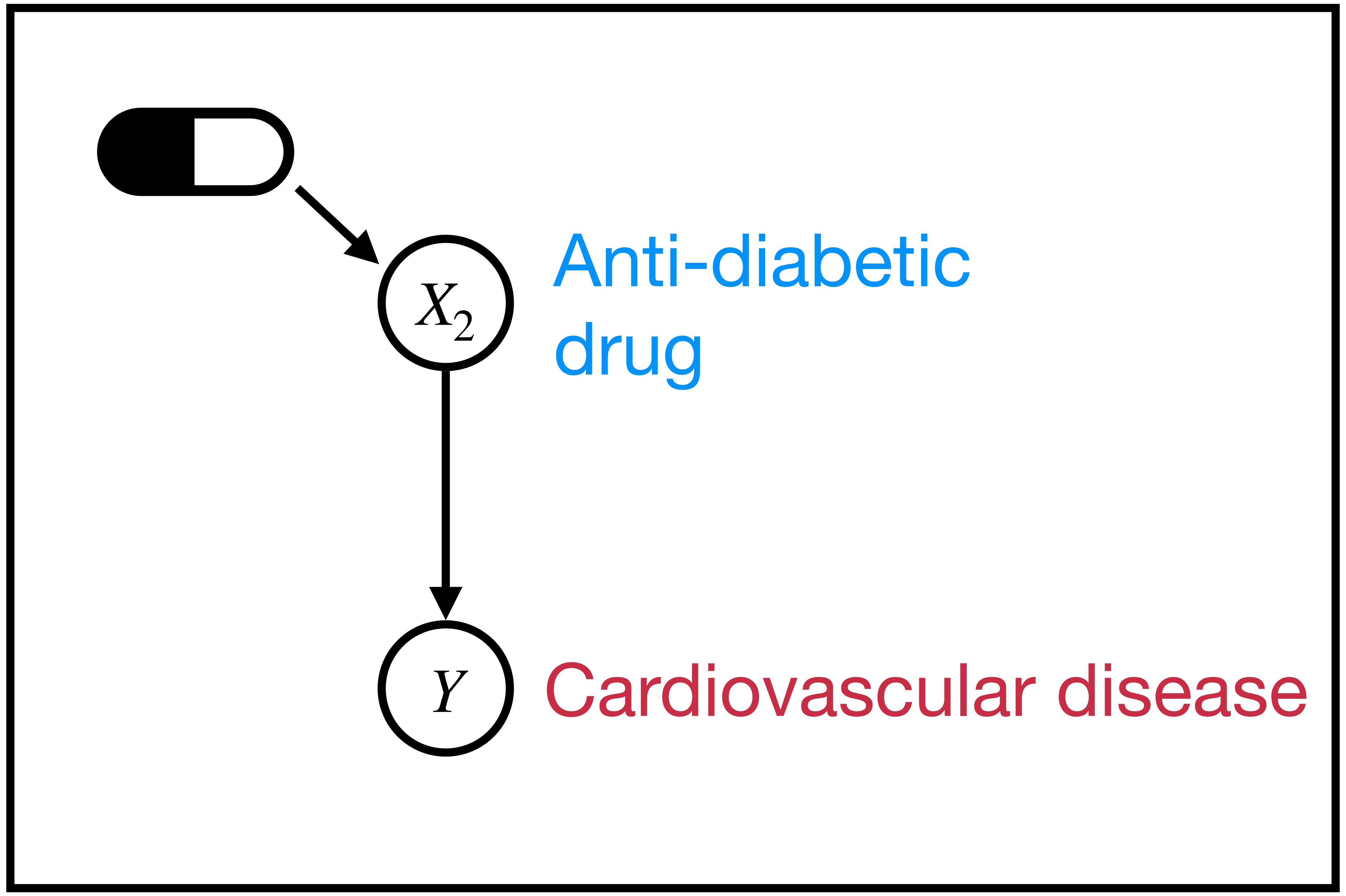
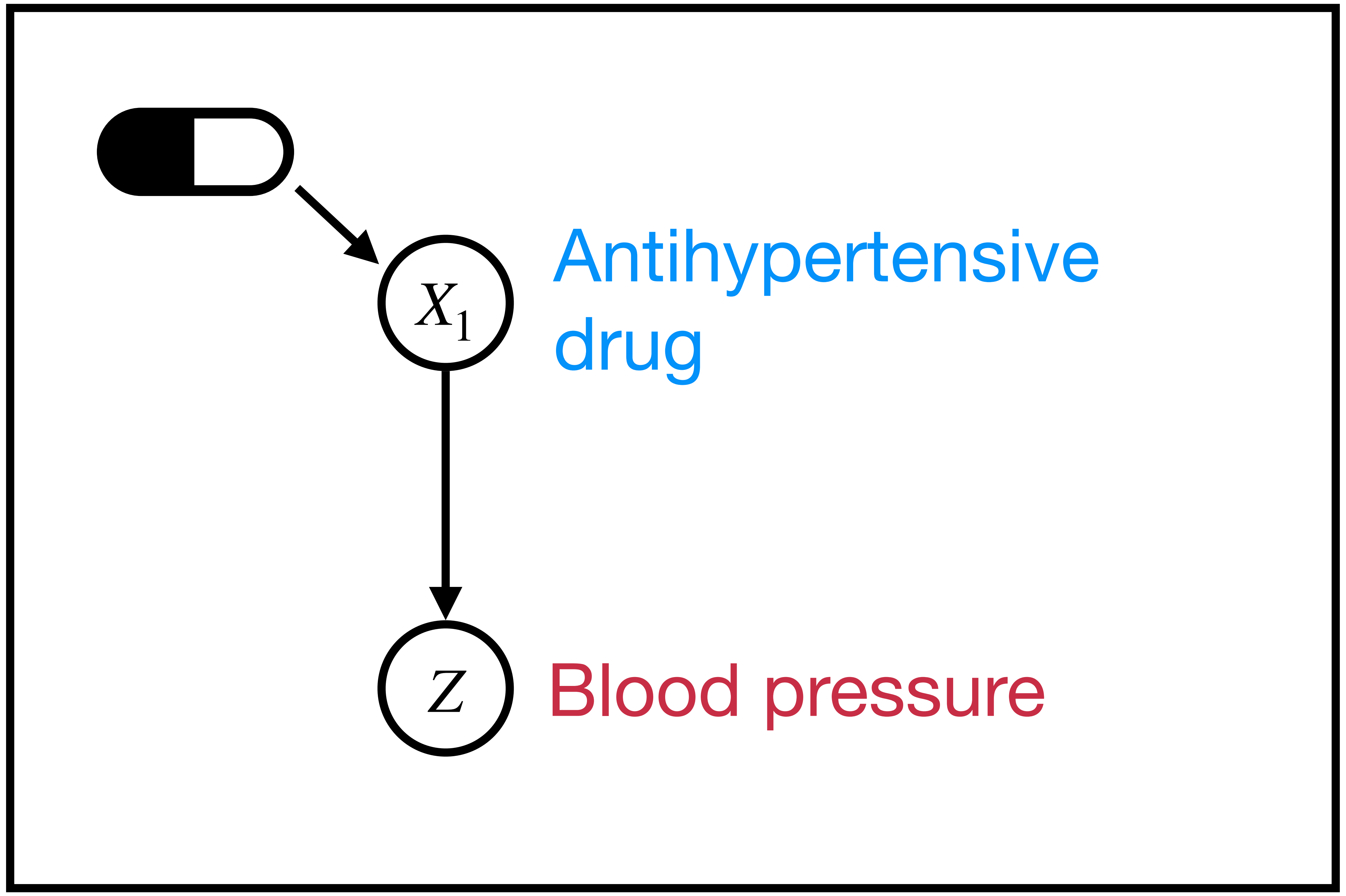


Experiment on X_2

Treatment-Treatment interaction



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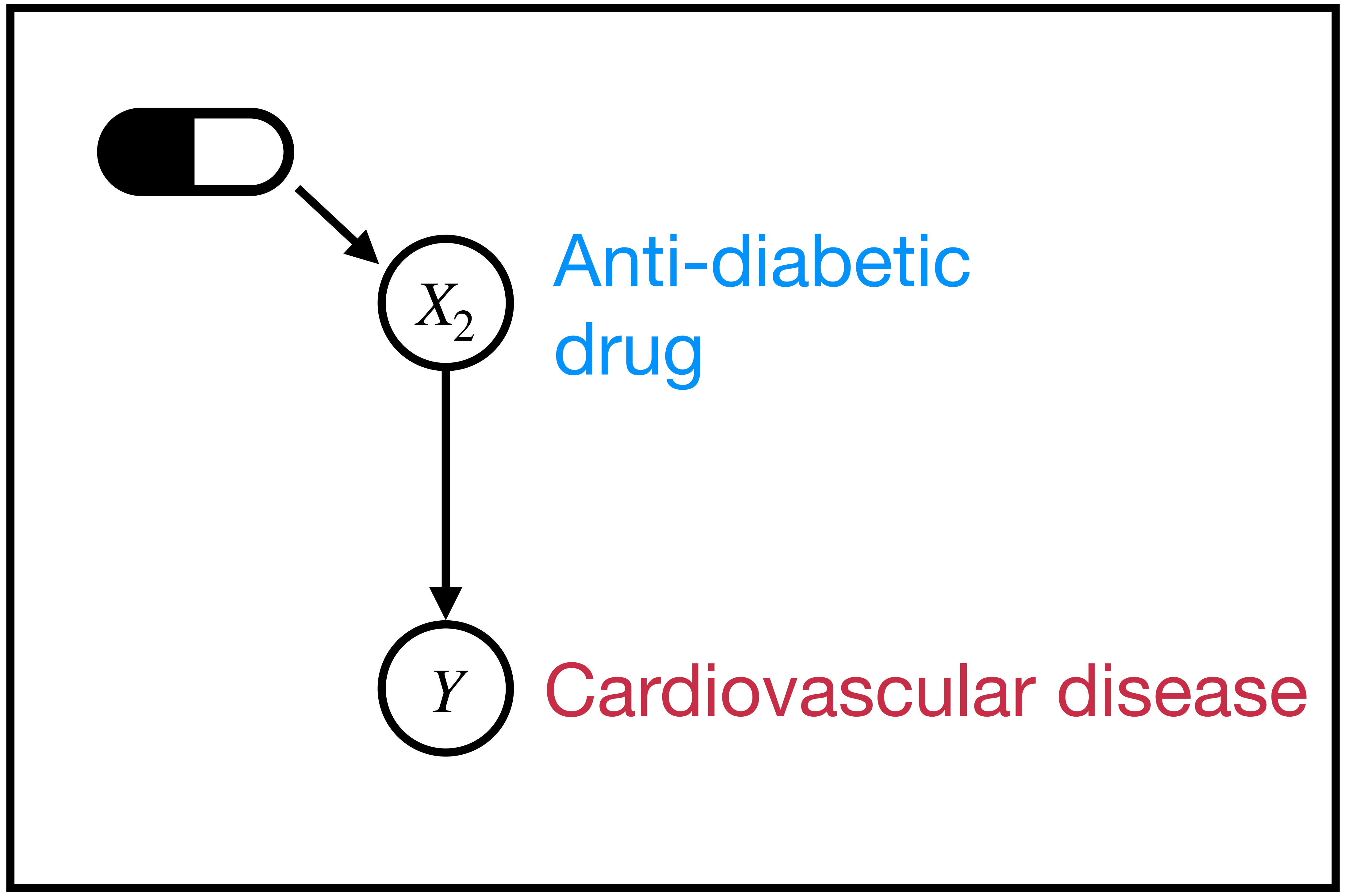

Experiment on X_2 😃

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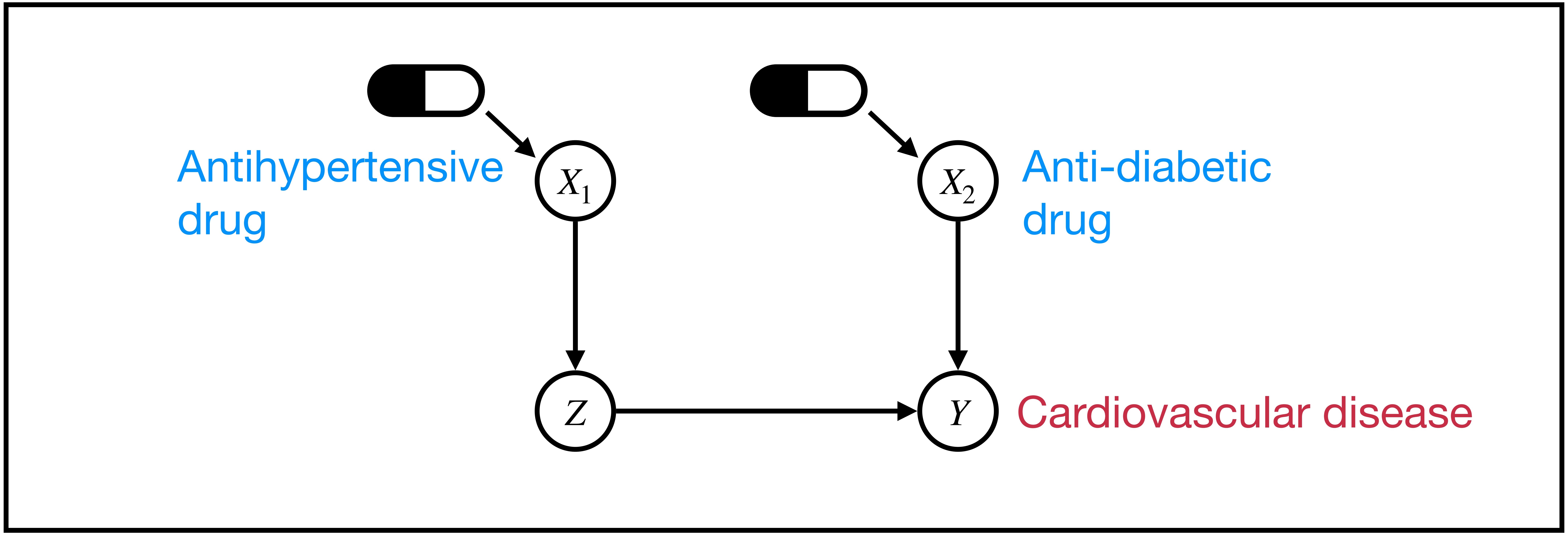
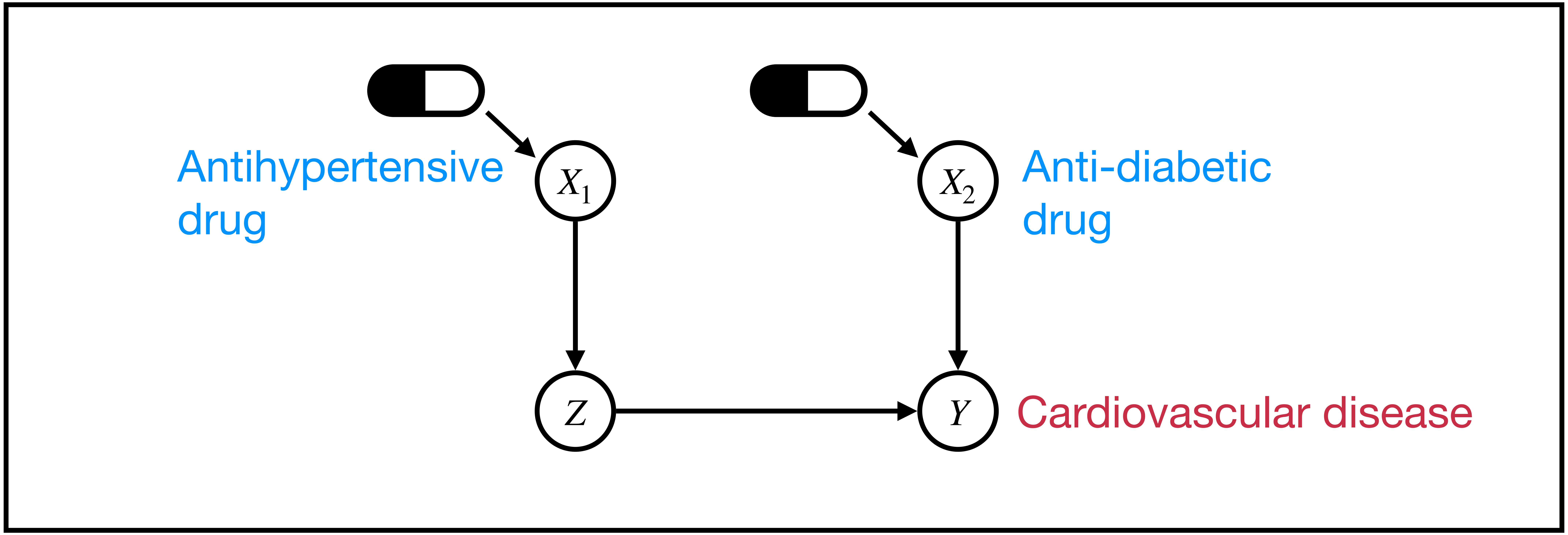


Experiment on X_1 and X_2

Treatment-Treatment interaction



Experiment on X_1 and X_2 😡

Multiple-Treatment interaction

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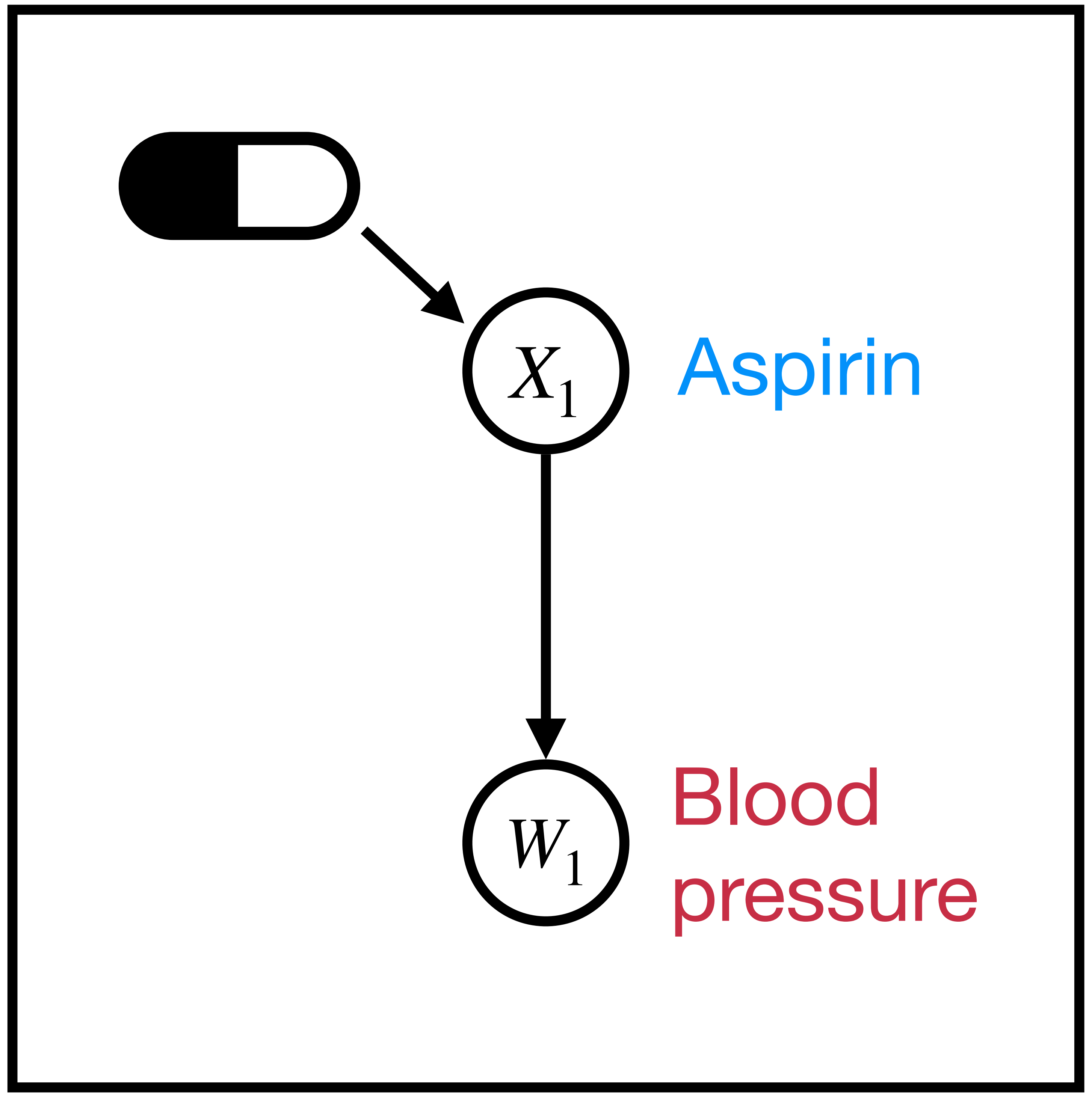
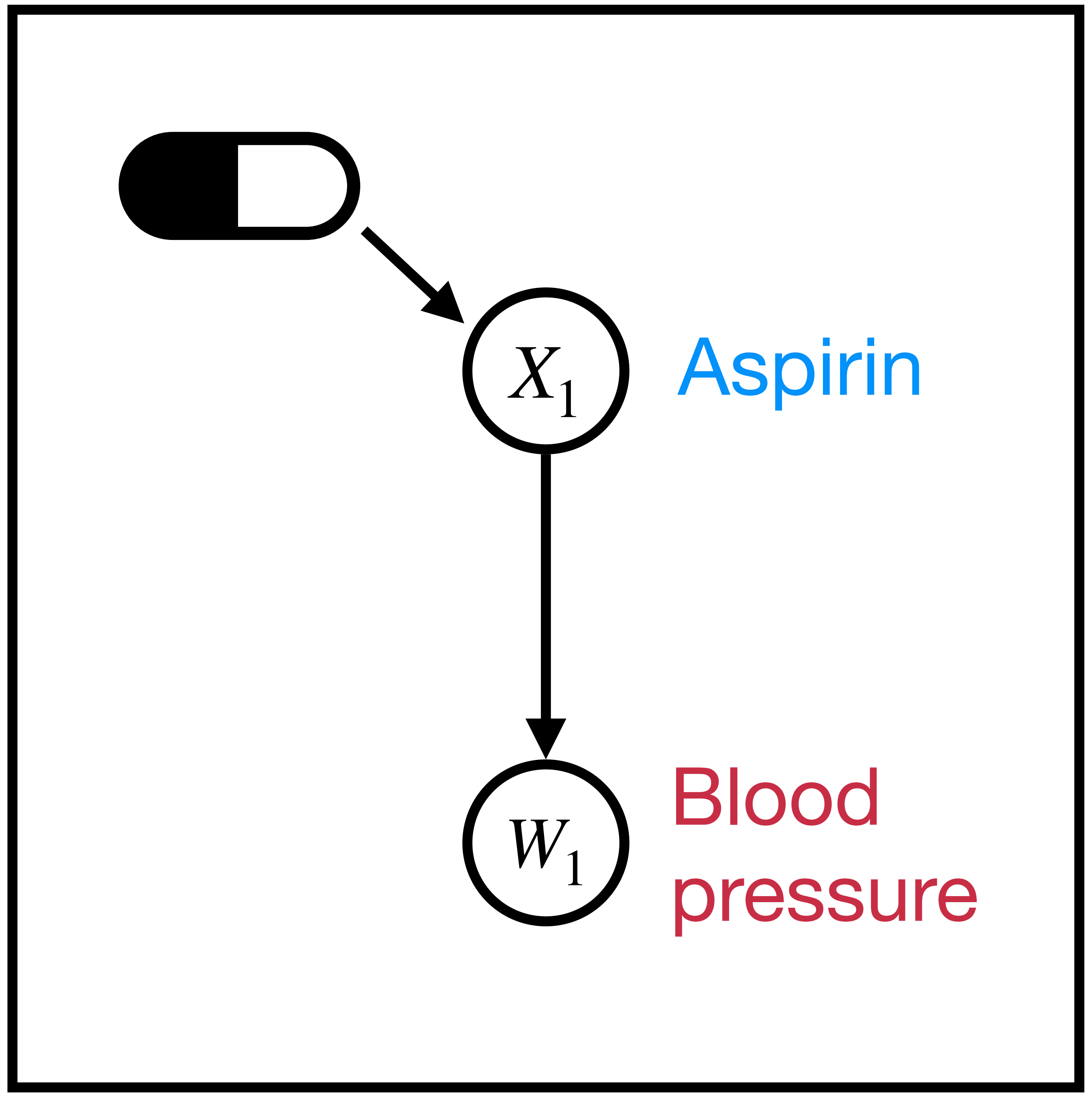


Experiment on X_1 😃

Multiple-Treatment interaction



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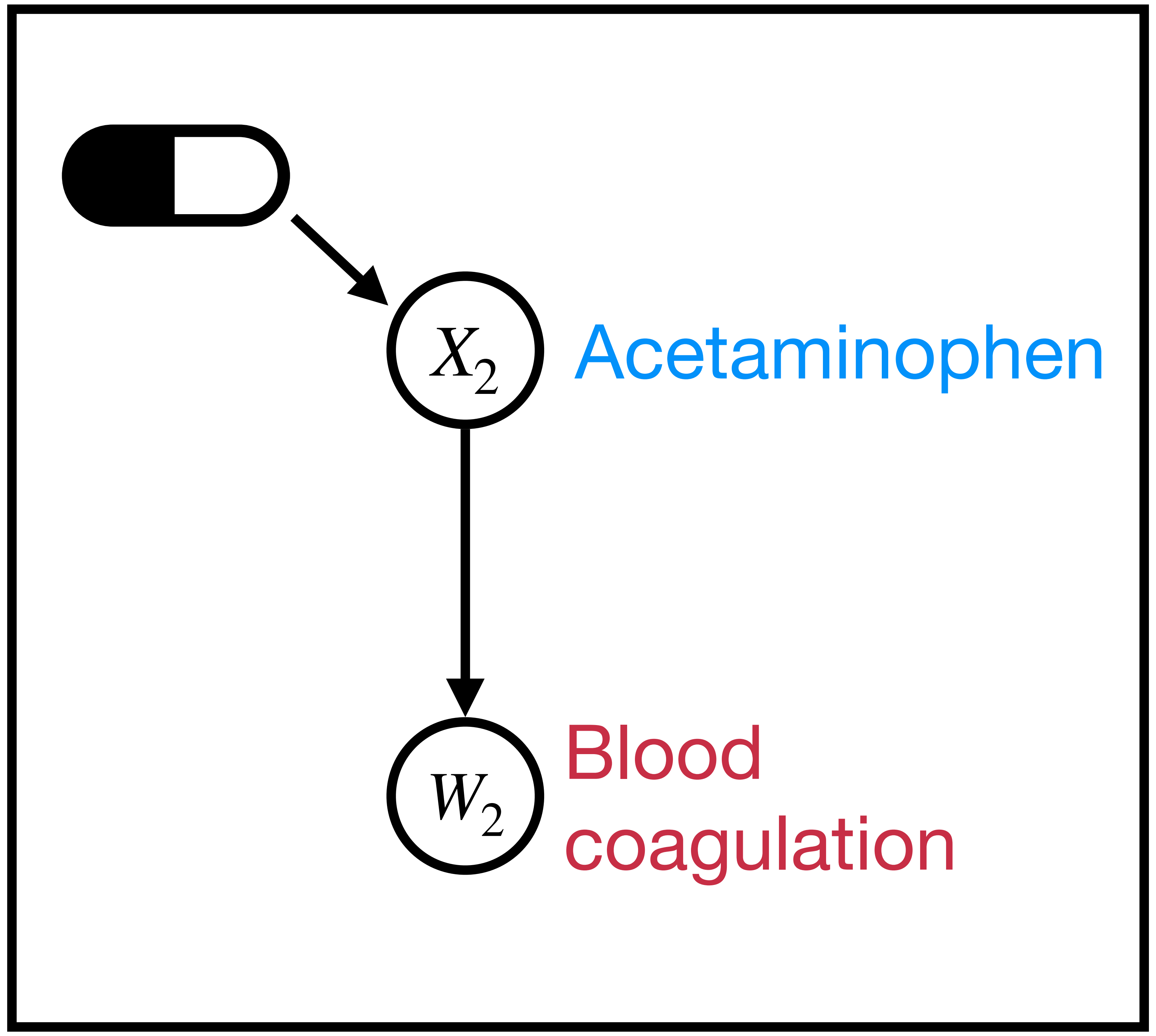
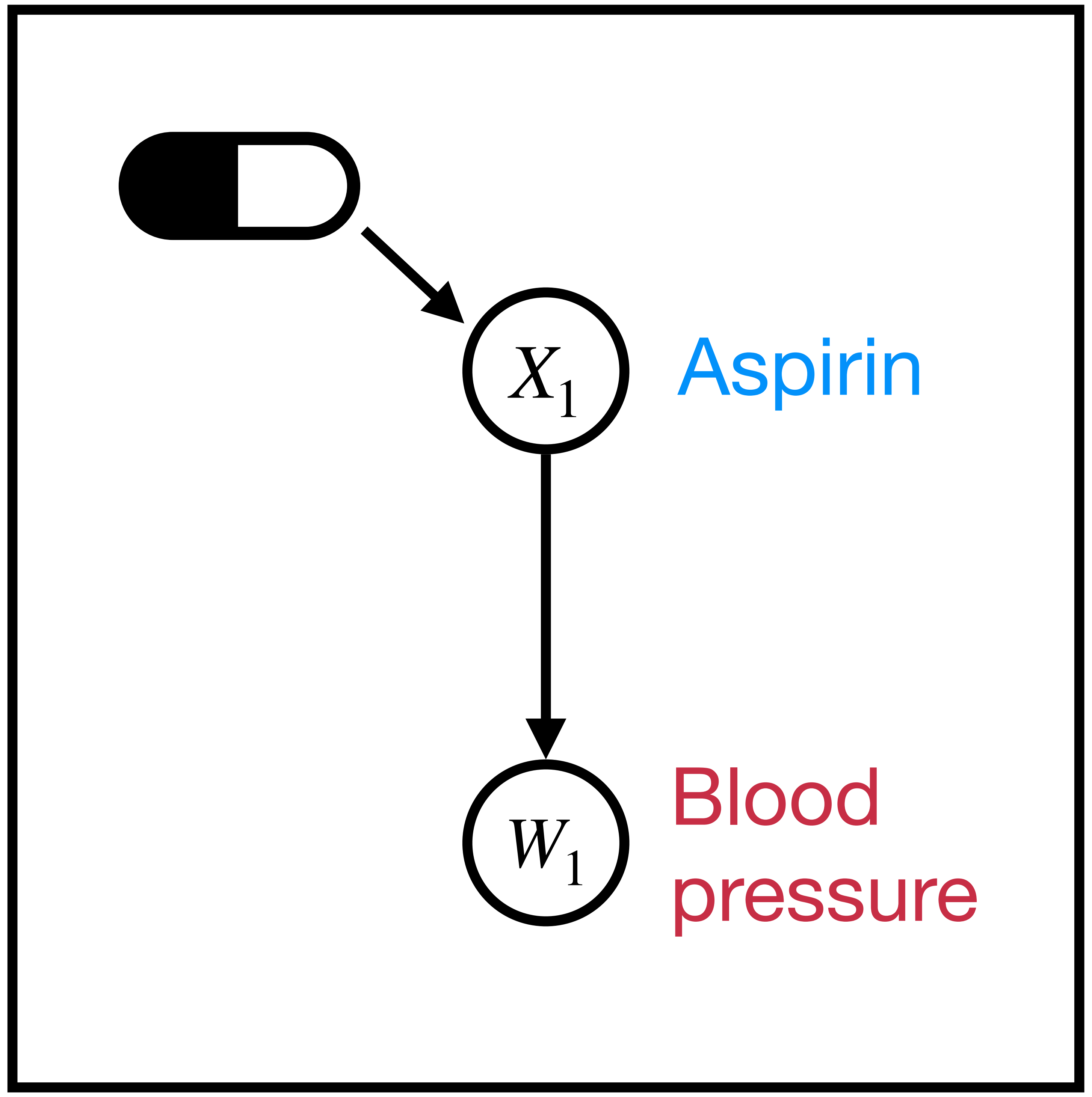


Experiment on X_2

Multiple-Treatment interaction



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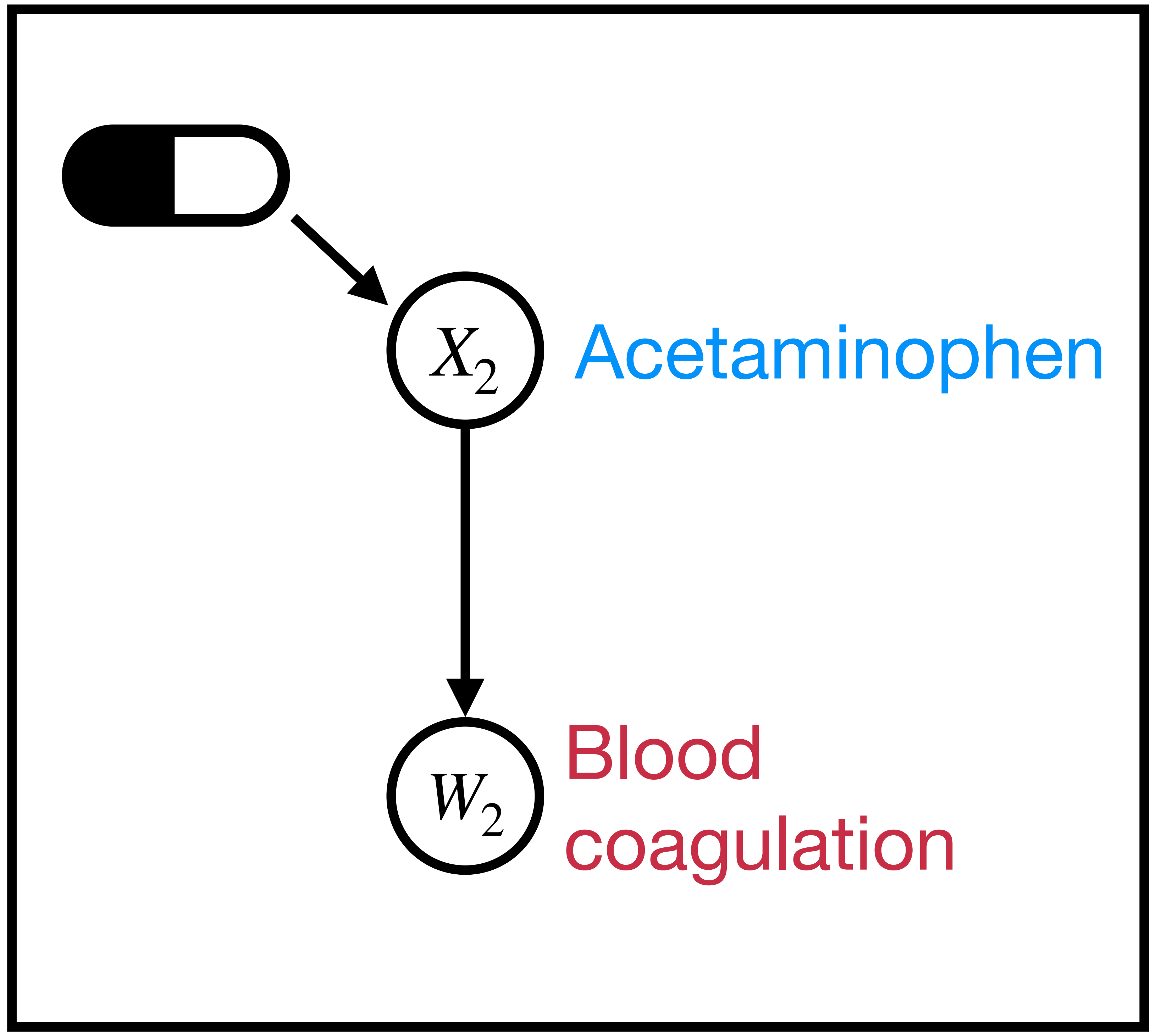
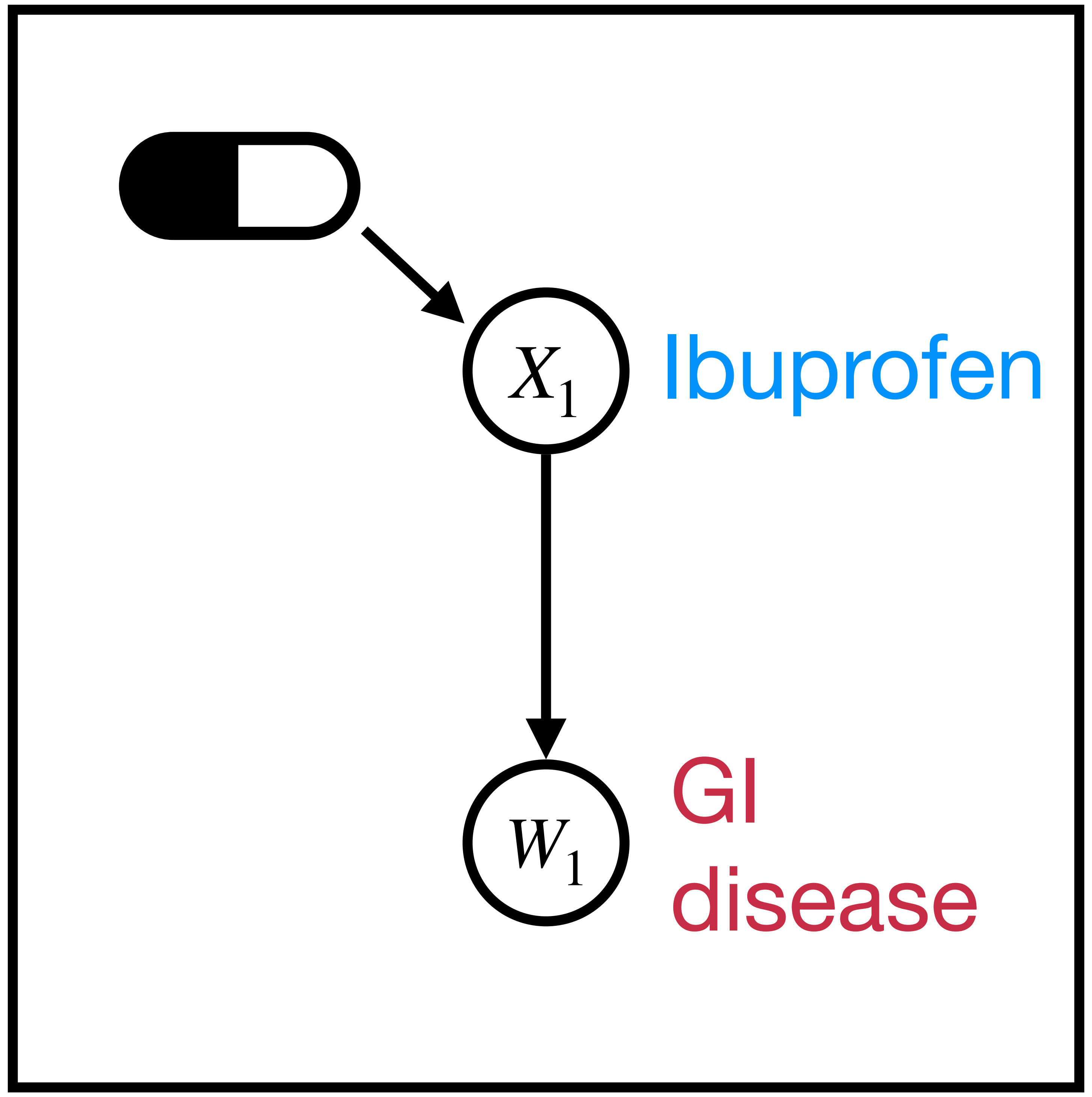


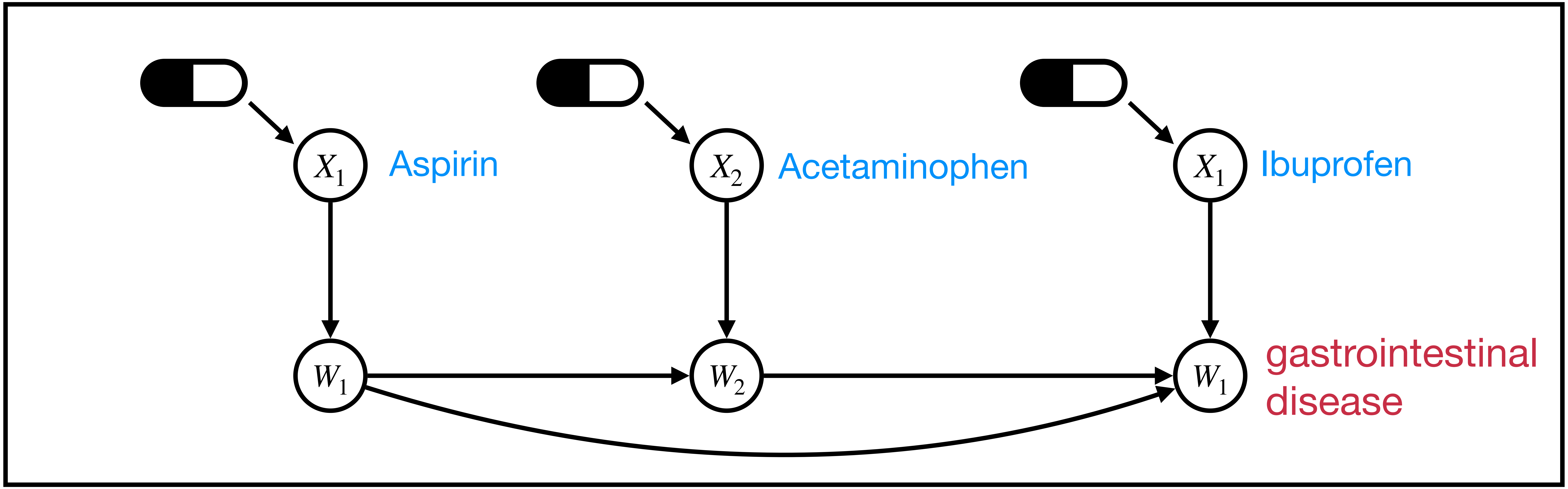
Experiment on X_2 😃



Experiment on X_3 😃

Multiple-Treatment interaction

Multiple-Treatment interaction



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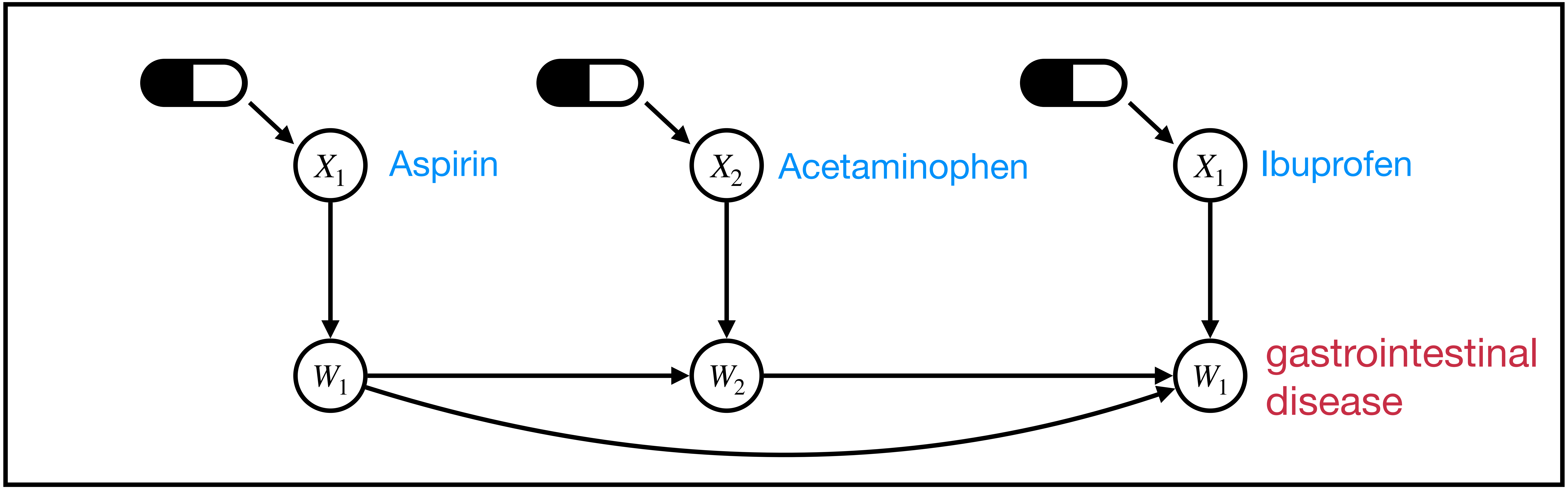
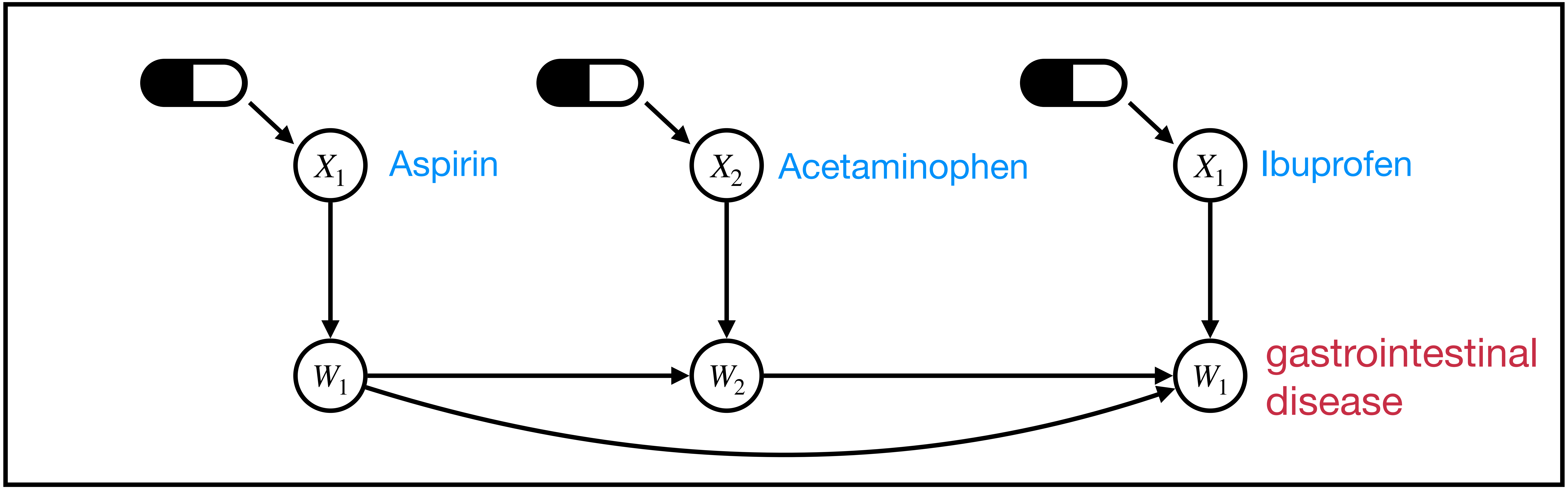


Experiment on X_1, X_2, X_3

Multiple-Treatment interaction



Experiment on X_1, X_2, X_3 😡

Practical challenges

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3. What about leveraging existing marginal experiments to estimate the joint treatment effect?

This talk: Combining marginal experiments to estimate joint treatment etfects

Outline of this talk

1. Preliminary and Problem Setup
   (1) Structural Causal Model
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   (1) Identification
   (2) Estimand
   (3) Estimation and Error Analysis
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3. Multiple-Treatment Interaction (+ Omitted Results)
4. Future directions & Summary

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Structural Causal Model: DGP

Structural Causal Model (SCM) $\langle \mathbf{V}, \mathbf{U}, \mathbf{F}, P(\mathbf{U}) \rangle$

Structural Causal Model: DGP

Structural Causal Model (SCM) $\langle \mathbf{V}, \mathbf{U}, \mathbf{F}, P(\mathbf{U}) \rangle$

- $\mathbf{V}$: A set of observable variables.
- $\mathbf{U}$: A set of latent variables.
- $\mathbf{F}$: A set of functions $\{f_{V_i}\}_{V_i \in \mathbf{V}}$ determining the value of $V_i \in \mathbf{V}$; i.e.,

  $V_i \leftarrow f_{v_i}(PA_{V_i}, U_{V_i})$ for some $PA_{V_i} \subseteq \mathbf{V}$ and $U_{V_i} \subseteq \mathbf{U}$.
- $P(\mathbf{U})$: A probability distribution for $\mathbf{U}$.

Causal Graphical Model

Causal Graphical Model

SCM

$$U_Z, U_X, U_Y \sim \text{normal}(0,1)$$
$$Z \leftarrow f_Z(U_Z)$$
$$X \leftarrow f_X(Z, U_X)$$
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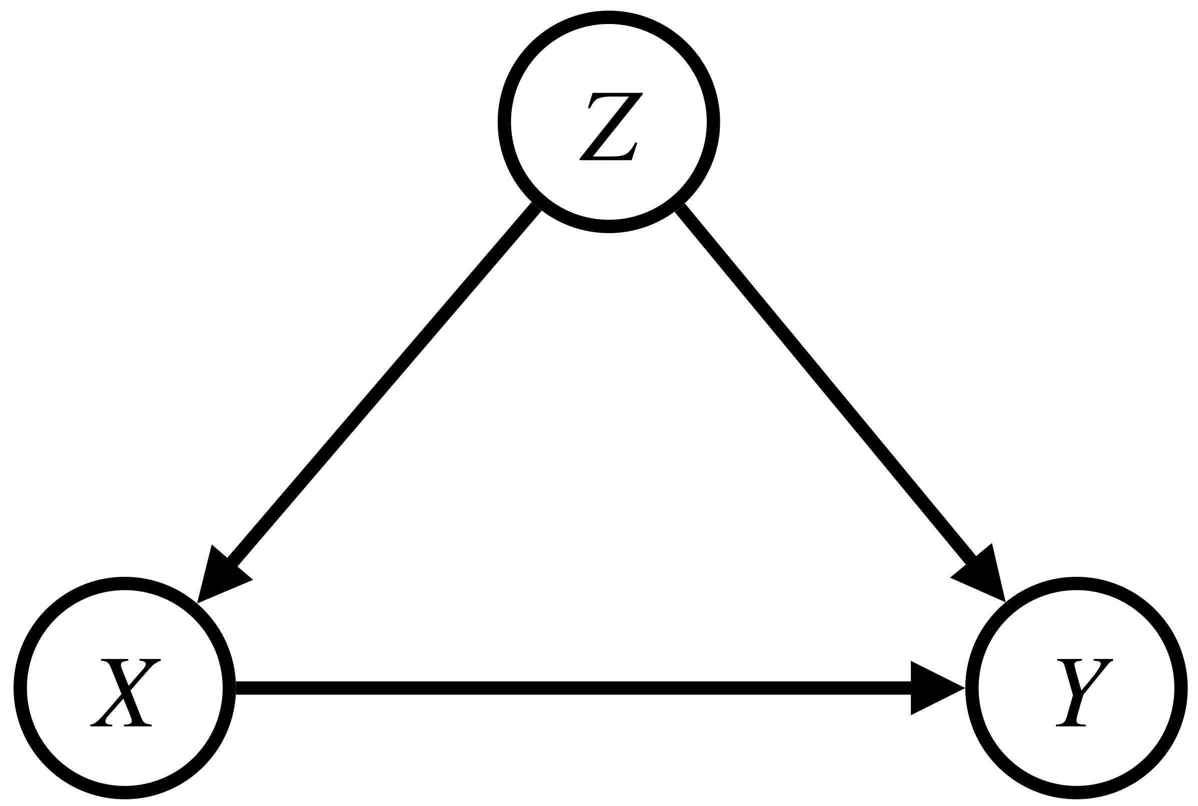
Graph



Causal Graphical Model: Intervention

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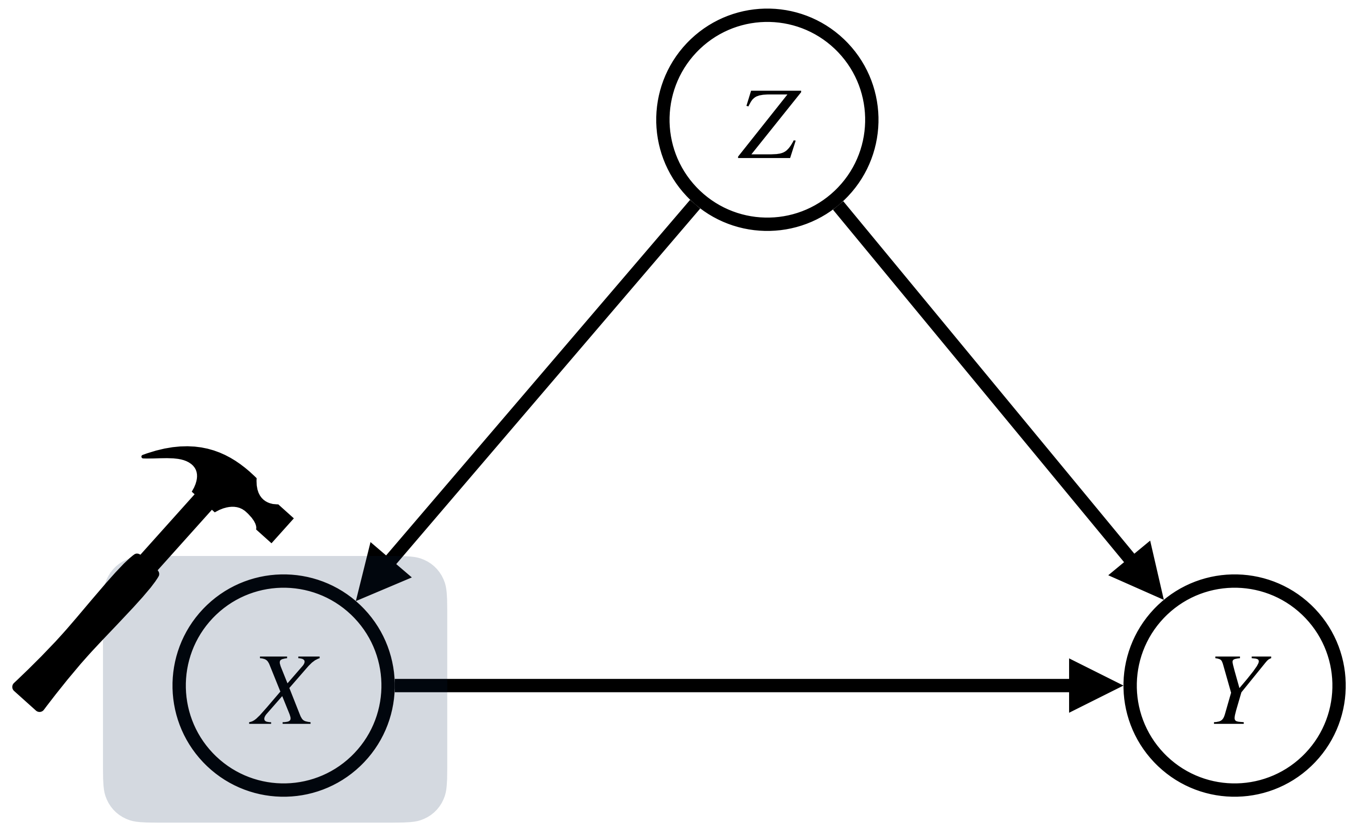

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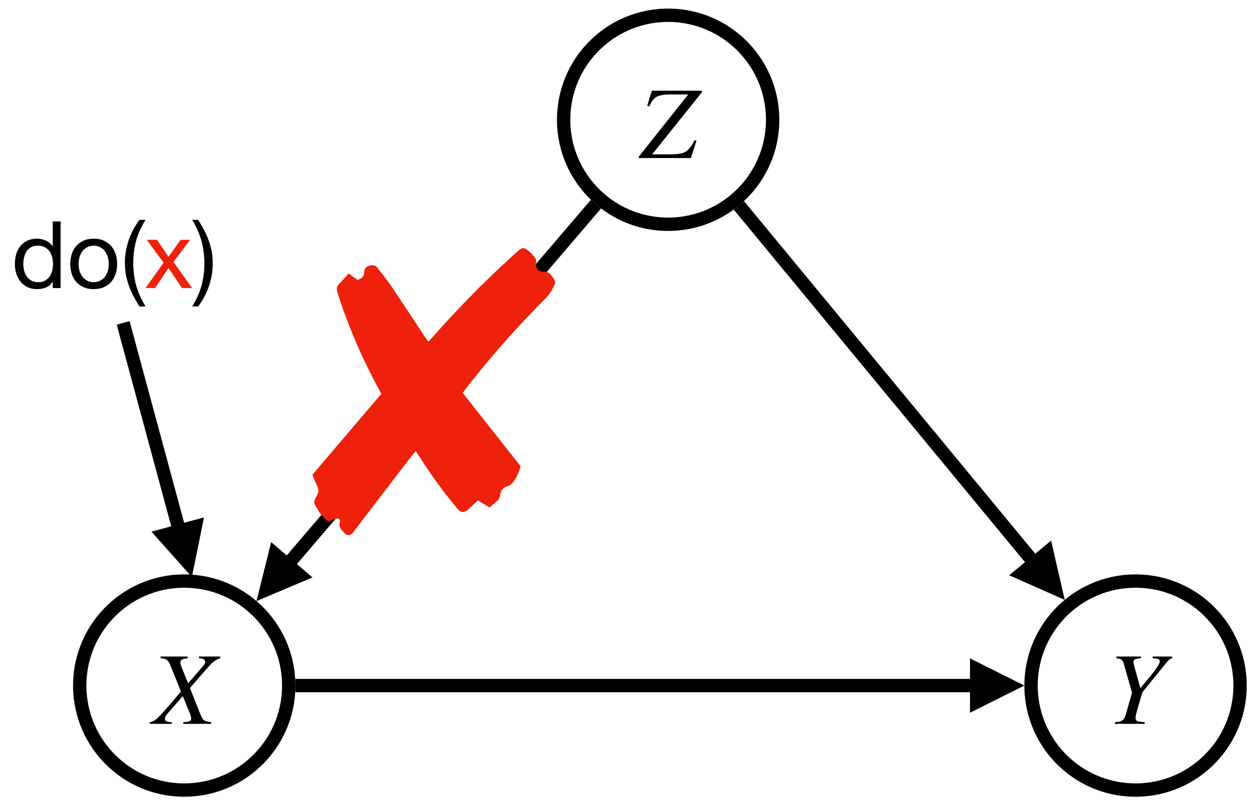

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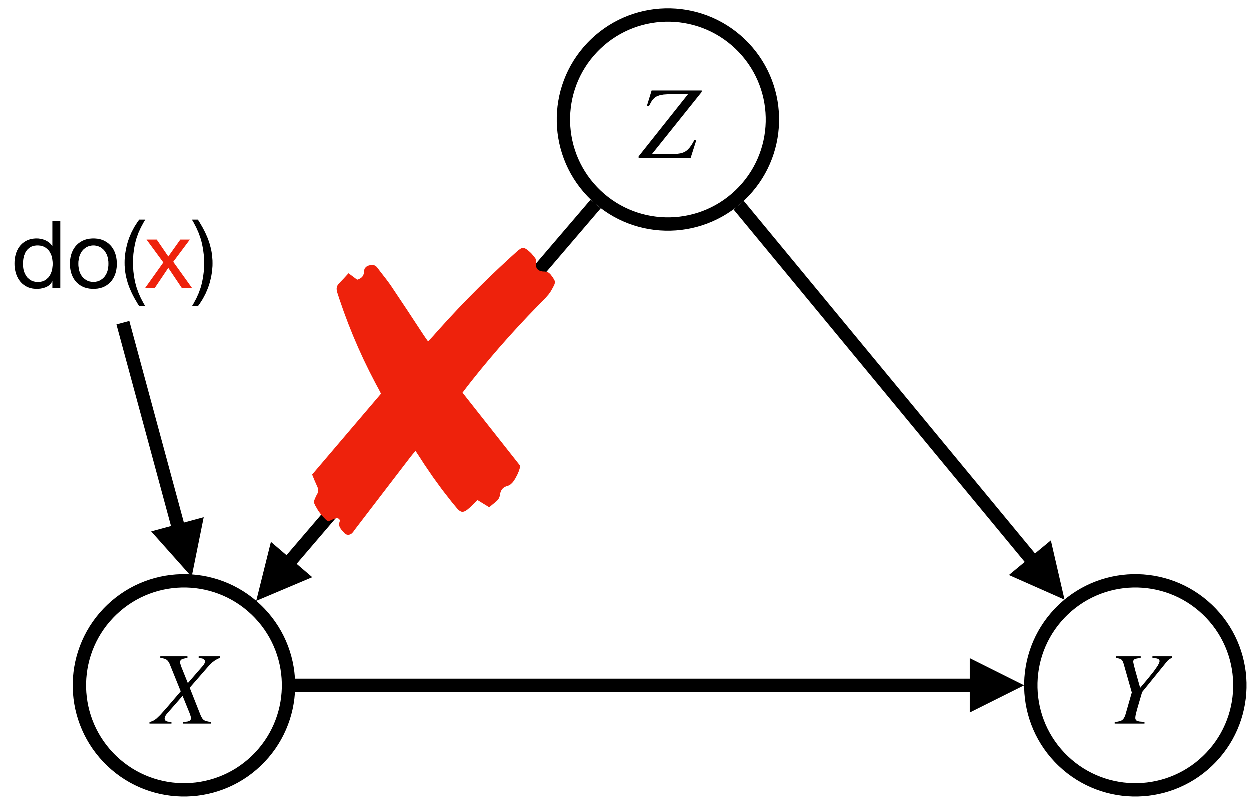

$G_{\overline{X}}$

Causal Graphical Model: Potential Outcome

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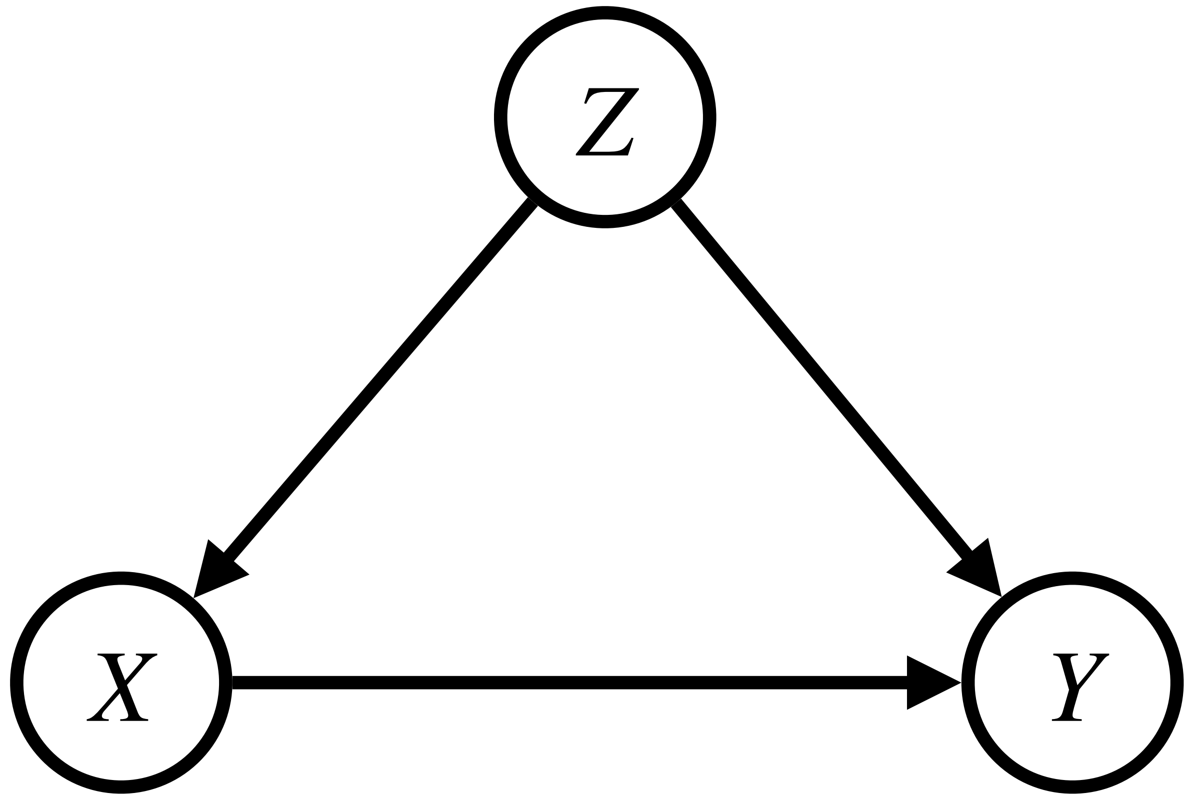

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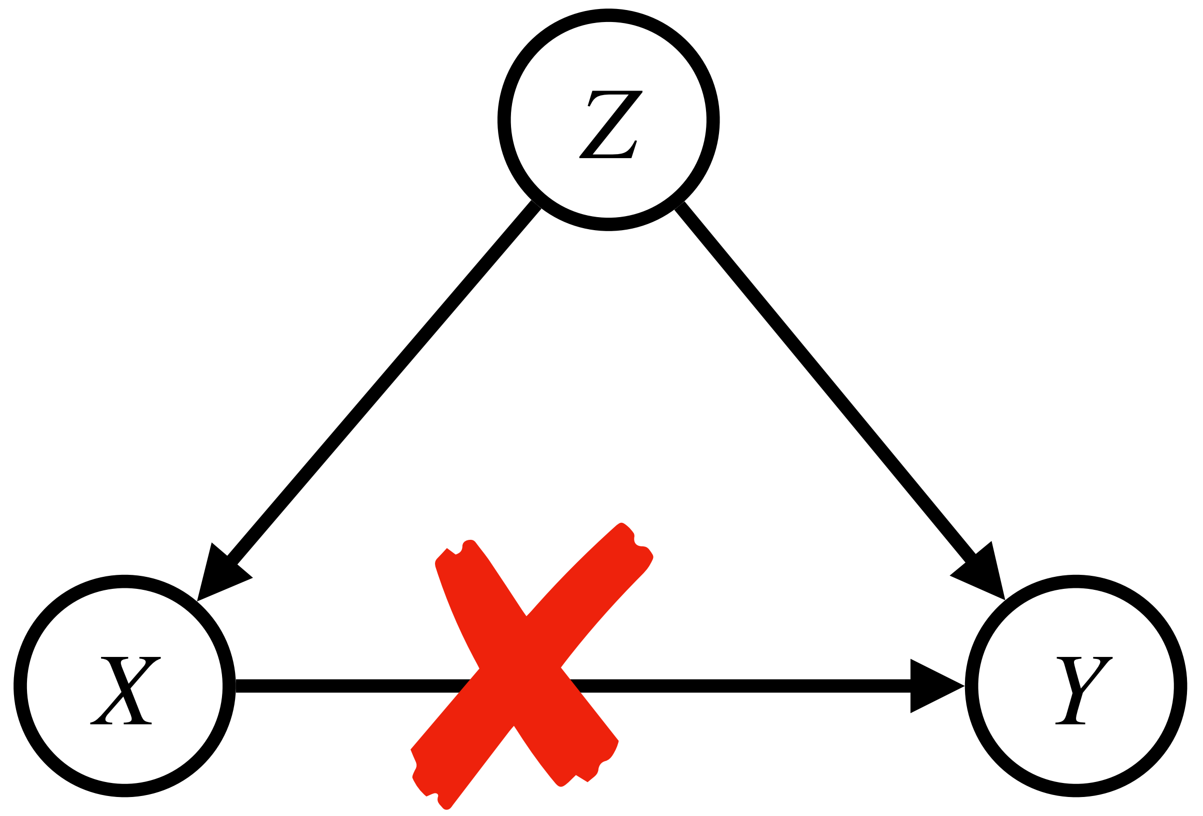

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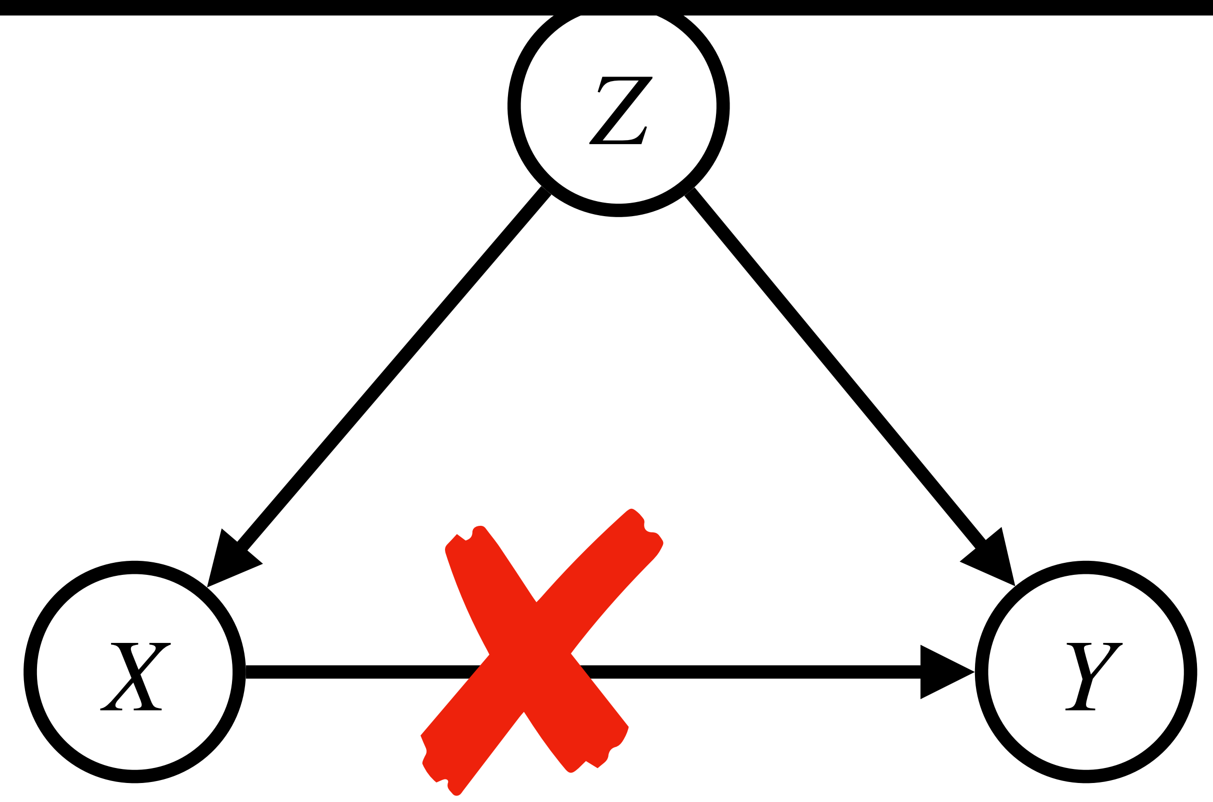
Potential outcome $Y(x)$

Y if X had been fixed to x in the DGP



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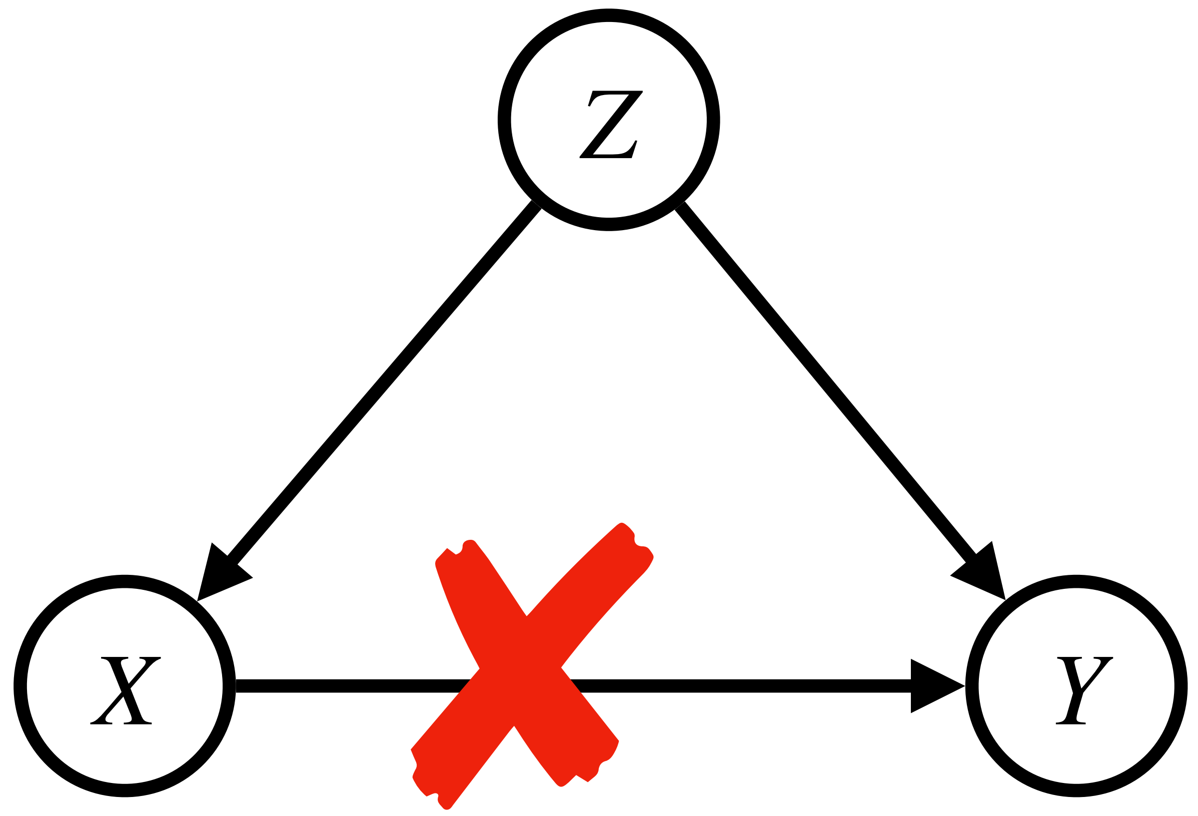

$G_{\underline{X}}$

do-Calculus: Rule 1
conditional independence

Conditional Independence after Intervention

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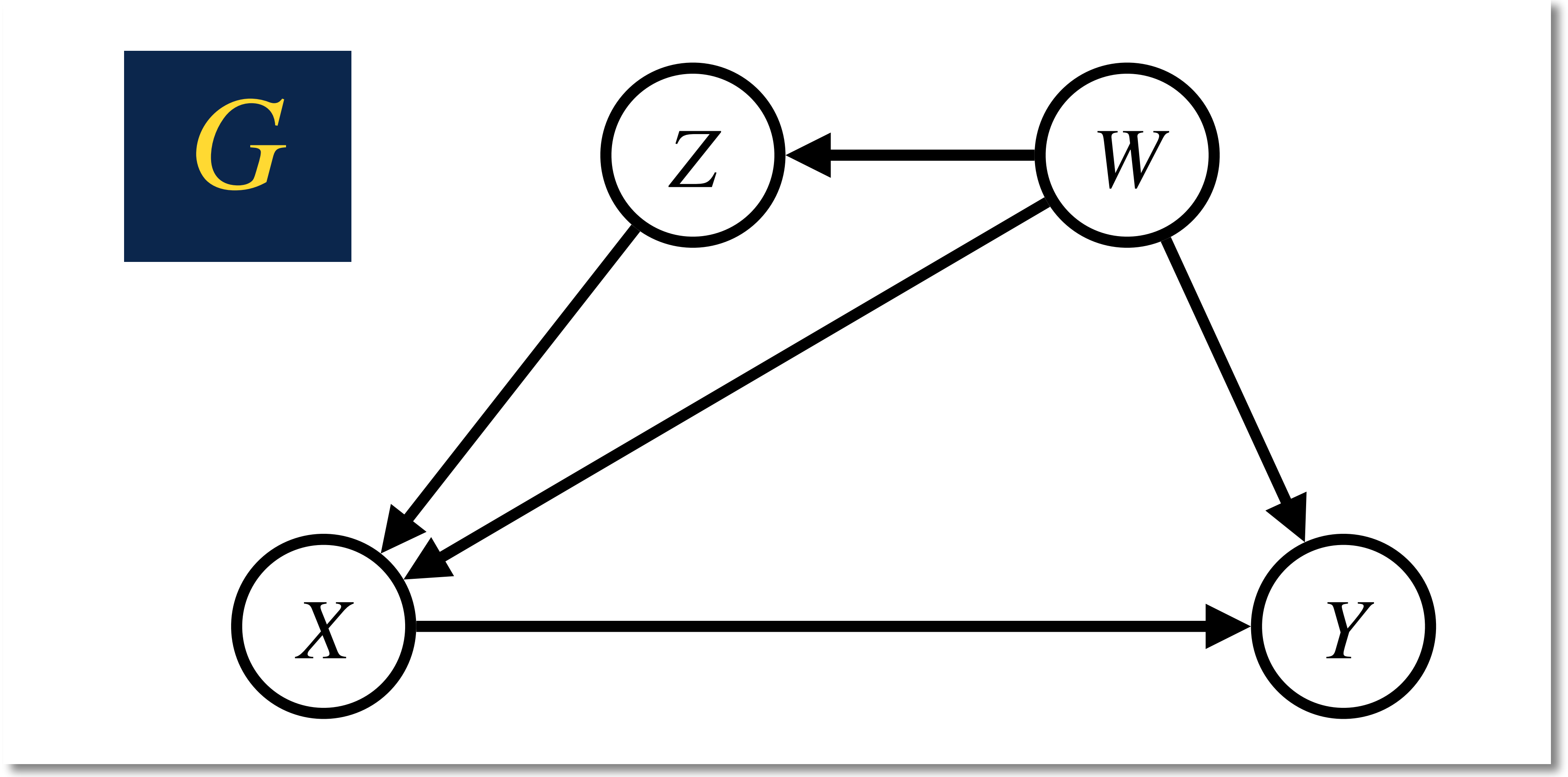
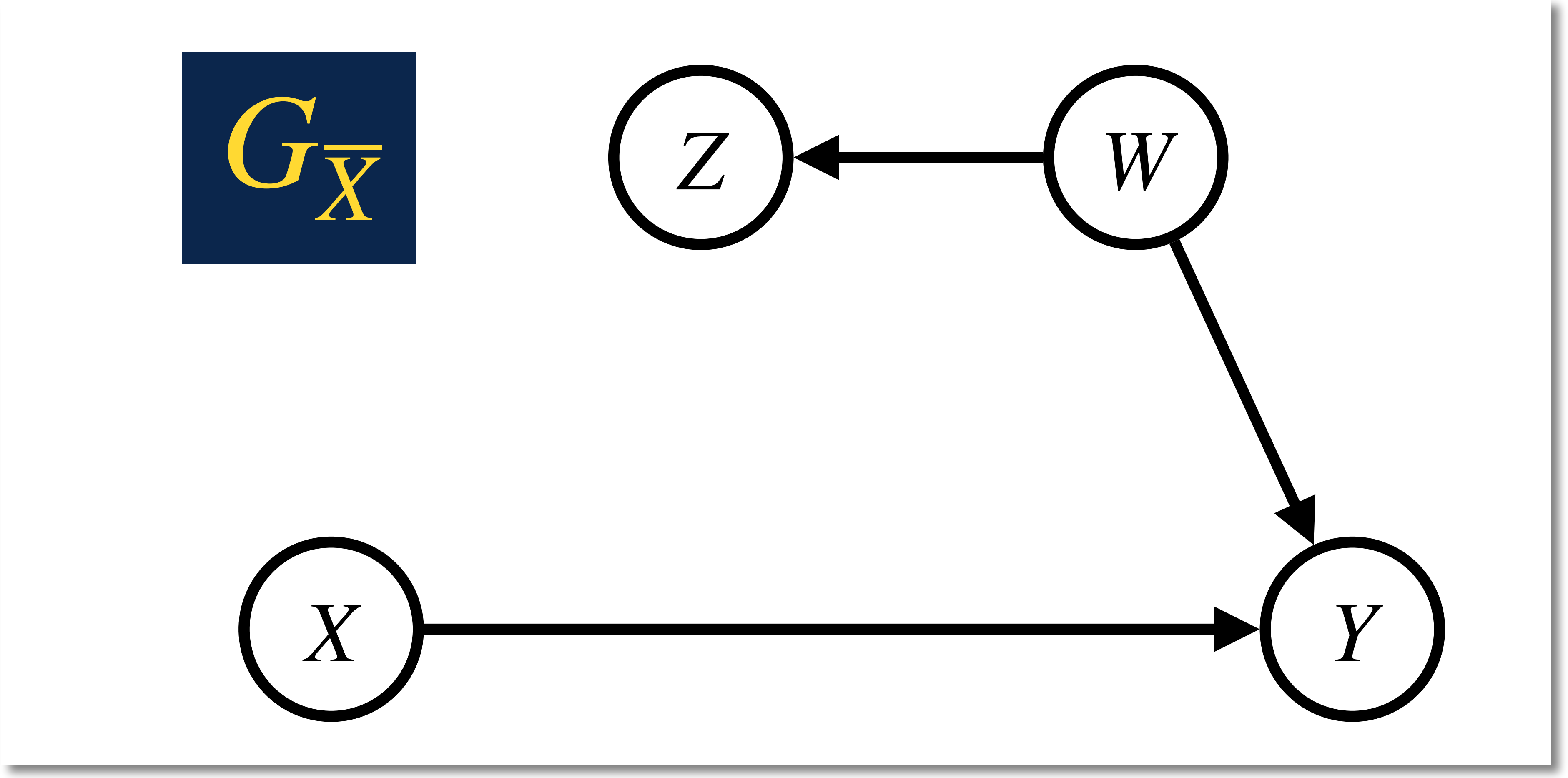
Conditional Independence after Intervention

$$(Y \perp\!\!\!\perp Z \mid X, W)_{G_{\overline{X}}}$$

Z doesn't have an information regarding Y given W after intervening on X.





do-Calculus: Rule 1
conditional independence

Conditional Independence after Intervention

$$(Y \perp\!\!\!\perp Z \mid X, W)_{G_{\overline{X}}} \quad \Rightarrow \quad P(Y \mid do(x), z, w) = P(Y \mid do(x), w)$$

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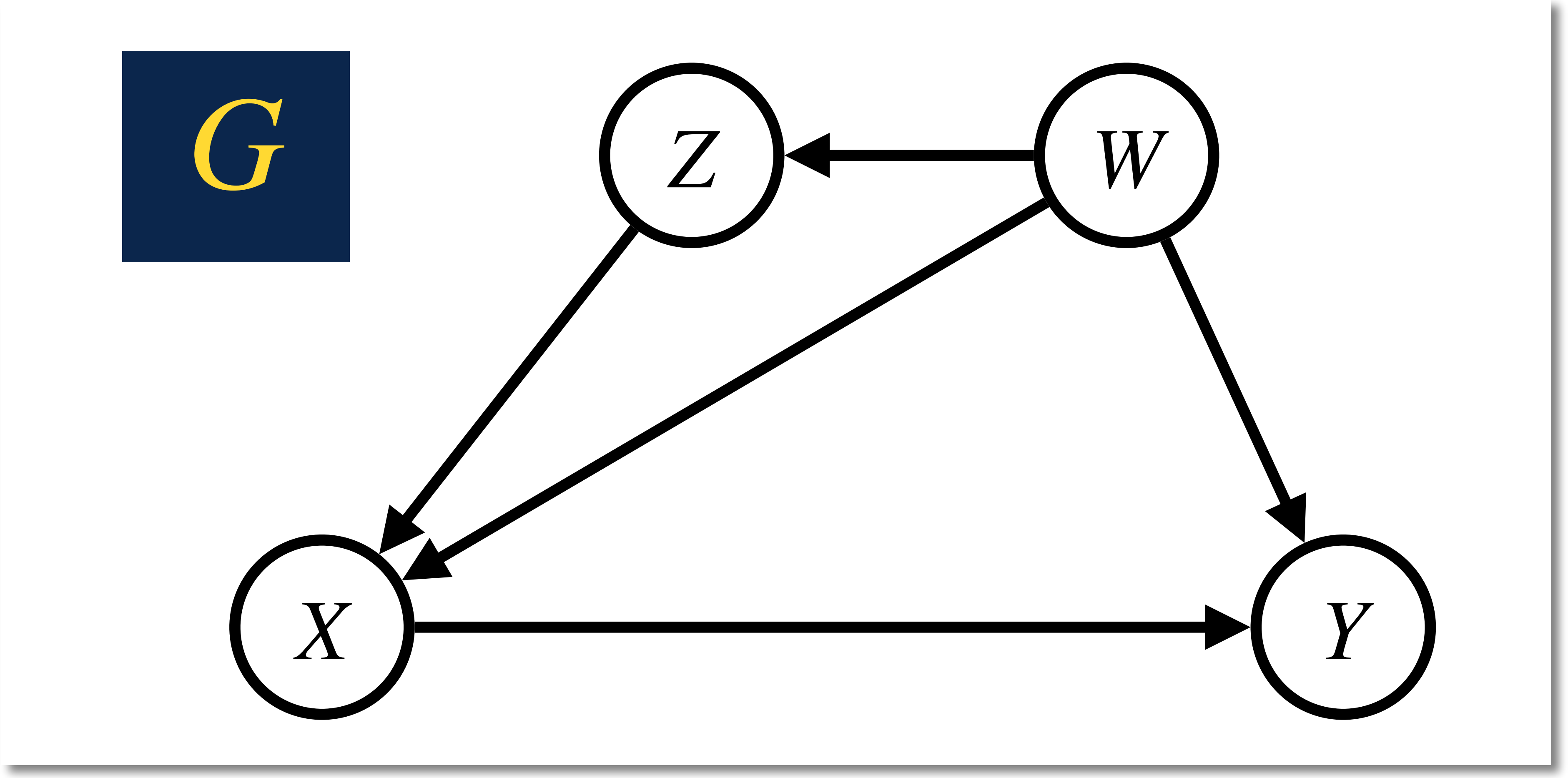

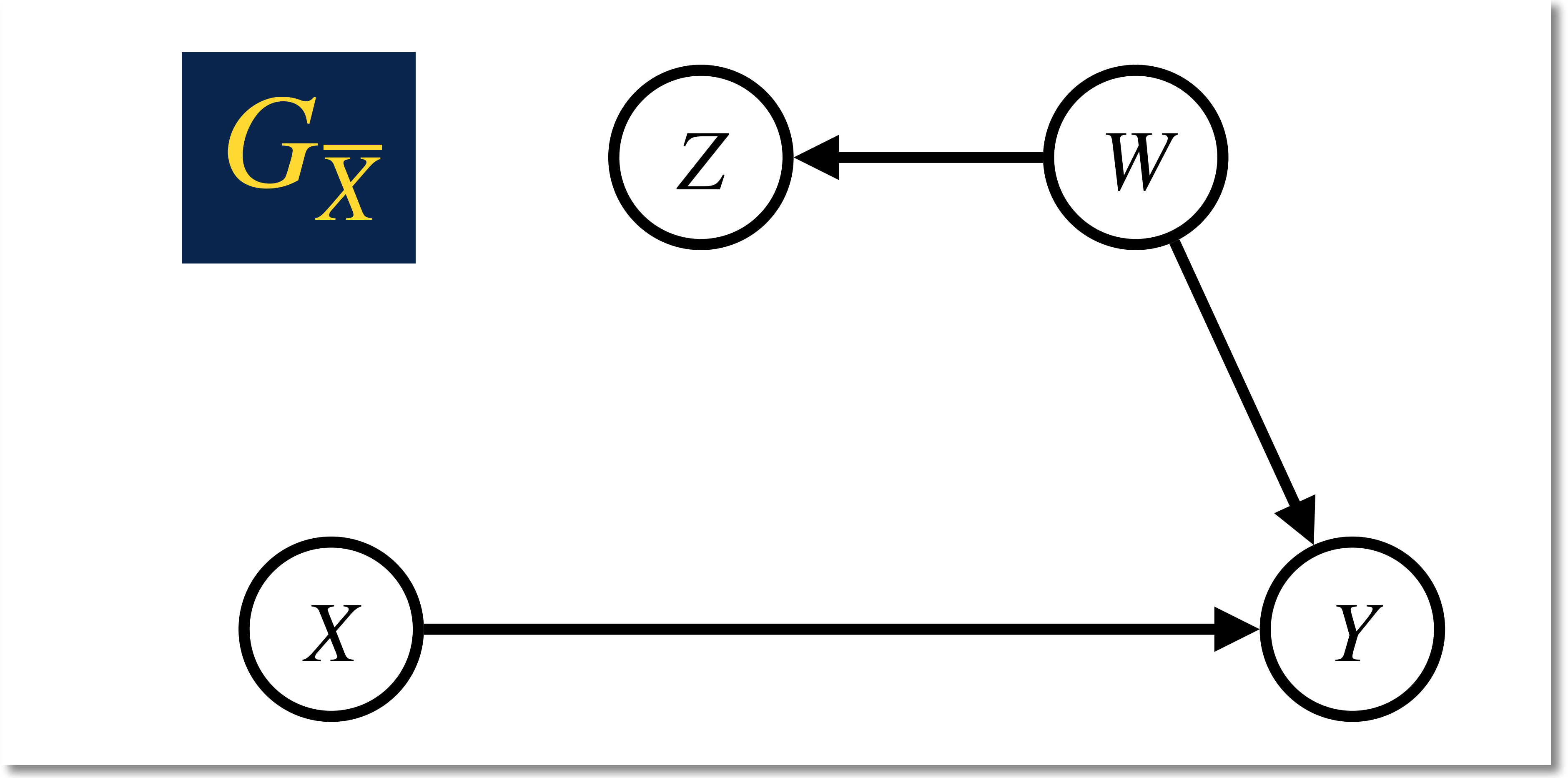

do-Calculus: Rule 2
No unmeasured confounders

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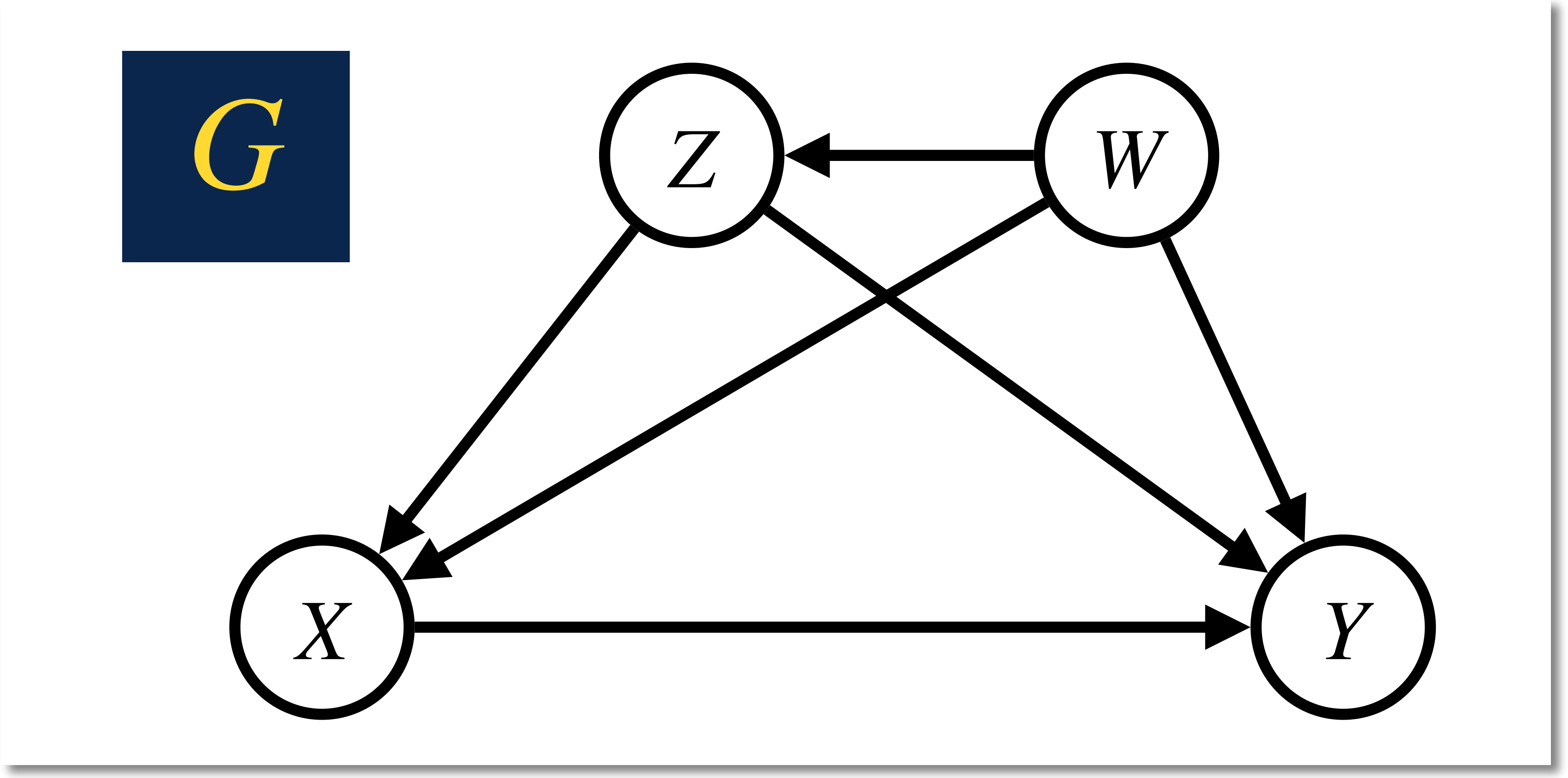
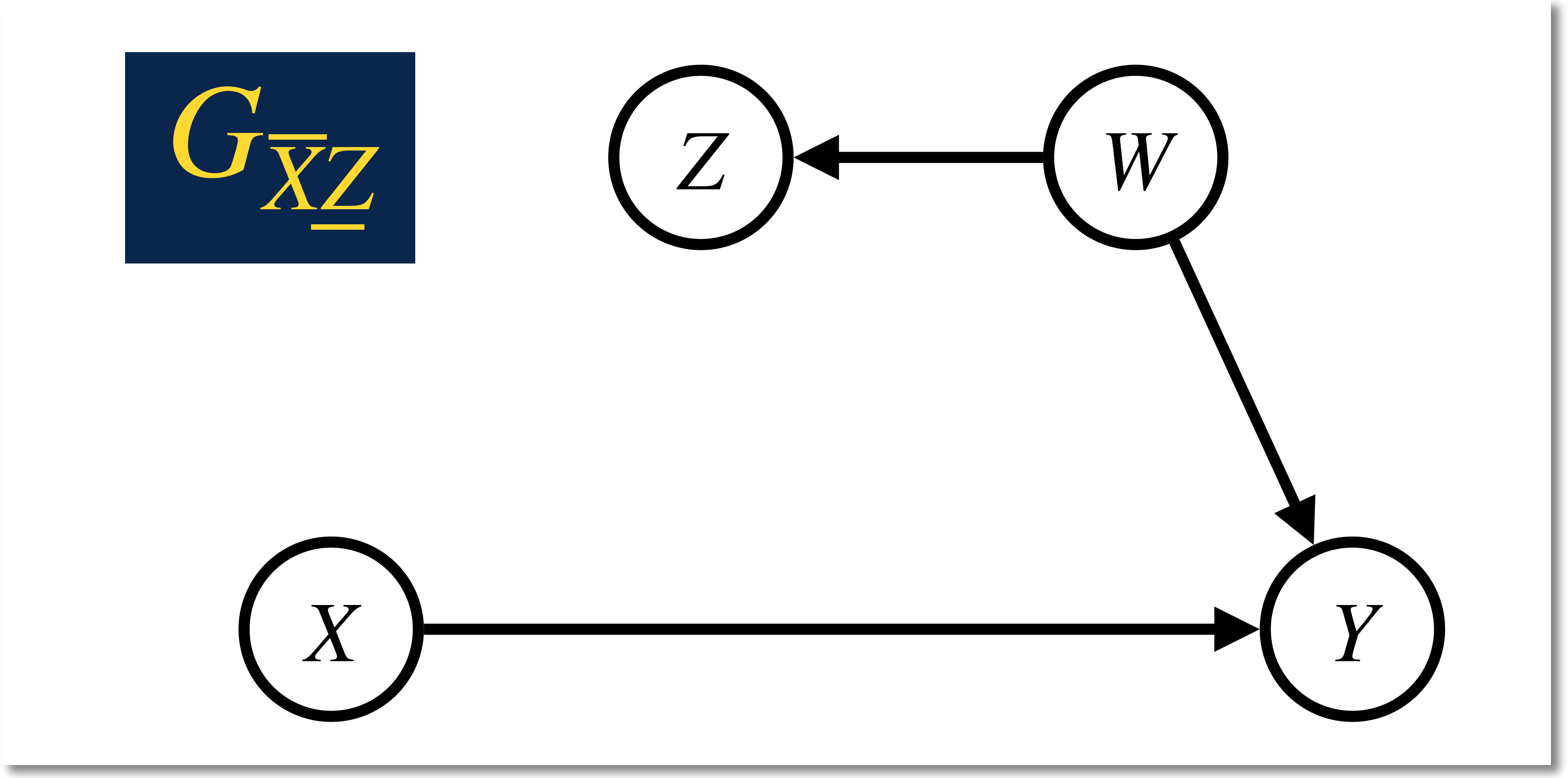
There are no unmeasured confounders between Y and Z conditioning on W.





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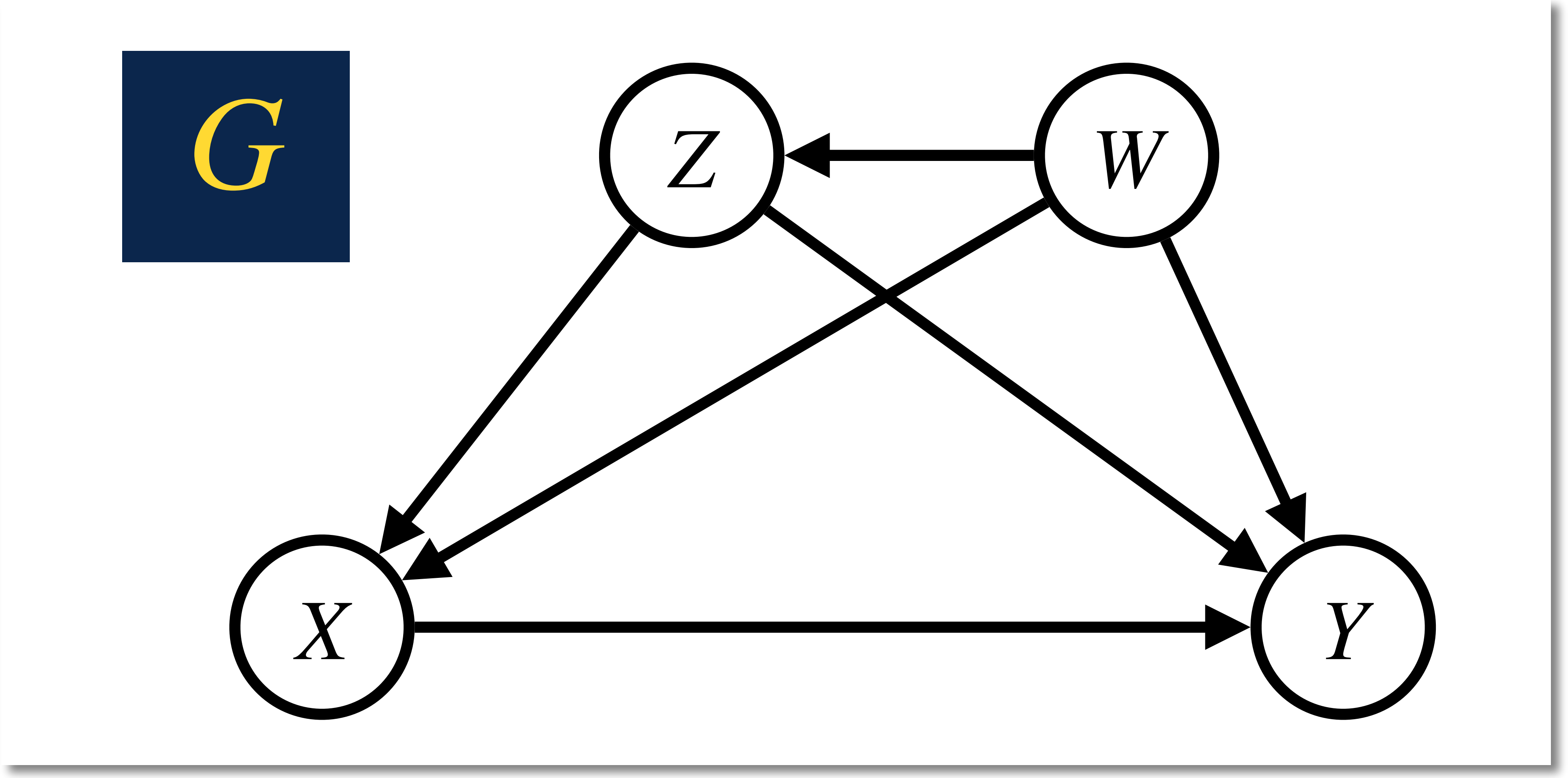

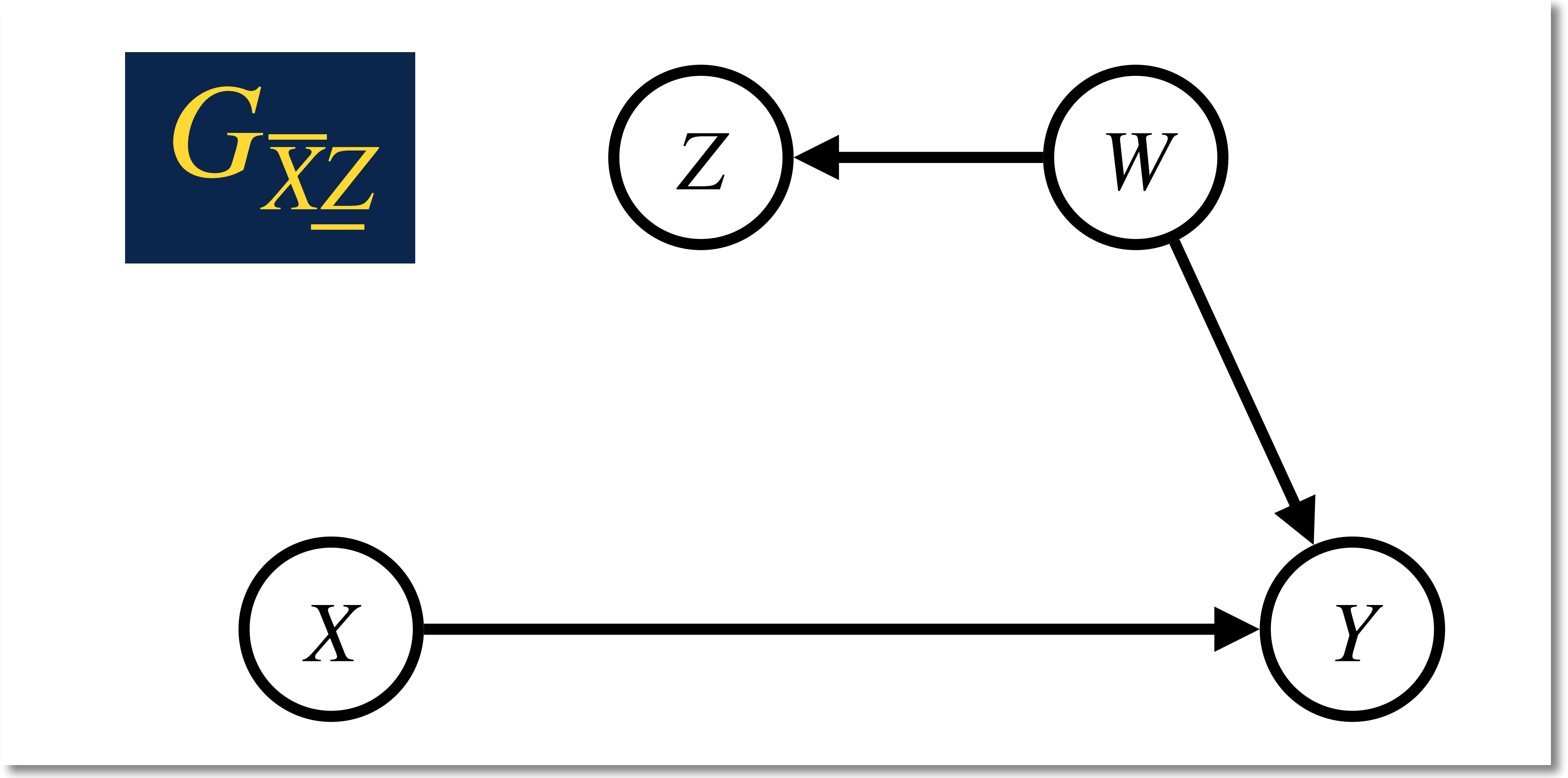

do-Calculus: Rule 3
Intervention independence

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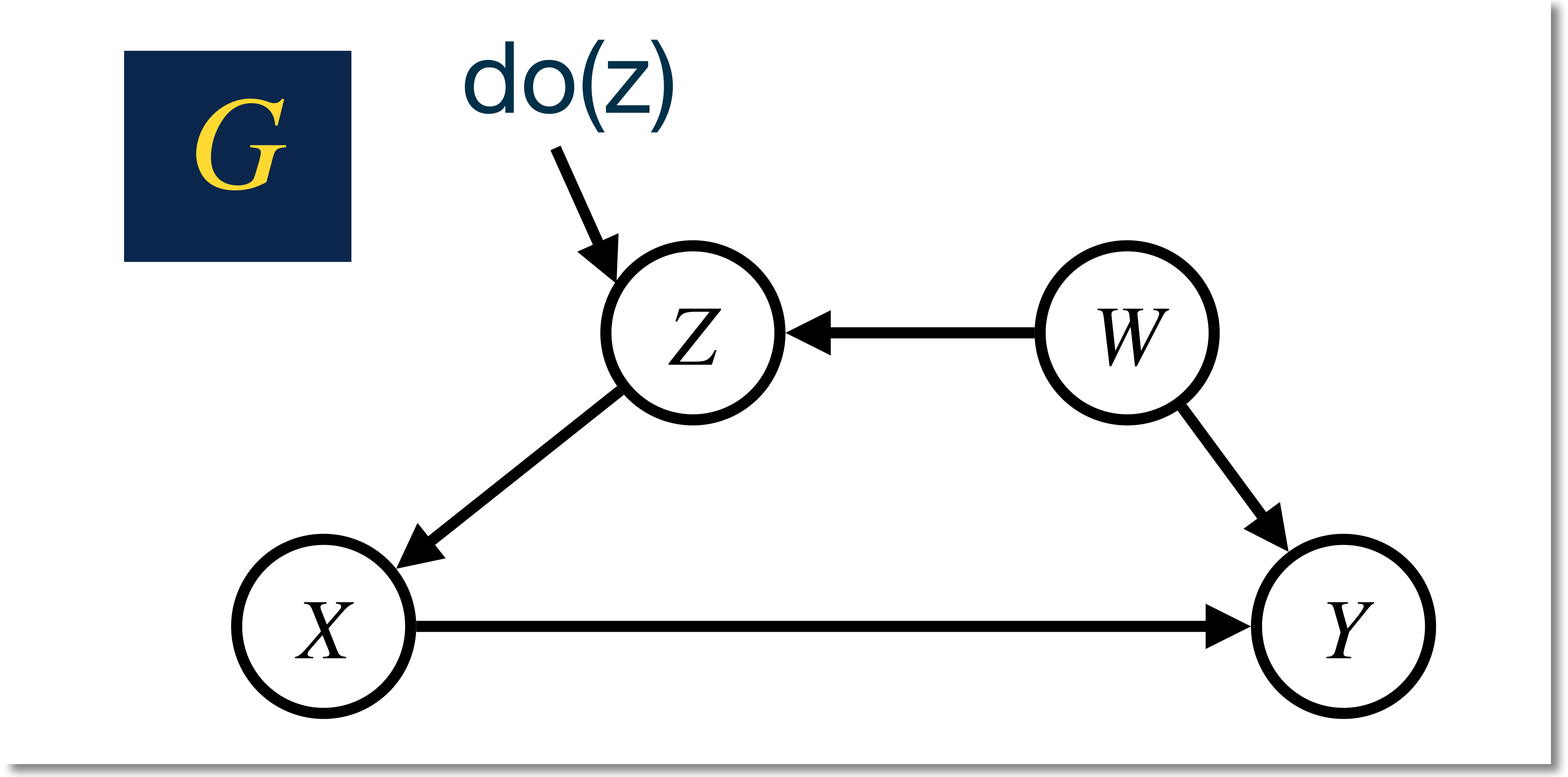
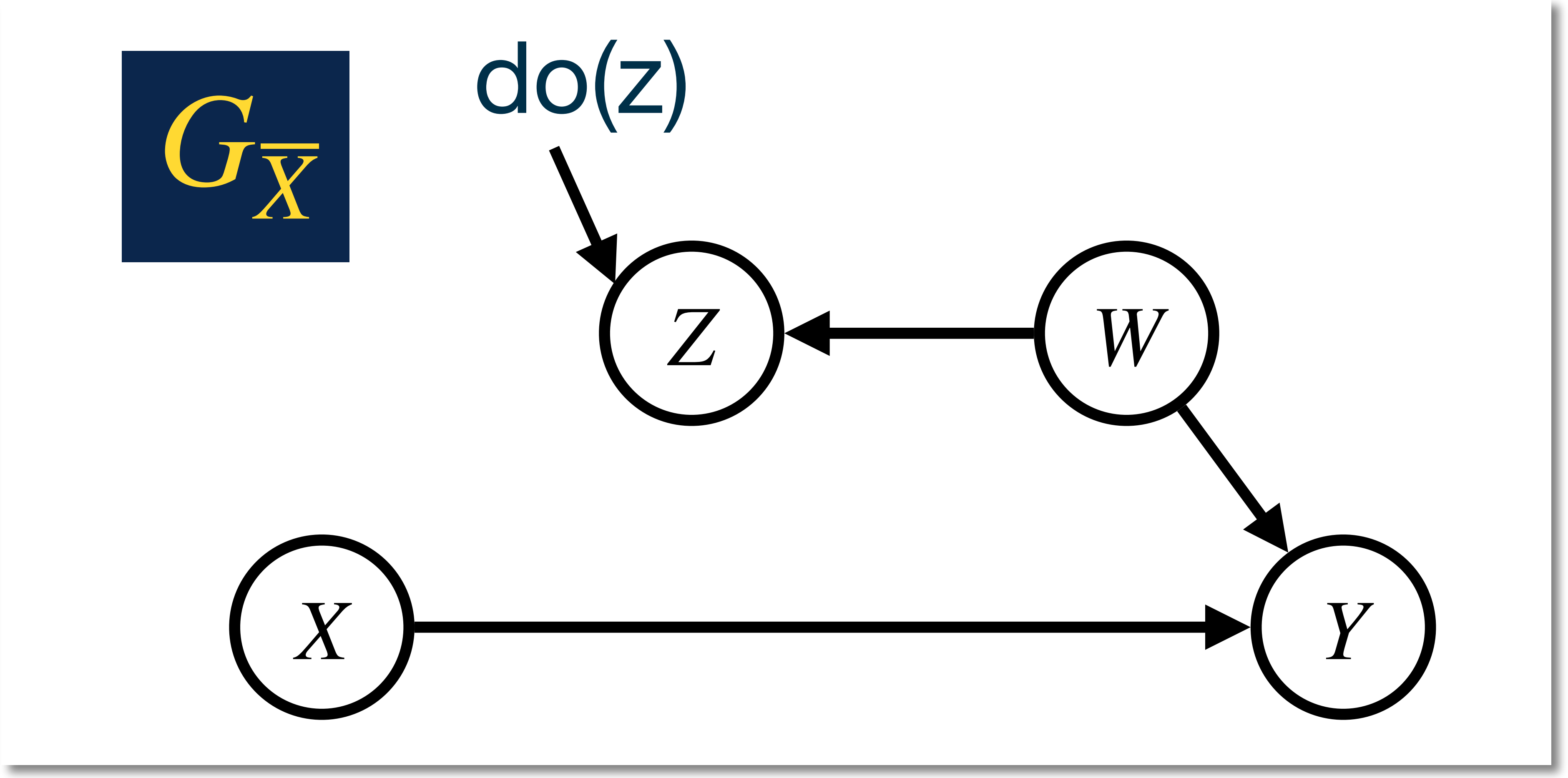
$$(Y \perp\!\!\!\perp \mathrm{do}(z) \mid X, W)_{G_{\overline{X}}}$$

After intervening on X, intervening on Z has no information regarding Y given W





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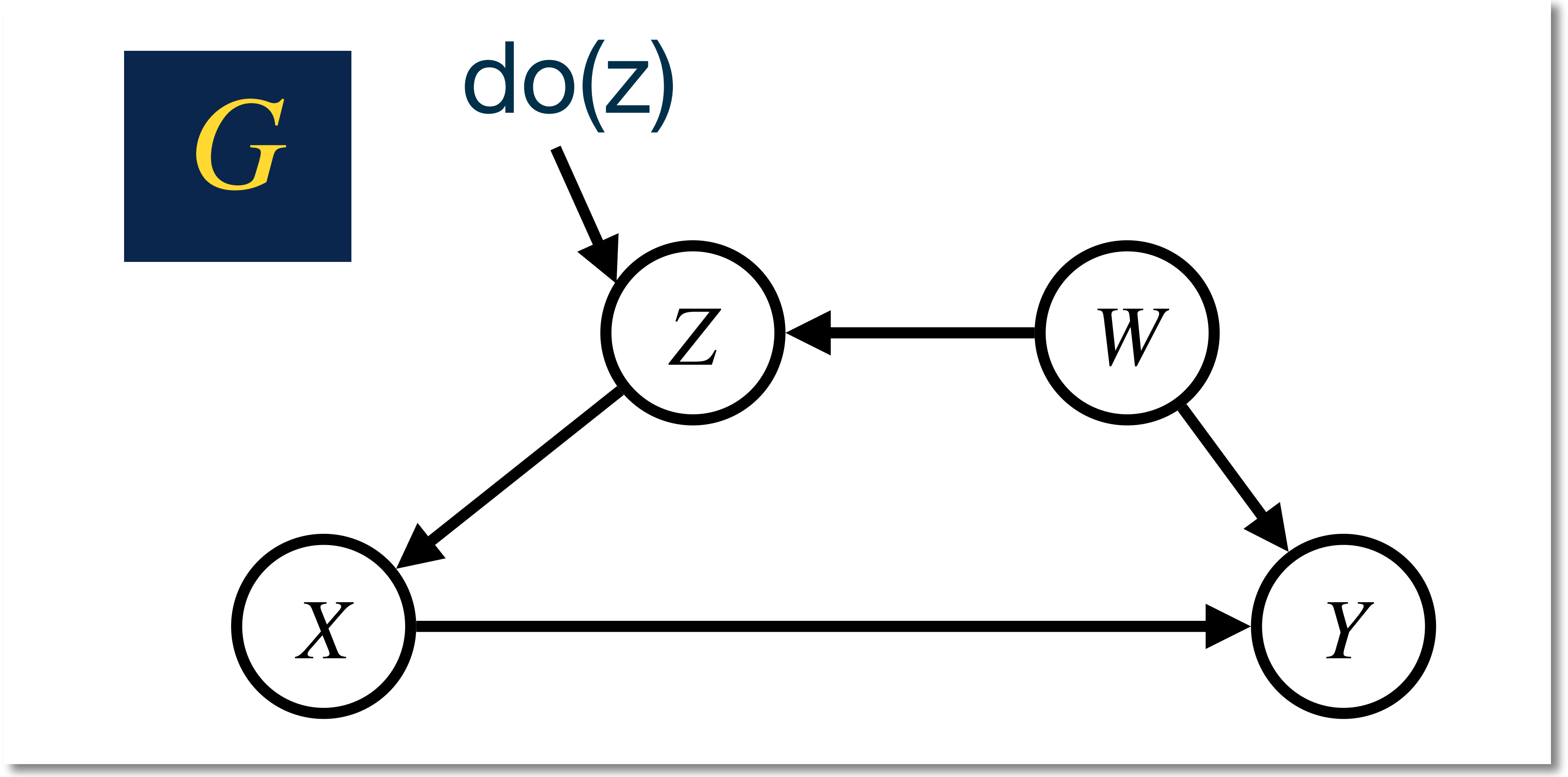

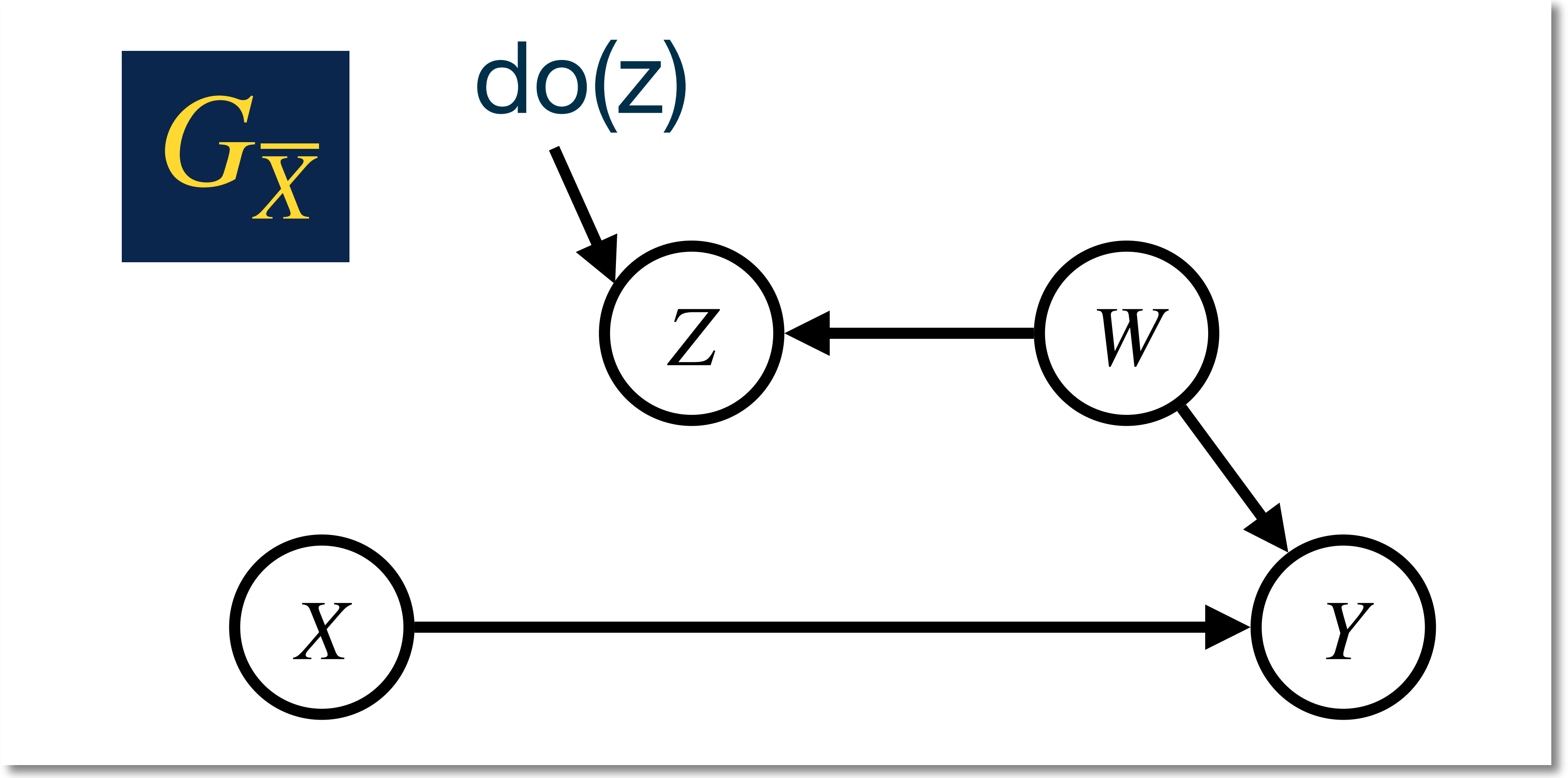

do-Calculus in Identification

Pearl's do-Calculus: Graphical criterion for conditional independences of POs in Graphs

1. $(Y \perp\!\!\!\perp Z \,|\, X, W)_{G_{\overline{X}}} \Rightarrow P(Y \,|\, do(x), z, w) = P(Y \,|\, do(x), w)$

2. $(Y \perp\!\!\!\perp X \,|\, X, W)_{G_{\overline{X}\underline{Z}}} \Rightarrow P(Y \,|\, do(x), do(z), w) = P(Y \,|\, do(x), z, w)$

3. $(Y \perp\!\!\!\perp \text{intv}(Z) \,|\, X, W)_{G_{\overline{X}}} \Rightarrow P(Y \,|\, do(x), do(z), w) = P(Y \,|\, do(x), w)$

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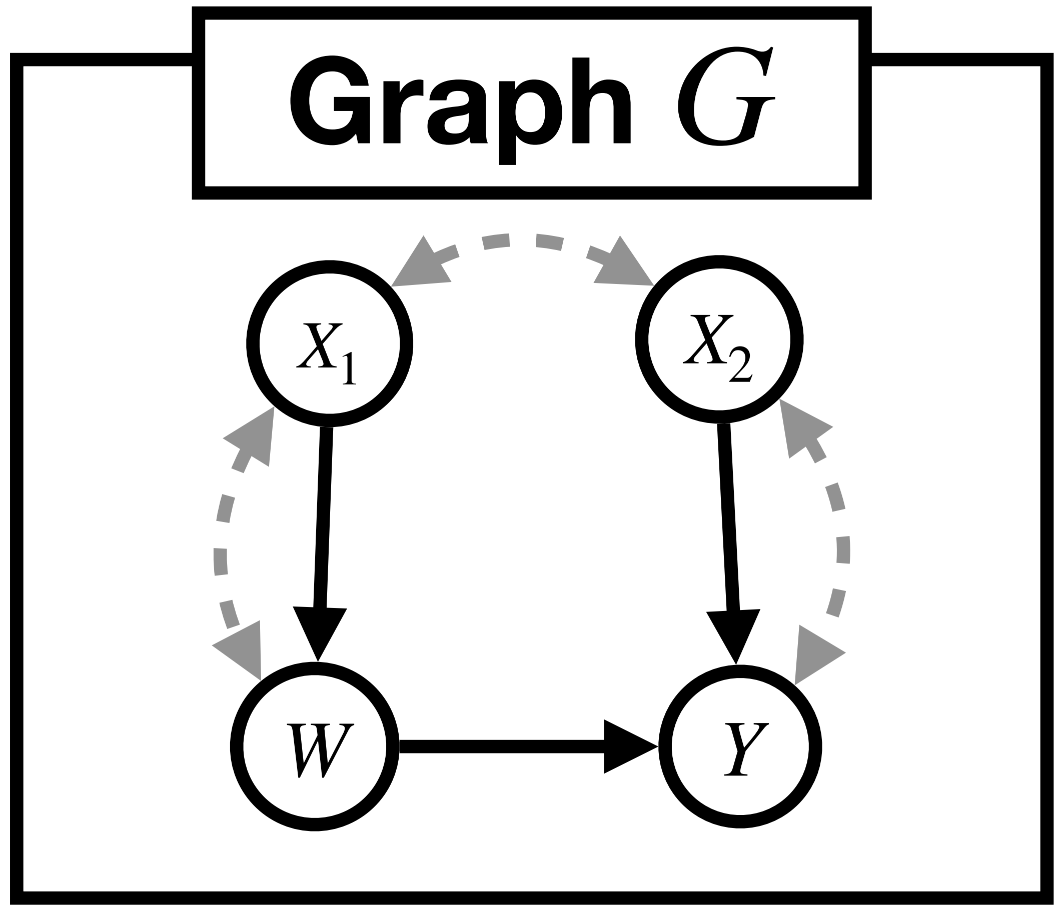
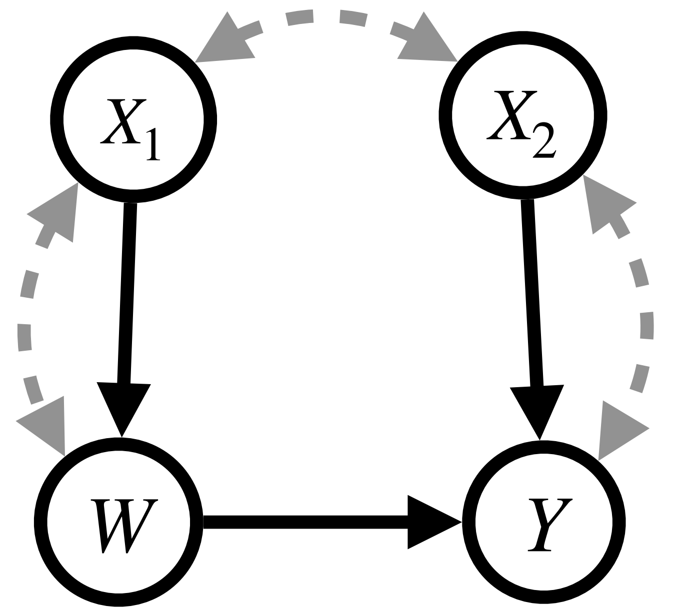


Treatment-Treatment Interaction

Graph G



Dist. $\{P\}$

$$\{P\} := \{P(\mathbf{V} \mid do(x_1)), P(\mathbf{V} \mid do(x_2))\}$$

Treatment-Treatment Interaction

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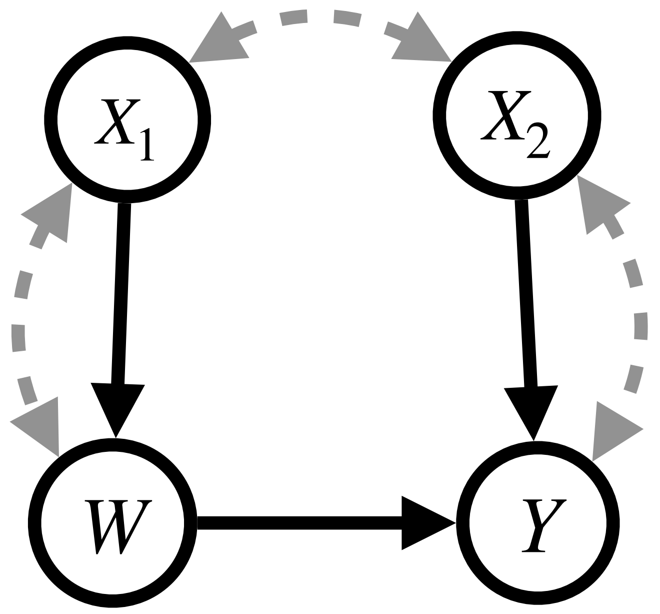

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Query $\mathbb{Q}$

$$\mathbb{E}[Y \mid do(x_1, x_2)]$$

Treatment-Treatment Interaction

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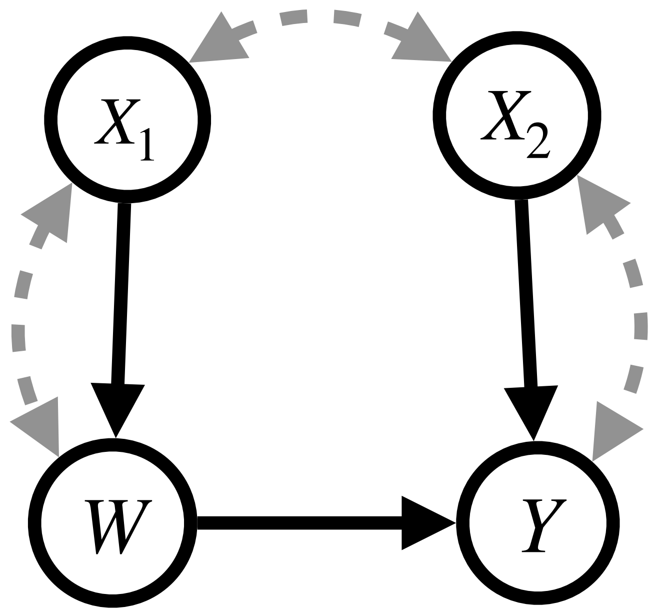

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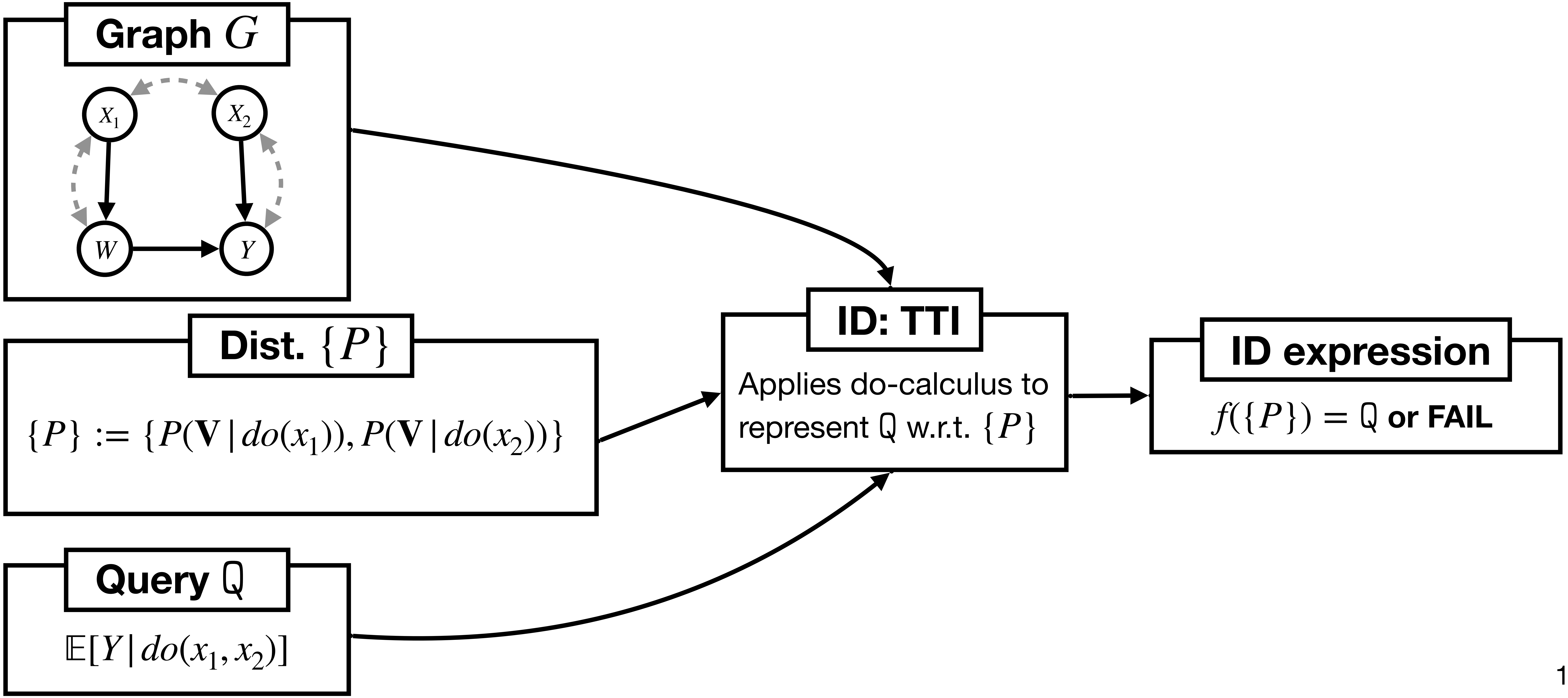
ID: TTI

Applies do-calculus to represent $\mathbb{Q}$ w.r.t. $\{P\}$

Treatment-Treatment Interaction



Sufficient ID Condition: TTI

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Identification Condition for Treatment-Treatment Interaction

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Identification Condition for Treatment-Treatment Interaction

There is a set of variables W s.t.

1. W is a pretreatment variable of the second treatment X_2.
2. There are no unmeasured confounders between X_1 and Y (other than W) in the post-treatment distribution $X_2 = x_2$

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Equivalently,

1. $(W \perp\!\!\!\perp X_2 \mid X_1)_{G_{\overline{X_1, X_2}}}$
2. $(Y \perp\!\!\!\perp X_1 \mid X_2, W)_{G_{\underline{X_1}\overline{X_2}}}$

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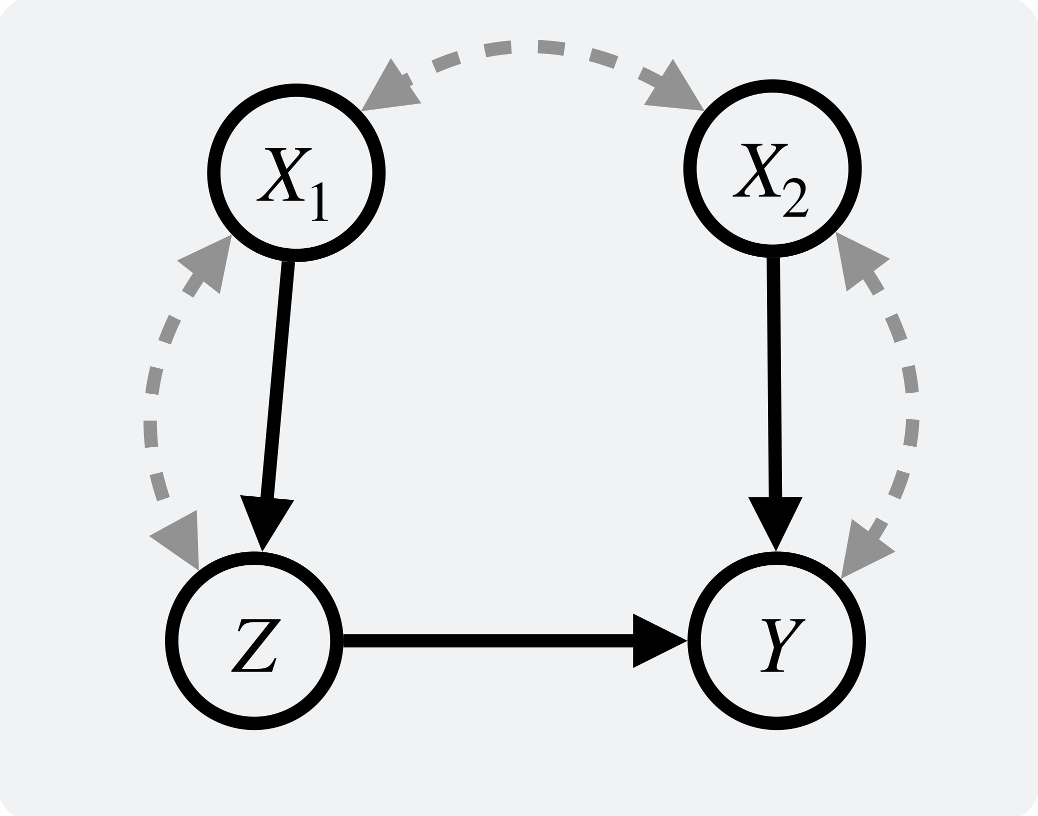

$$W = \{Z\}$$

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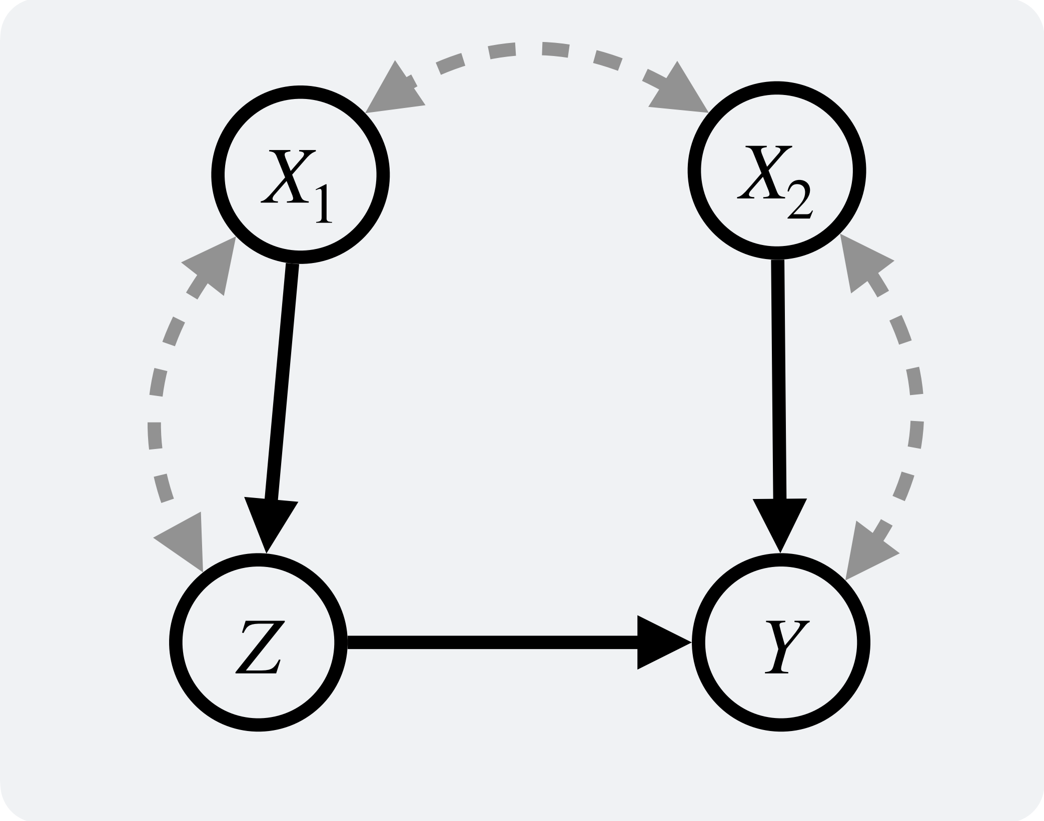
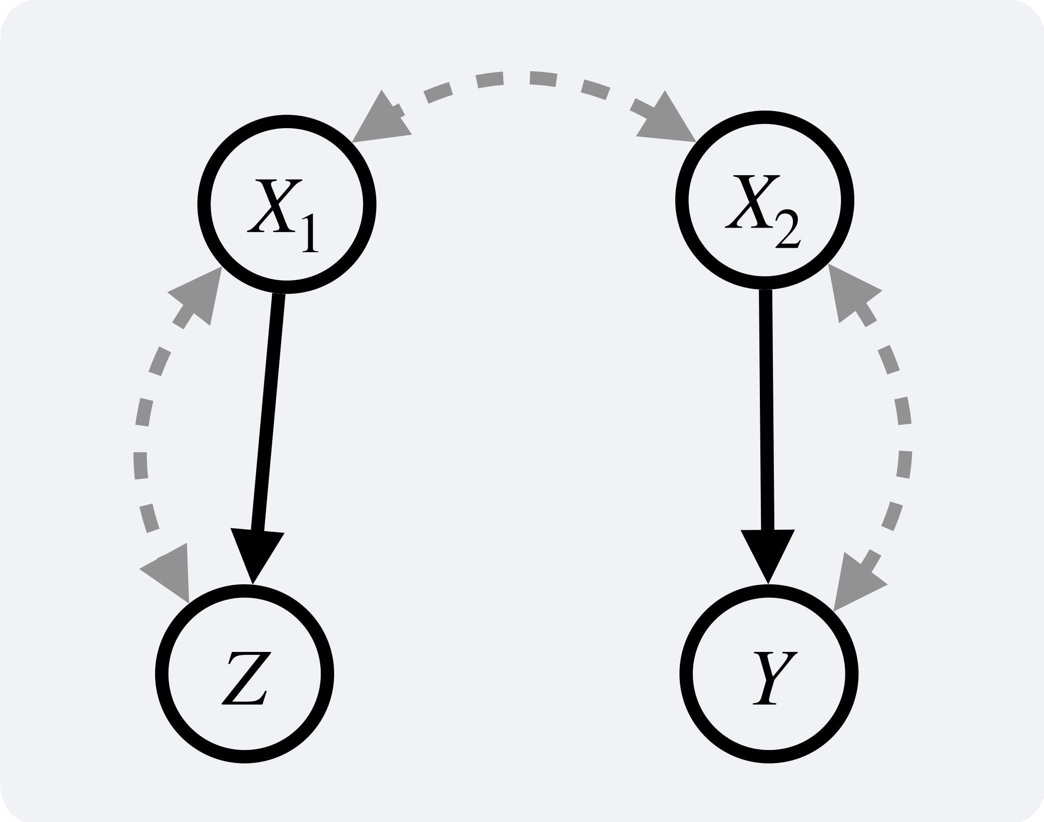


$W = \{Z\}$

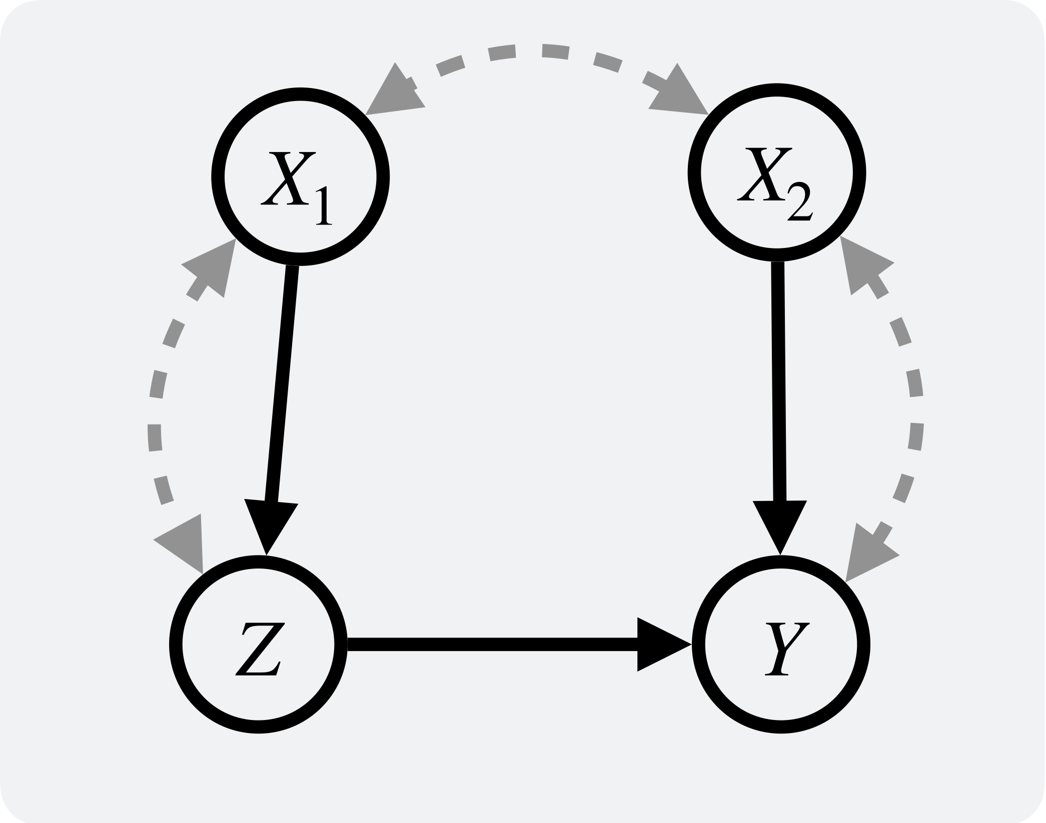


$W = \{Z\}$

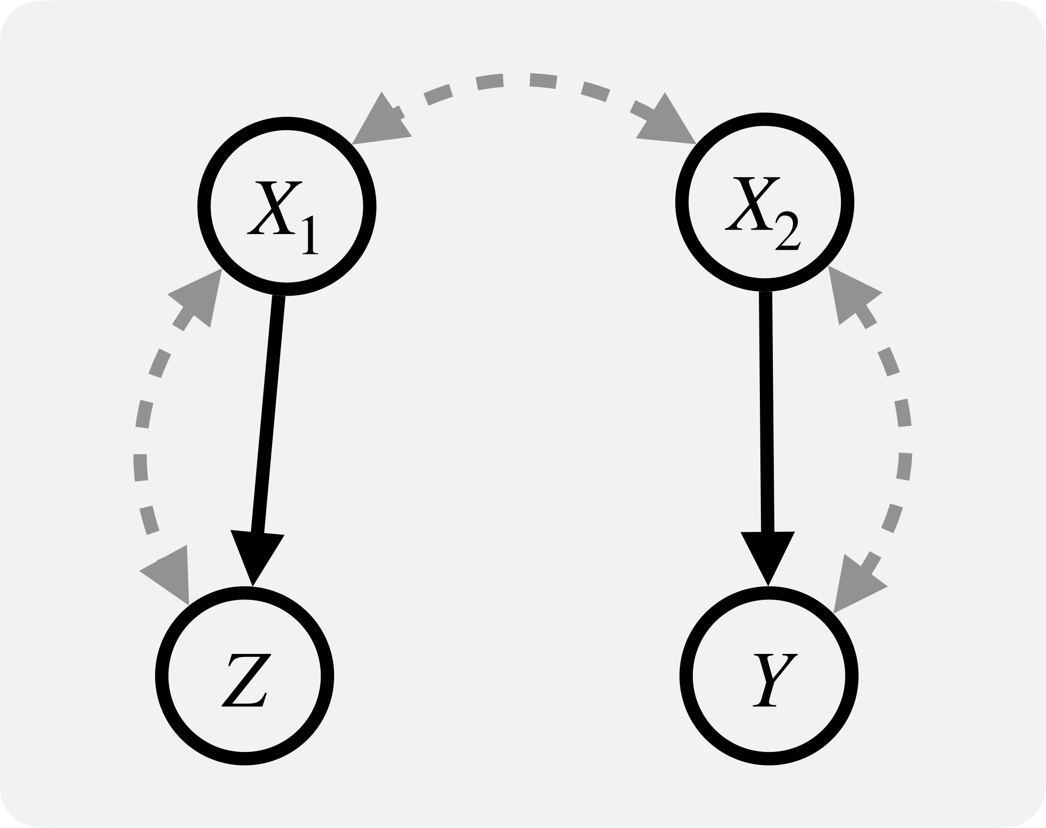
Sufficient ID Condition: TTI

There is a set of variables W s.t.

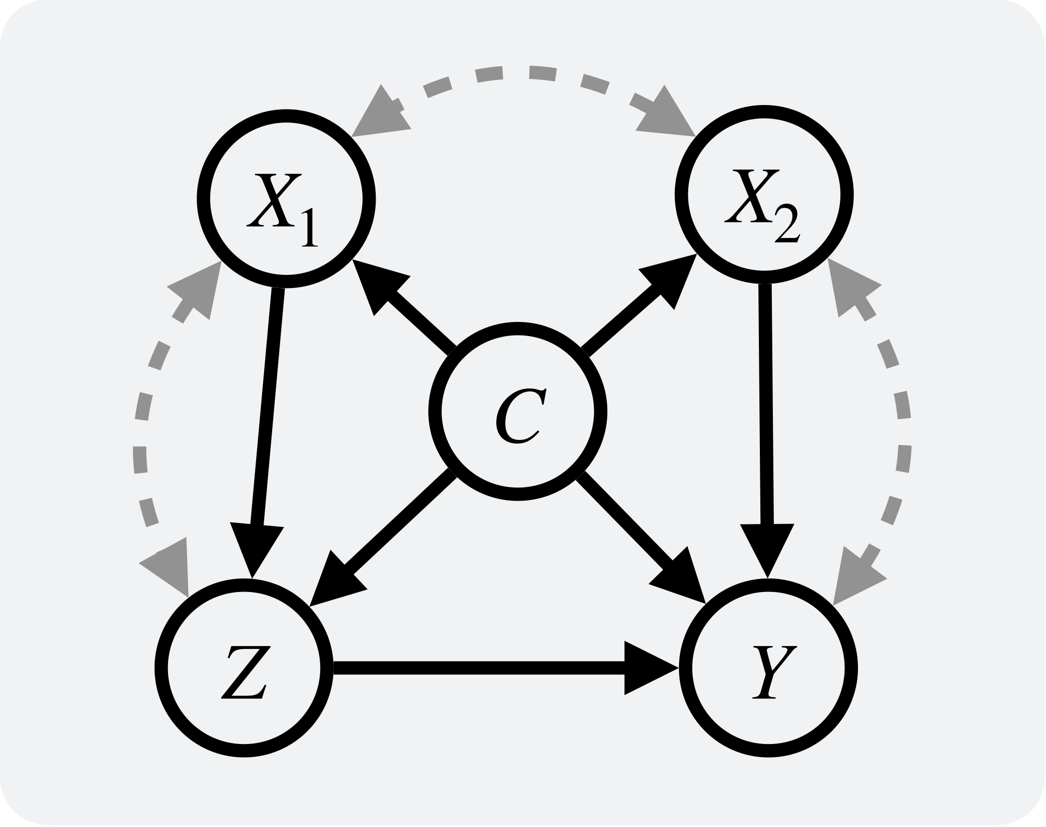
1. W is a pretreatment variable of the second treatment X_2.
2. There are no unmeasured confounders between X_1 and Y other than W in the post-treatment distribution $X_2 = x_2$



$W = \{Z\}$



$W = \{Z\}$



$W = \{Z, C\}$

Treatment-Treatment Interaction

Identification Condition for Treatment-Treatment Interaction

There is a set of variables W s.t.

1. W is a pretreatment variable of the second treatment X_2.
2. There are no unmeasured confounders between X_1 and Y in the post-treatment distribution $X_2 = x_2$

Treatment-Treatment Interaction

Identification Condition for Treatment-Treatment Interaction

There is a set of variables W s.t.

1. W is a pretreatment variable of the second treatment X_2.
2. There are no unmeasured confounders between X_1 and Y in the post-treatment distribution $X_2 = x_2$

Then, $\mathbb{E}[Y \mid do(x_1, x_2)]$ is identifiable from two experimental distributions $P(V \mid do(x_1))$ and $P(V \mid do(x_2))$ as follows:

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \sum_{w \in \mathcal{W}} \mathbb{E}[Y \mid do(x_2), w, x_1] P(w \mid do(x_1)).$$

Treatment-Treatment Interaction

Identification Condition for Treatment-Treatment Interaction

There is a set of variables W s.t.

1. W is a pretreatment variable of the second treatment X_2.
2. There are no unmeasured confounders between X_1 and Y in the post-treatment distribution $X_2 = x_2$

Then, $\mathbb{E}[Y \mid do(x_1, x_2)]$ is identifiable from two experimental distributions $P(V \mid do(x_1))$ and $P(V \mid do(x_2))$ as follows:

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Treatment-Treatment Interaction

Identification Condition for Treatment-Treatment Interaction

There is a set of variables W s.t.

1. W is a pretreatment variable of the second treatment X_2.
2. There are no unmeasured confounders between X_1 and Y in the post-treatment distribution $X_2 = x_2$

Then, $\mathbb{E}[Y \mid do(x_1, x_2)]$ is identifiable from two experimental distributions $P(V \mid do(x_1))$ and $P(V \mid do(x_2))$ as follows:

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Treatment-Treatment Interaction: Intuition

Identification Condition for Treatment-Treatment Interaction

1. $(W \perp\!\!\!\perp X_2 \mid X_1)_{G_{\overline{X_1, X_2}}}$
2. $(Y \perp\!\!\!\perp X_1 \mid X_2, W)_{G_{\underline{X_1}\overline{X_2}}}$

Treatment-Treatment Interaction: Intuition

Identification Condition for Treatment-Treatment Interaction

1. $(W \perp\!\!\!\perp X_2 \mid X_1)_{G_{\overline{X_1, X_2}}}$
2. $(Y \perp\!\!\!\perp X_1 \mid X_2, W)_{G_{\underline{X_1}\overline{X_2}}}$

$$P(y \mid do(x_1, x_2)) = \sum_w P(y \mid do(x_1, x_2), w) P(w \mid do(x_1, x_2))$$

Treatment-Treatment Interaction: Intuition

Identification Condition for Treatment-Treatment Interaction

1. $(W \perp\!\!\!\perp X_2 \mid X_1)_{G_{\overline{X_1, X_2}}}$
2. $(Y \perp\!\!\!\perp X_1 \mid X_2, W)_{G_{\underline{X_1}\overline{X_2}}}$

$$P(y \mid do(x_1, x_2)) = \sum_w P(y \mid do(x_1, x_2), w) P(w \mid do(x_1, x_2))$$

$$= \sum_w P(y \mid do(x_1, x_2), w) P(w \mid do(x_1))$$

do-calc. 3 (ID condition 1)

Treatment-Treatment Interaction: Intuition

Identification Condition for Treatment-Treatment Interaction

1. $(W \perp\!\!\!\perp X_2 \mid X_1)_{G_{\overline{X_1, X_2}}}$
2. $(Y \perp\!\!\!\perp X_1 \mid X_2, W)_{G_{\underline{X_1}\overline{X_2}}}$

$$P(y \mid do(x_1, x_2)) = \sum_w P(y \mid do(x_1, x_2), w) P(w \mid do(x_1, x_2))$$

$$= \sum_w P(y \mid do(x_1, x_2), w) P(w \mid do(x_1))$$

do-calc. 3 (ID condition 1)

$$= \sum_w P(y \mid do(x_2), x_1, w) P(w \mid do(x_1))$$

do-calc. 2 (ID condition 2)

Outline of this talk

1. Preliminary and Problem Setup
   (1) Structural Causal Model
2. **Treatment-Treatment Interaction**
   (1) Identification
   (2) Estimand
   (3) Estimation and Error Analysis
   (4) Simulation Results
3. Multiple-Treatment Interaction (+ Omitted Results)
4. Future directions & Summary

Joint Treatment Effect Estimation : Regression

ID Expression of the Joint Treatment Effect:

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \sum_w \mathbb{E}[Y \mid do(x_2), x_1, w] P(w \mid do(x_1))$$

Joint Treatment Effect Estimation : Regression

ID Expression of the Joint Treatment Effect:

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \sum_w \mathbb{E}[Y \mid do(x_2), x_1, w] P(w \mid do(x_1))$$

Regression-based Representation

Joint Treatment Effect Estimation : Regression

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Regression-based Representation

$$\mu_0(W, X_1) := \mathbb{E}[Y \mid do(x_2), x_1, W]$$

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\mu_0(W, x_1) \mid do(x_1)]$$

Joint Treatment Effect Estimation : Regression

ID Expression of the Joint Treatment Effect:

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$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\boxed{\mu_0(W, x_1)} \mid do(x_1)]$$

Estimable from $D_2 \sim P(\mathbf{V} \mid do(x_2))$

Joint Treatment Effect Estimation : Regression

ID Expression of the Joint Treatment Effect:

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \sum_w \mathbb{E}[Y \mid do(x_2), x_1, w] P(w \mid do(x_1))$$

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Joint Treatment Effect Estimation : Regression

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$$= \mathbb{E}[\mu_0(W, x_1) \mid do(x_1)]$$

Joint Treatment Effect Estimation : Probability Weighting

ID Expression of the Joint Treatment Effect:

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Joint Treatment Effect Estimation : Probability Weighting

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Probability Weighting-based Representation

$$\pi_0(W, X_1) := \frac{P(W \mid do(x_1))}{P(W, X_1 \mid do(x_2))}$$

Joint Treatment Effect Estimation : Probability Weighting

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$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)]$$

Joint Treatment Effect Estimation : Probability Weighting

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Estimable from $D_1 \sim P(\mathbf{V} \mid do(x_1))$

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Estimable from $D_2 \sim P(\mathbf{V} \mid do(x_2))$

Joint Treatment Effect Estimation : Probability Weighting

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)]$$

where $\pi_0(W, X_1) := \dfrac{P(W \mid do(x_1))}{P(W, X_1 \mid do(x_2))}$

Joint Treatment Effect Estimation : Probability Weighting

$\mathbb{E}[Y|do(x_1, x_2)]$

$$\mathbb{E}[Y|do(x_1, x_2)] = \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y | do(x_2)]$$

where $\pi_0(W, X_1) := \dfrac{P(W|do(x_1))}{P(W, X_1|do(x_2))}$

Joint Treatment Effect Estimation : Probability Weighting

$\mathbb{E}[Y \mid do(x_1, x_2)]$

$= \sum_w \mathbb{E}[Y \mid do(x_2), x_1, w] P(w \mid do(x_1))$

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)]$$

where $\pi_0(W, X_1) := \dfrac{P(W \mid do(x_1))}{P(W, X_1 \mid do(x_2))}$

Joint Treatment Effect Estimation : Probability Weighting

$$\mathbb{E}[Y \mid do(x_1, x_2)]$$

$$= \sum_w \mathbb{E}[Y \mid do(x_2), x_1, w] P(w \mid do(x_1))$$

$$= \sum_{w,y} y P(y \mid do(x_2), x_1, w) P(w \mid do(x_1))$$

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)]$$

where $\pi_0(W, X_1) := \dfrac{P(W \mid do(x_1))}{P(W, X_1 \mid do(x_2))}$

Joint Treatment Effect Estimation : Probability Weighting

$$\mathbb{E}[Y \,|\, do(x_1, x_2)]$$

$$= \sum_{w} \mathbb{E}[Y \,|\, do(x_2), x_1, w] P(w \,|\, do(x_1))$$

$$= \sum_{w,y} y P(y \,|\, do(x_2), x_1, w) P(w \,|\, do(x_1))$$

$$= \sum_{w,y} y \frac{P(y, x_1, w \,|\, do(x_2))}{P(x_1, w \,|\, do(x_2))} P(w \,|\, do(x_1))$$

$$\mathbb{E}[Y \,|\, do(x_1, x_2)] = \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \,|\, do(x_2)]$$

where $\pi_0(W, X_1) := \dfrac{P(W \,|\, do(x_1))}{P(W, X_1 \,|\, do(x_2))}$

Joint Treatment Effect Estimation : Probability Weighting

$$\mathbb{E}[Y \mid do(x_1, x_2)]$$

$$= \sum_w \mathbb{E}[Y \mid do(x_2), x_1, w] P(w \mid do(x_1))$$

$$= \sum_{w,y} y P(y \mid do(x_2), x_1, w) P(w \mid do(x_1))$$

$$= \sum_{w,y} y \frac{P(y, x_1, w \mid do(x_2))}{P(x_1, w \mid do(x_2))} P(w \mid do(x_1))$$

$$= \sum_{w,y,x_1'} \underbrace{\frac{P(w \mid do(x_1))}{P(x_1', w \mid do(x_2))}}_{=\pi_0(w, x_1')} I_{x_1}(x_1') y P(y, x_1', w \mid do(x_2))$$

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)]$$

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Joint Treatment Effect Estimation : Probability Weighting

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$$= \sum\nolimits_{w,y,x_1'} \underbrace{\frac{P(w \mid do(x_1))}{P(x_1', w \mid do(x_2))}}_{=\pi_0(w, x_1')} I_{x_1}(x_1') y P(y, x_1', w \mid do(x_2)) \quad = \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)]$$

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)]$$

where $\pi_0(W, X_1) := \dfrac{P(W \mid do(x_1))}{P(W, X_1 \mid do(x_2))}$

Joint Treatment Effect Estimation : DR Estimand

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \begin{cases} \mathbb{E}[\mu_0(W, x_1) \mid do(x_1)] \\ \\ \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)] \end{cases}$$

Joint Treatment Effect Estimation : DR Estimand

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \begin{cases} \mathbb{E}[\mu_0(W, x_1) \mid do(x_1)] \\ \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)] \end{cases}$$

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$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\mu_0(W, x_1) \mid do(x_1)]$$

REG, $= \mathbb{E}[Y \mid do(x_1, x_2)]$

Joint Treatment Effect Estimation : DR Estimand

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \begin{cases} \mathbb{E}[\mu_0(W, x_1) \mid do(x_1)] \\ \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)] \end{cases}$$

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\mu_0(W, x_1) \mid do(x_1)]$$

$$+ \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)]$$

IPW, $= \mathbb{E}[Y \mid do(x_1, x_2)]$

Joint Treatment Effect Estimation : DR Estimand

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \begin{cases} \mathbb{E}[\mu_0(W, x_1) \mid do(x_1)] \\ \\ \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)] \end{cases}$$

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\mu_0(W, x_1) \mid do(x_1)]$$

$$+ \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)]$$

$$- \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) \mu_0(W, X_1) \mid do(x_2)]$$

Bias correcting term

$= \mathbb{E}[Y \mid do(x_1, x_2)]$

Joint Treatment Effect Estimation : DR Estimand

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \begin{cases} \mathbb{E}[\mu_0(W, x_1) \mid do(x_1)] \\ \\ \mathbb{E}[\pi_0(W, X_1) I_{x_1}(X_1) Y \mid do(x_2)] \end{cases}$$

DR Representation

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}\left[\pi_0 I_{x_1}(X_1)(Y - \mu_0) \middle| do(x_2) \right] + \mathbb{E}\left[\mu_0(W, x_1) \middle| do(x_1)) \right]$$

Joint Treatment Effect Estimation : DR Estimand

Joint Treatment Effect Estimation : DR Estimand

$$\mathbb{E}[Y \mid do(x_1, x_2)] := \Psi(\pi_0, \mu_0) = \mathbb{E}\left[\pi_0 I_{x_1}(X_1)(Y - \mu_0) \middle| do(x_2)\right] + \mathbb{E}\left[\mu_0(W, x_1) \middle| do(x_1))\right]$$

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Doubly Robustness of DR representation

Joint Treatment Effect Estimation : DR Estimand

$$\mathbb{E}[Y \mid do(x_1, x_2)] := \Psi(\pi_0, \mu_0) = \mathbb{E}\left[\pi_0 I_{x_1}(X_1)(Y - \mu_0) \middle| do(x_2)\right] + \mathbb{E}\left[\mu_0(W, x_1) \middle| do(x_1)\right]$$

Doubly Robustness of DR representation

The DR representation above $\Psi(\pi_0, \mu_0)$ is unbiased when π_0 or μ_0 is misspecified; i.e., for arbitrary $\pi, \mu \in L_2$,

$$\Psi(\pi_0, \mu_0) = \Psi(\pi, \mu_0) = \Psi(\pi_0, \mu)$$

Outline of this talk

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DML Estimator

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\pi_0 I_{x_1}(X_1)(Y - \mu_0) \mid do(x_2)] + \mathbb{E}[\mu_0(W, x_1) \mid do(x_1))]$$

DML Estimator

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\pi_0 I_{x_1}(X_1)(Y - \mu_0) \mid do(x_2)] + \mathbb{E}[\mu_0(W, x_1) \mid do(x_1))]$$

DML estimator T^{dml}

DML Estimator

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\pi_0 I_{x_1}(X_1)(Y - \mu_0) \mid do(x_2)] + \mathbb{E}[\mu_0(W, x_1) \mid do(x_1))]$$

DML estimator T^{dml}

1. Randomly split the sample $D_1 := D_{1,a} \cup D_{1,b}$ and $D_2 := D_{2,a} \cup D_{2,b}$.

DML Estimator

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\pi_0 I_{x_1}(X_1)(Y - \mu_0) \mid do(x_2)] + \mathbb{E}[\mu_0(W, x_1) \mid do(x_1))]$$

DML estimator T^{dml}

1. Randomly split the sample $D_1 := D_{1,a} \cup D_{1,b}$ and $D_2 := D_{2,a} \cup D_{2,b}$.
2. Estimate (μ^a, π^a) for $\mu_0(W, X_1) := \mathbb{E}[Y \mid do(x_2), X_1, W]$ and $\pi_0(W, X_1) := P(W \mid do(x_1))/P(W, X_1 \mid do(x_2))$ using $D_{1,a}, D_{2,a}$.

DML Estimator

$$\mathbb{E}[Y \mid do(x_1, x_2)] = \mathbb{E}[\pi_0 I_{x_1}(X_1)(Y - \mu_0) \mid do(x_2)] + \mathbb{E}[\mu_0(W, x_1) \mid do(x_1))]$$

DML estimator T^{dml}

1. Randomly split the sample $D_1 := D_{1,a} \cup D_{1,b}$ and $D_2 := D_{2,a} \cup D_{2,b}$.
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3. Evaluate $T^b := \mathbb{E}_{D_{2,b}}[\pi^a I_{x_1}(X_1)\{Y - \mu^a(W, X_1)\}] + \mathbb{E}_{D_{1,b}}[\mu^a(W, x_1)]$ using $D_{1,b}, D_{2,b}$.

DML Estimator

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4. Repeat 2-3 by switching $D_{1,a}, D_{2,a}$ and $D_{1,b}, D_{2,b}$ and obtain T^a with (μ^b, π^b).

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4. Repeat 2-3 by switching $D_{1,a}, D_{2,a}$ and $D_{1,b}, D_{2,b}$ and obtain T^a with (μ^b, π^b).
5. $T^{dml} := (T^a + T^b)/2$.

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2. Augment (W_i, X_i, λ_i). Then,

$$\pi_0(W, X_1) = \frac{P(\lambda = 1)}{P(\lambda = 0)} \frac{P(\lambda = 0 \mid W, X_1)}{P(\lambda = 1 \mid W, X_1)}$$

Finite Sample Error Analysis

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With a probability $1 - 4\epsilon$

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where $n_1 := |D_1|$, $n_2 := |D_2|$, $\|\|_P$ is a $L_2(P)$ norm.

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$$(b) := \sum_{k \in \{a,b\}} \mathbb{E}_{P_2}[\{\mu^k - \mu_0\}\{\pi_0 - \pi^k\}]$$

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$$T^{dml} - \mathbb{E}[Y \mid do(x_1, x_2)] = (R_1 + R_2) + \sum_{k \in \{a,b\}} O_{P_2}\left(\|\pi^k - \pi_0\|_{P_2}\|\mu^k - \mu_0\|_{P_2}\right)$$

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where R_i (for $i \in \{1,2\}$) is a random variable s.t. $\sqrt{n_i}R_i \xrightarrow{d} Z_i \sim$ normal$(0,\sigma_i^2)$, where $\sigma_1^2 := \mathbb{V}_{P_1}[\mu_0(W, x_1)]$ and $\sigma_0^2 := \mathbb{V}_{P_2}[\pi_0(Y - \mu_0)]$.

Robustness of DML estimator

$$T^{dml} - \mathbb{E}[Y \mid do(x_1, x_2)] = (R_1 + R_2) + \sum_{k \in \{a,b\}} O_{P_2}\left(\|\pi^k - \pi_0\|_{P_2} \|\mu^k - \mu_0\|_{P_2} \right)$$

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Doubly Robustness & Debiasedness

1. **Doubly Robustness (DR):** T^{dml} converges to $\mathbb{E}[Y \mid do(x_1, x_2)]$ at $n^{-1/2}$-rate (where $n := \min(n_1, n_2)$) if $\pi = \pi_0$ or $\mu = \mu_0$.

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2. **Debiasedness (DB):** T^{dml} converges to $\mathbb{E}[Y \mid do(x_1, x_2)]$ at $n^{-1/2}$-rate if π, μ converges to π_0, μ_0 at least at $n^{-1/4}$ rate.

Efficiency of DML estimator

$$T^{dml} - \mathbb{E}[Y \mid do(x_1, x_2)] = (R_1 + R_2) + \sum_{k \in \{a,b\}} O_{P_2}\left(\|\pi^k - \pi_0\|_{P_2} \|\mu^k - \mu_0\|_{P_2} \right)$$

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CAN and Efficiency

Efficiency of DML estimator

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CAN and Efficiency

1. **Consistency and Asymptotic Normality (CAN):** Under the {DR,DB} conditions, T^{dml} achieves consistency and asymptotic normality (CAN).

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CAN and Efficiency

1. **Consistency and Asymptotic Normality (CAN):** Under the {DR,DB} conditions, T^{dml} achieves consistency and asymptotic normality (CAN).

2. **Statistical Efficiency:** Under the {DR,DB} conditions, T^{dml} achieves the nonparametric efficiency bound (i.e., achieves the minimum variance).

Outline of this talk

1. Preliminary and Problem Setup
   (1) Structural Causal Model
2. **Treatment-Treatment Interaction**
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   (3) Estimation and Error Analysis
   (4) Simulation Results
3. Multiple-Treatment Interaction (+ Omitted Results)
4. Future directions & Summary

Simulations: Synthetic Dataset

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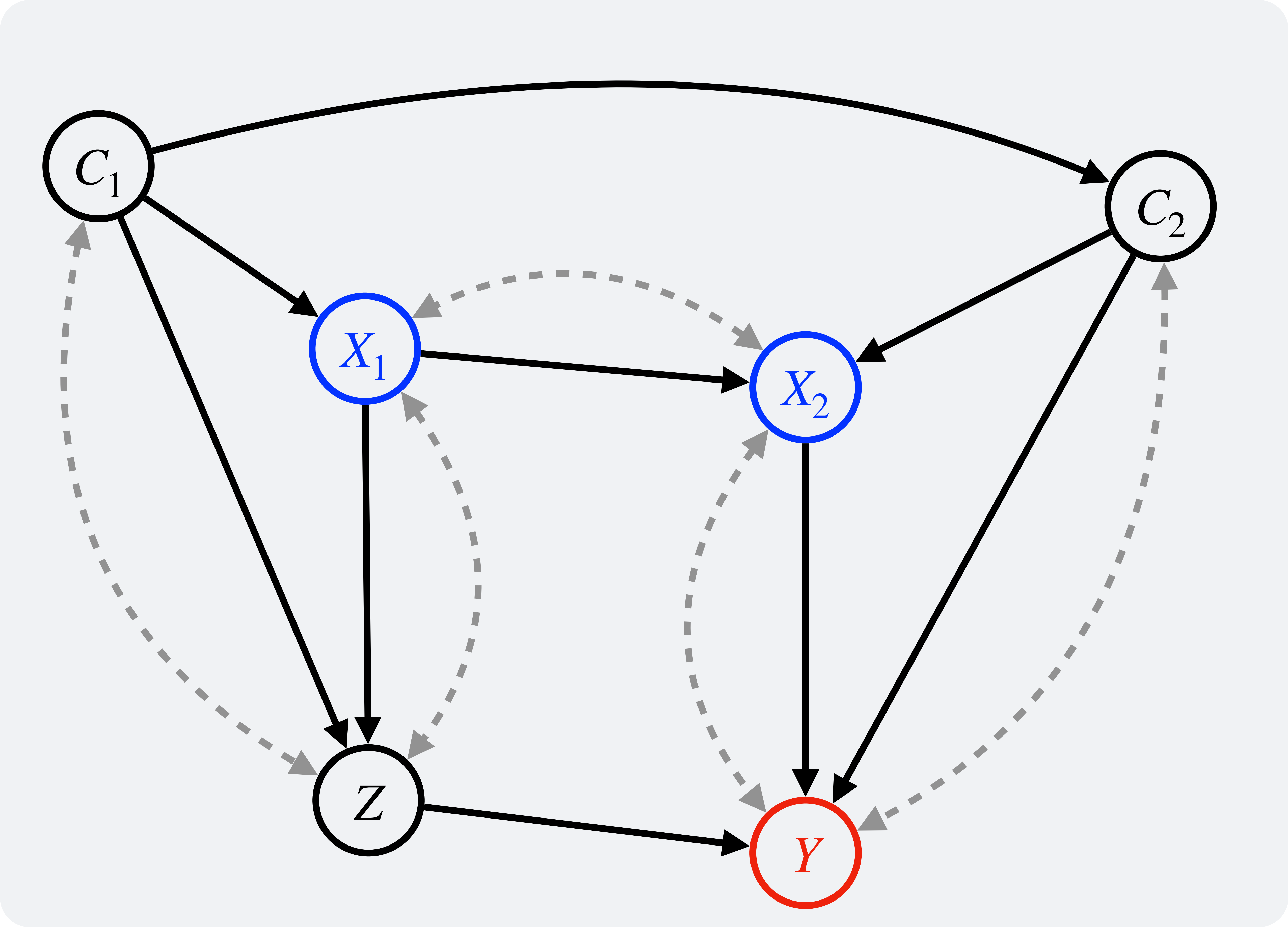

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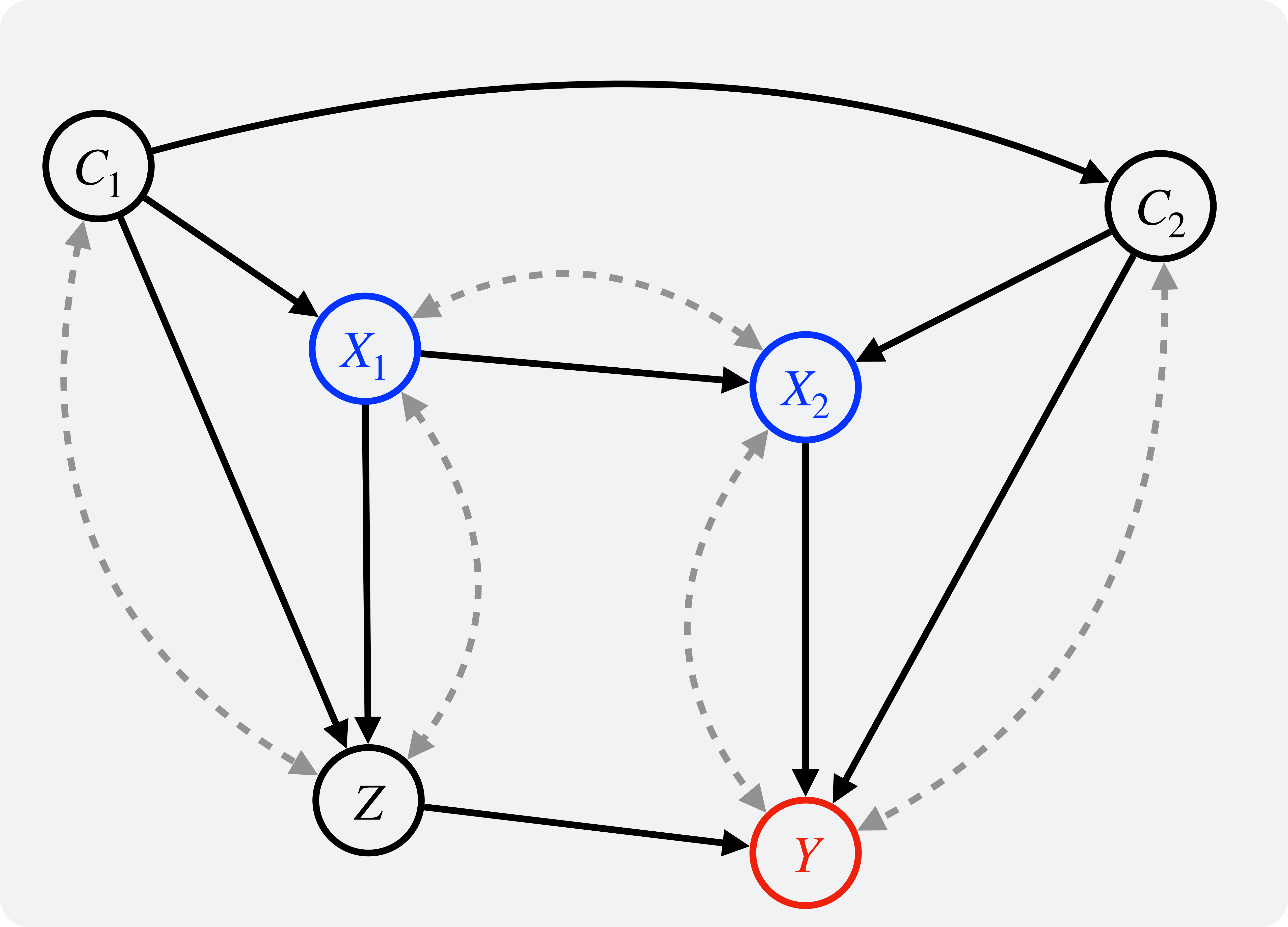

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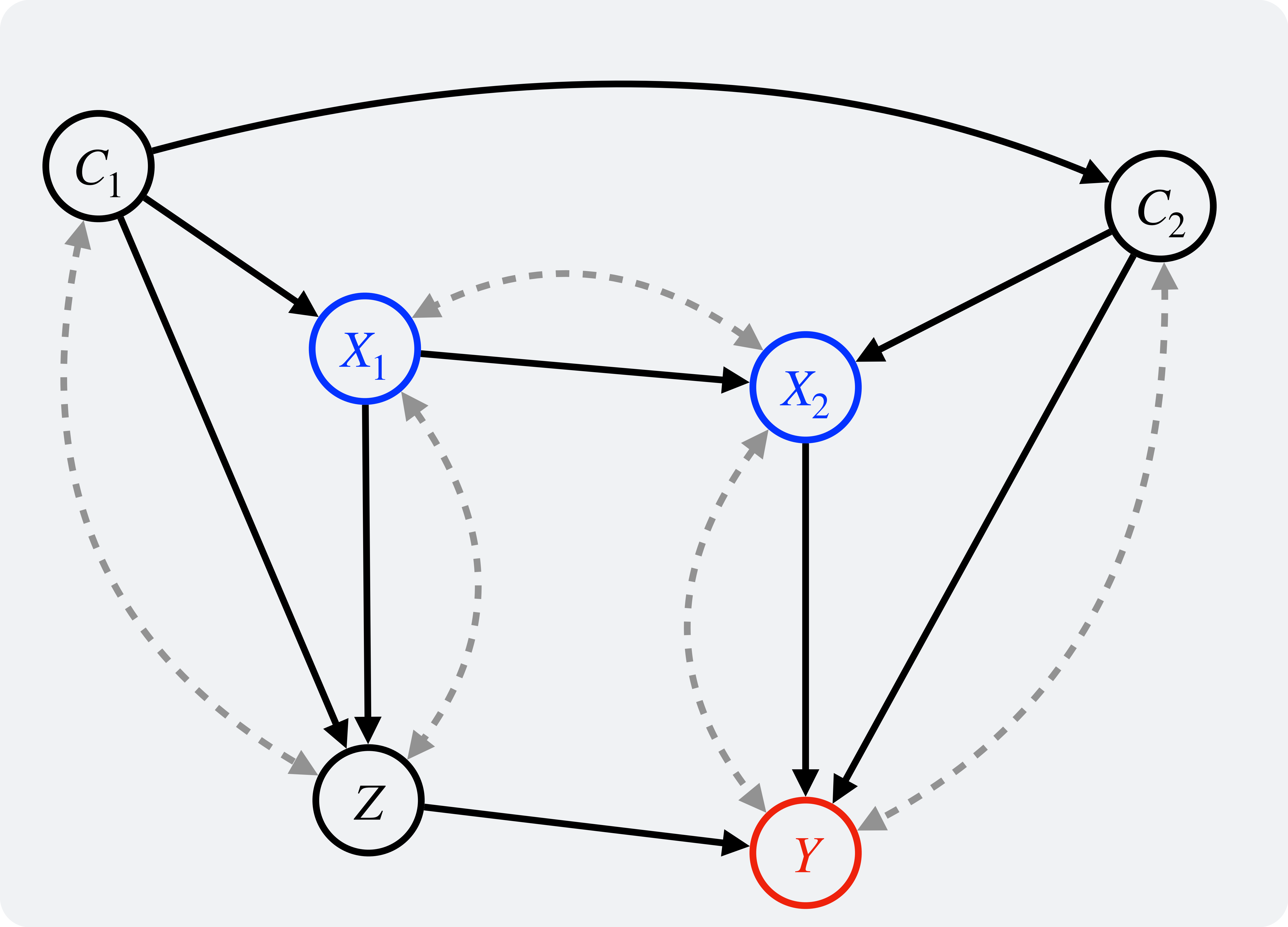

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Input:

1. $D_1 \overset{iid}{\sim} P(C_1, C_2, X_2, Z, Y \mid do(x_1))$
2. $D_2 \overset{iid}{\sim} P(C_1, C_2, X_1, Z, Y \mid do(x_2))$

Simulation Scenario

$$\mathbb{E}[Y \,|\, do(x_1, x_2)] = \mathbb{E}[\pi_0 I_{x_1}(X_1)(Y - \mu_0) \,|\, do(x_2)] + \mathbb{E}[\mu_0(W, x_1) \,|\, do(x_1))]$$

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Scenario 1. Regular learning environment.

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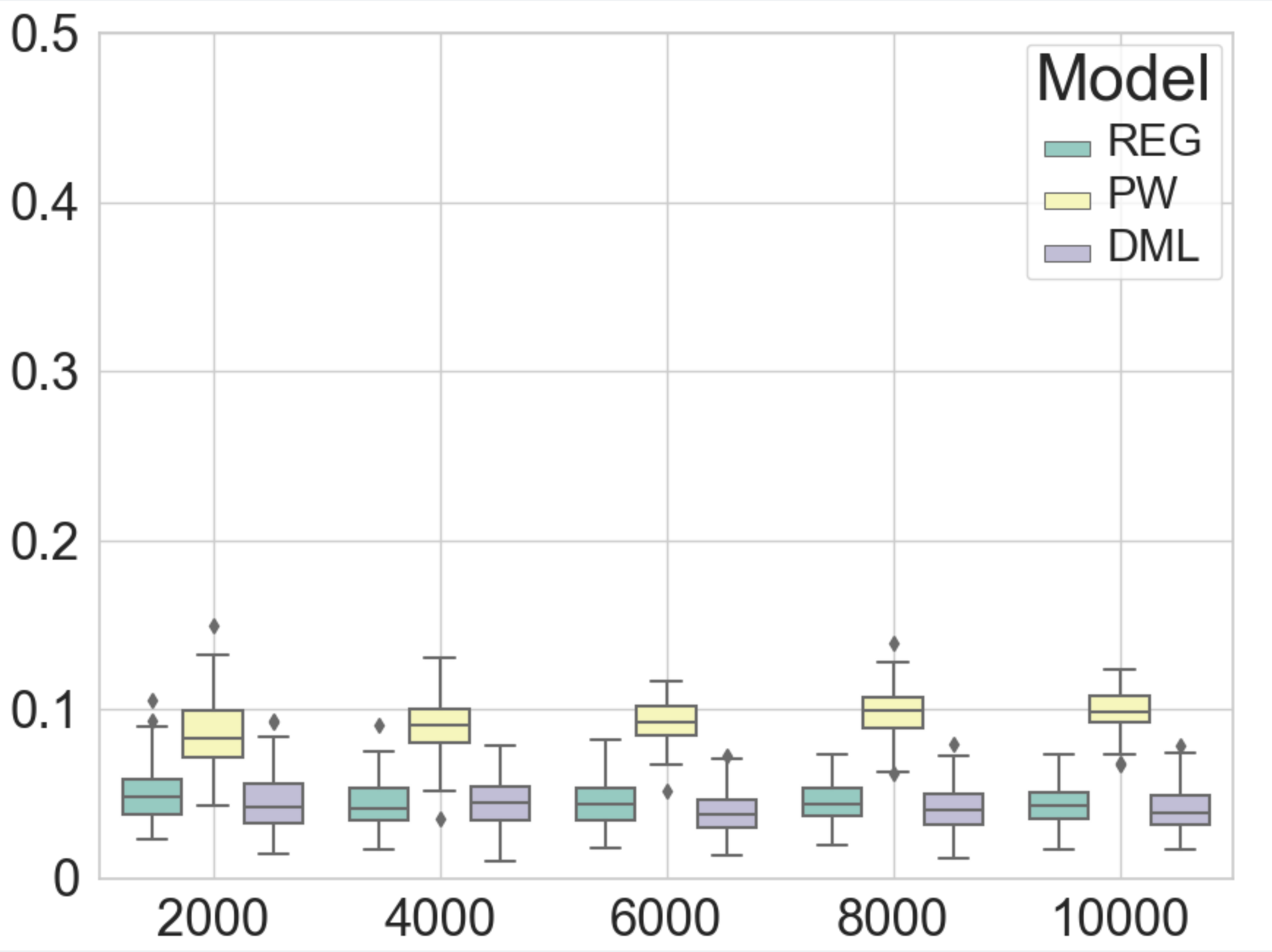
Scenario 3. π is wrongly estimated .

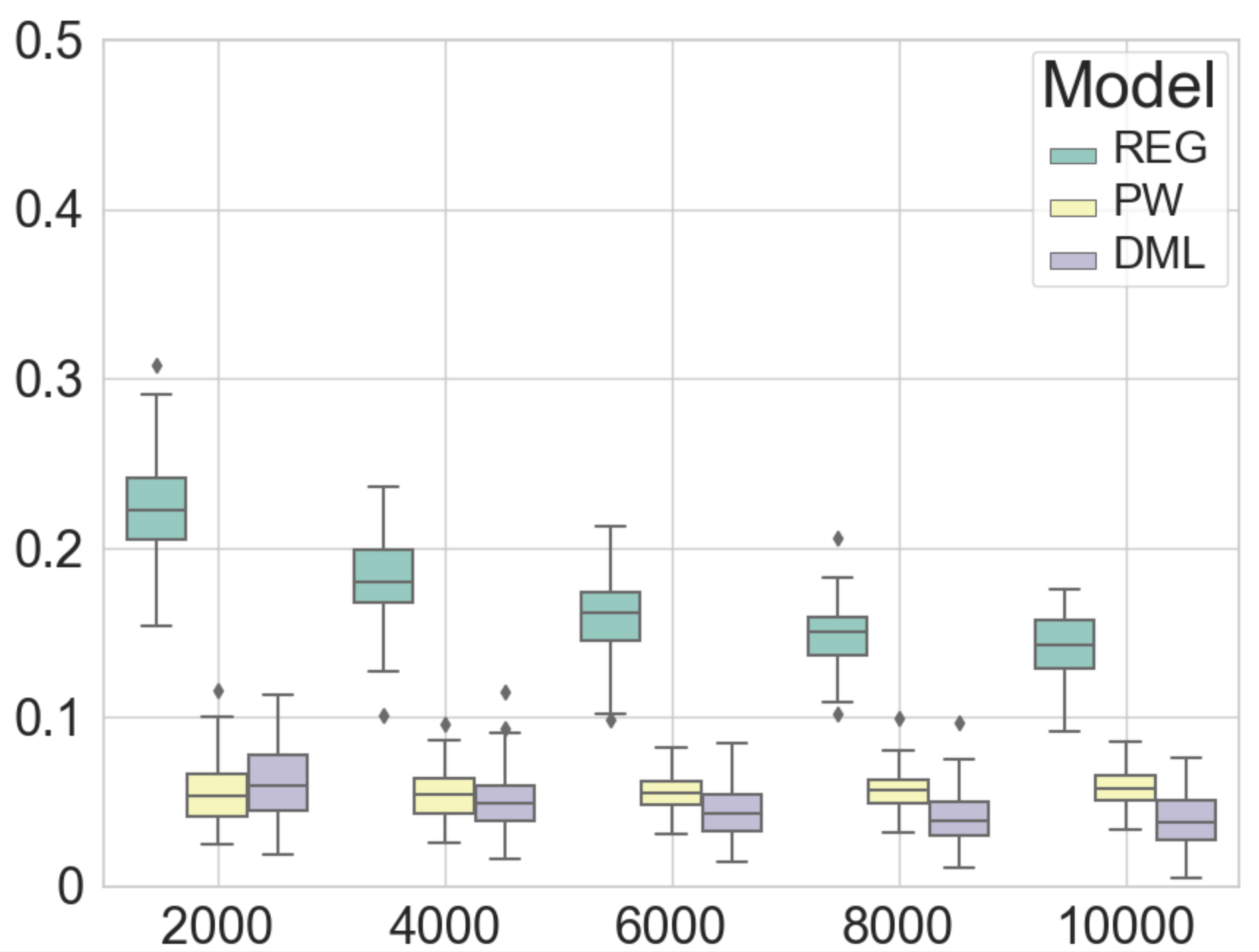
Scenario 4. μ is wrongly estimated .

Simulation Results

Scenario 1

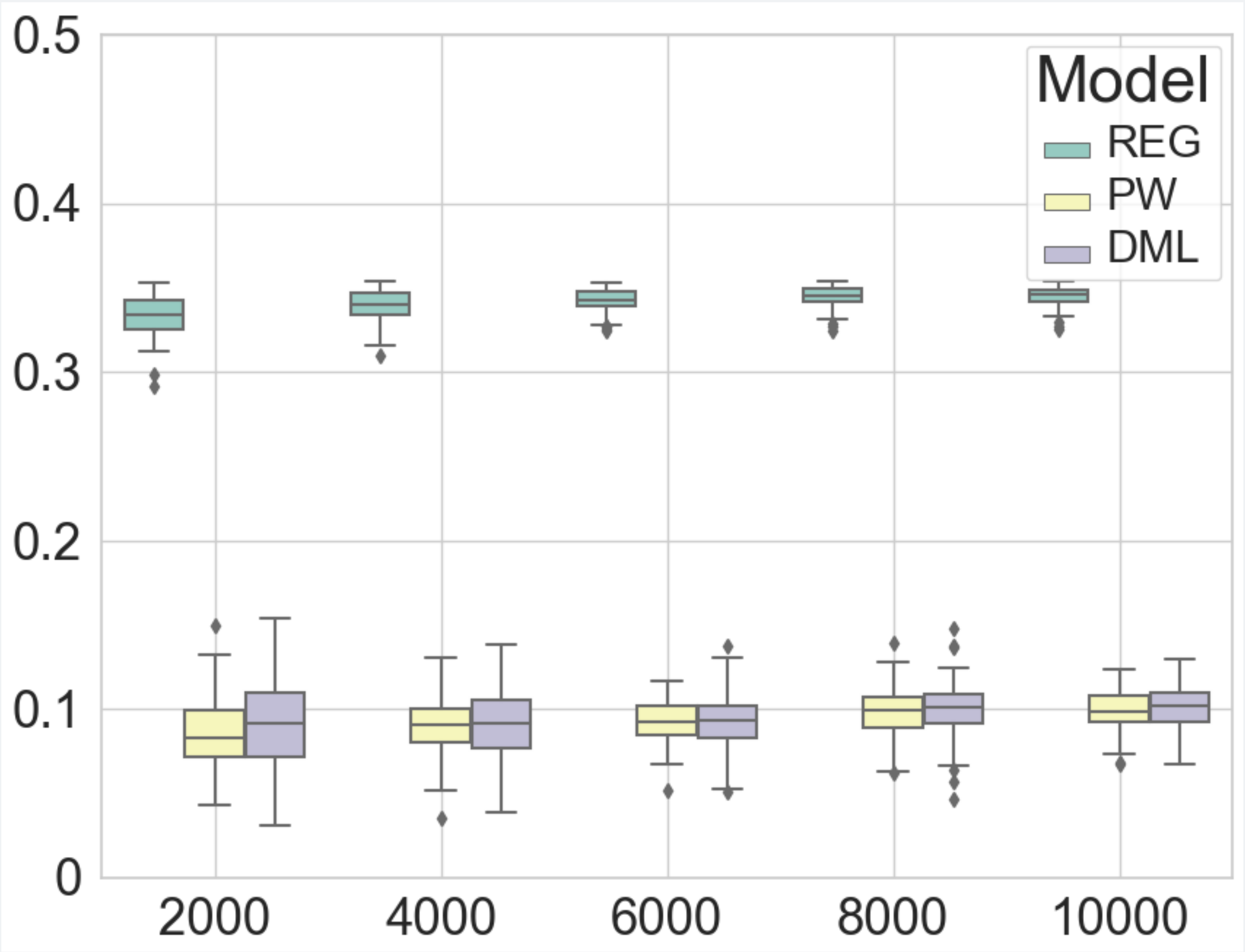


Scenario 2

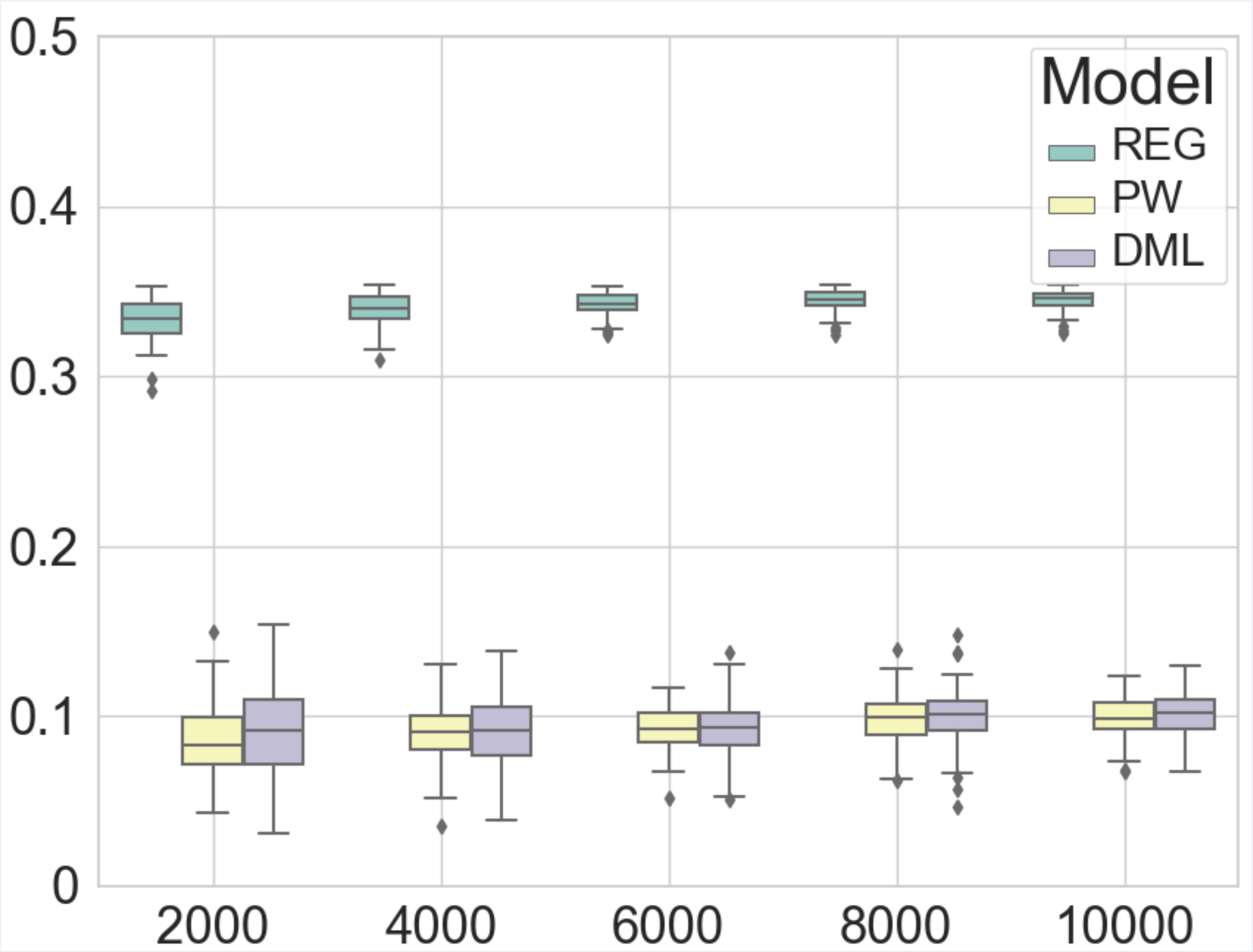


Simulation Results

Scenario 3



Scenario 4



Simulations: Project STAR

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3. We only focus on the student/teacher ratio for the pre-K students (X_1) and the third-grade students (X_2) as well as their testing scores at the third grade (Y).

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3. We only focus on the student/teacher ratio for the pre-K students (X_1) and the third-grade students (X_2) as well as their testing scores at the third grade (Y).

4. We created confounders to to ensure that the data aligns with our graphical settings.

Data Generating Process for STAR

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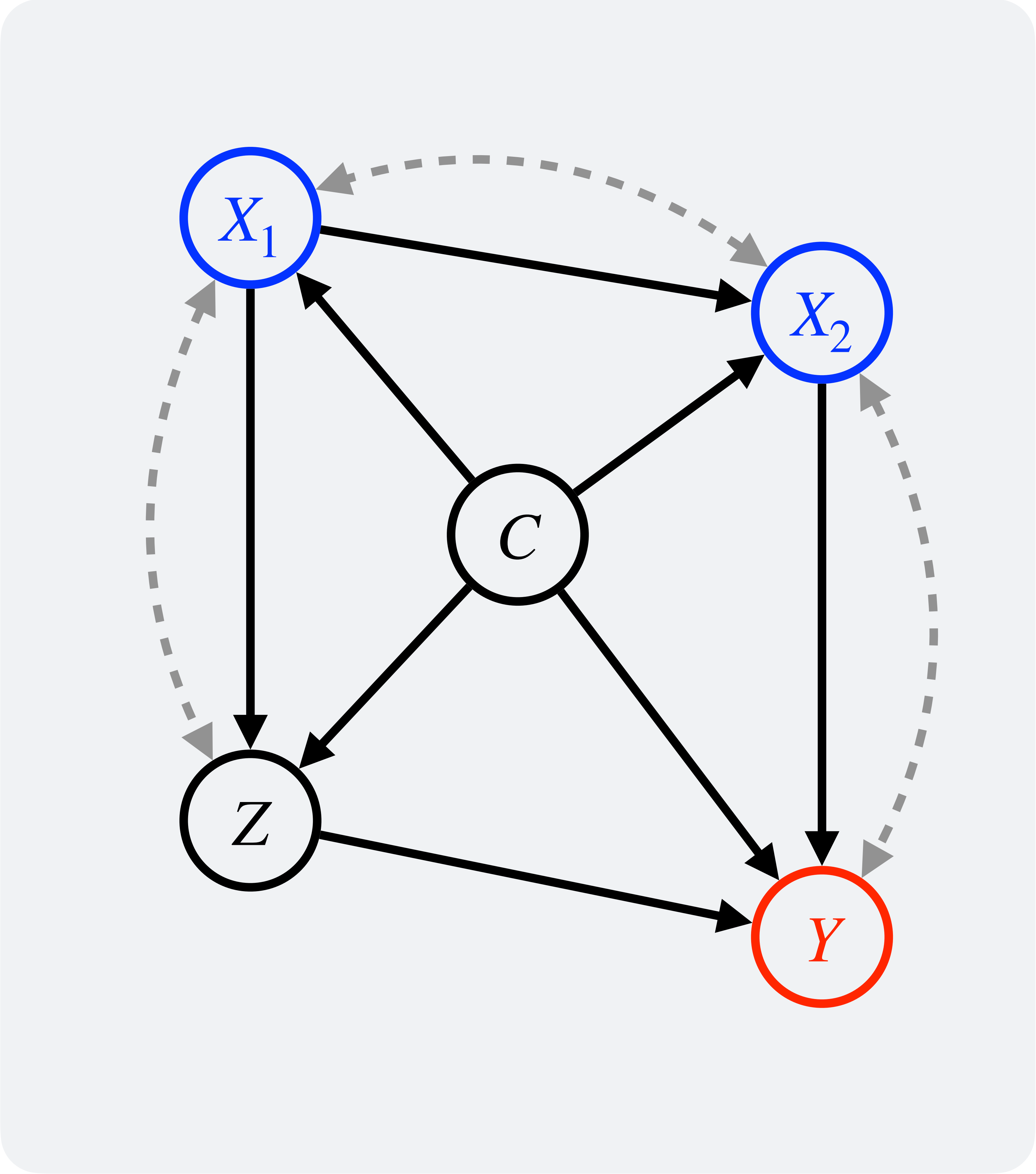

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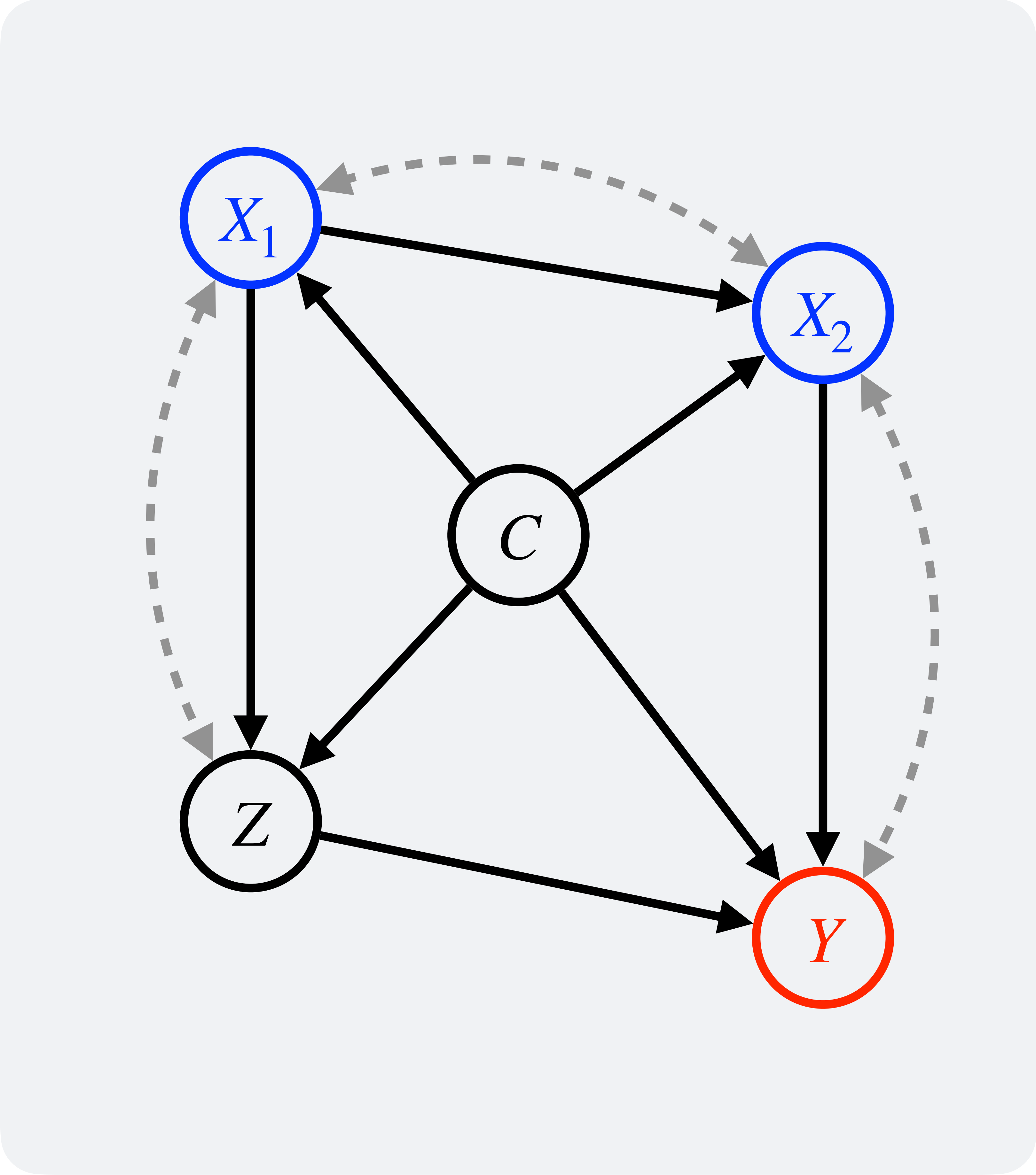
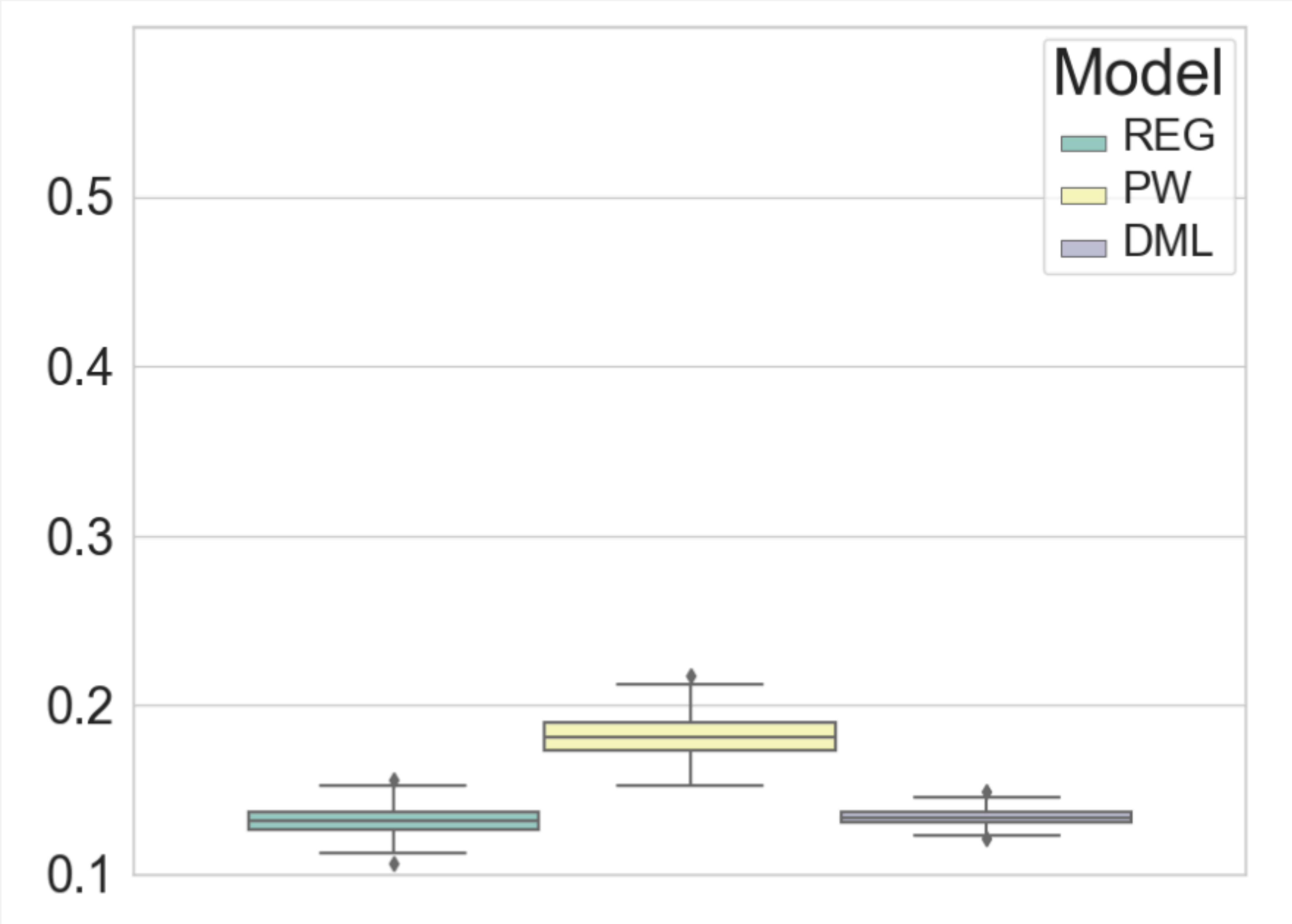


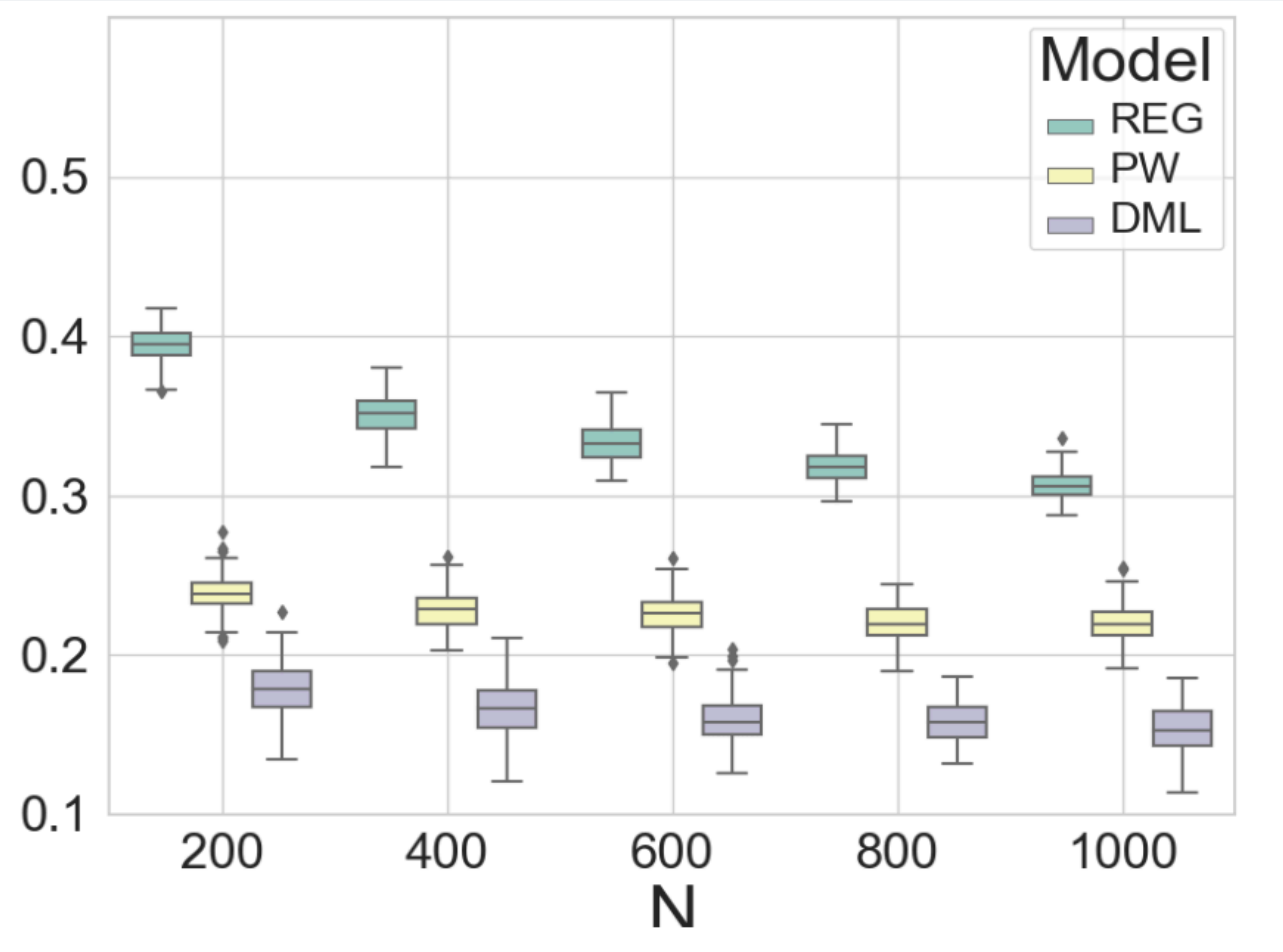
- X_1: The student/teacher ratio for the pre-K students.
- X_2: The student/teacher ratio for the 3rd grade students.
- Y: The testing score of the third grade students.
- C: Baseline covariates (socioeconomic factors, teacher's education, etc.)
- Z: The testing score of the pre-K students.

Simulation Results: STAR

Scenario 1

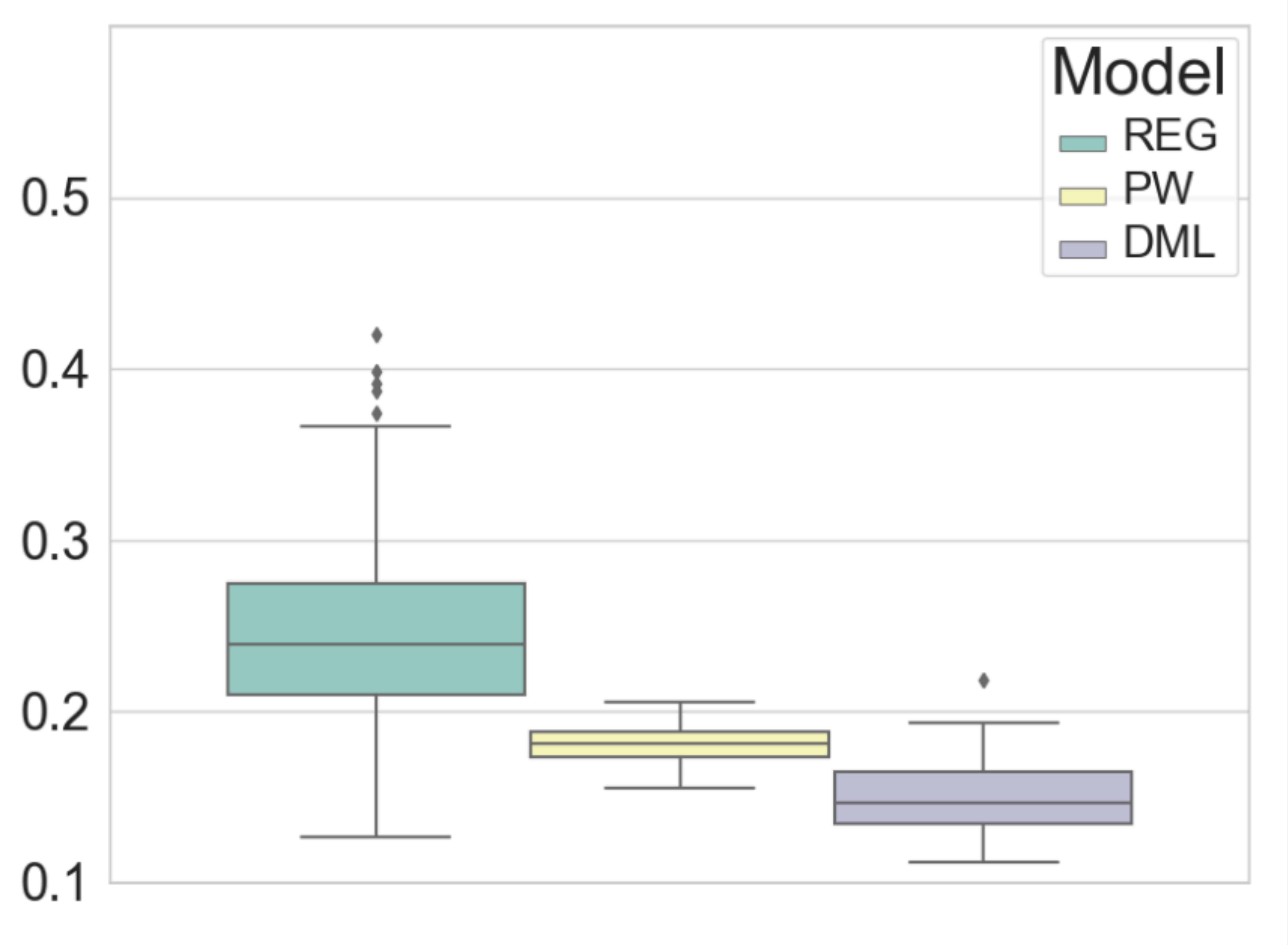


Scenario 2

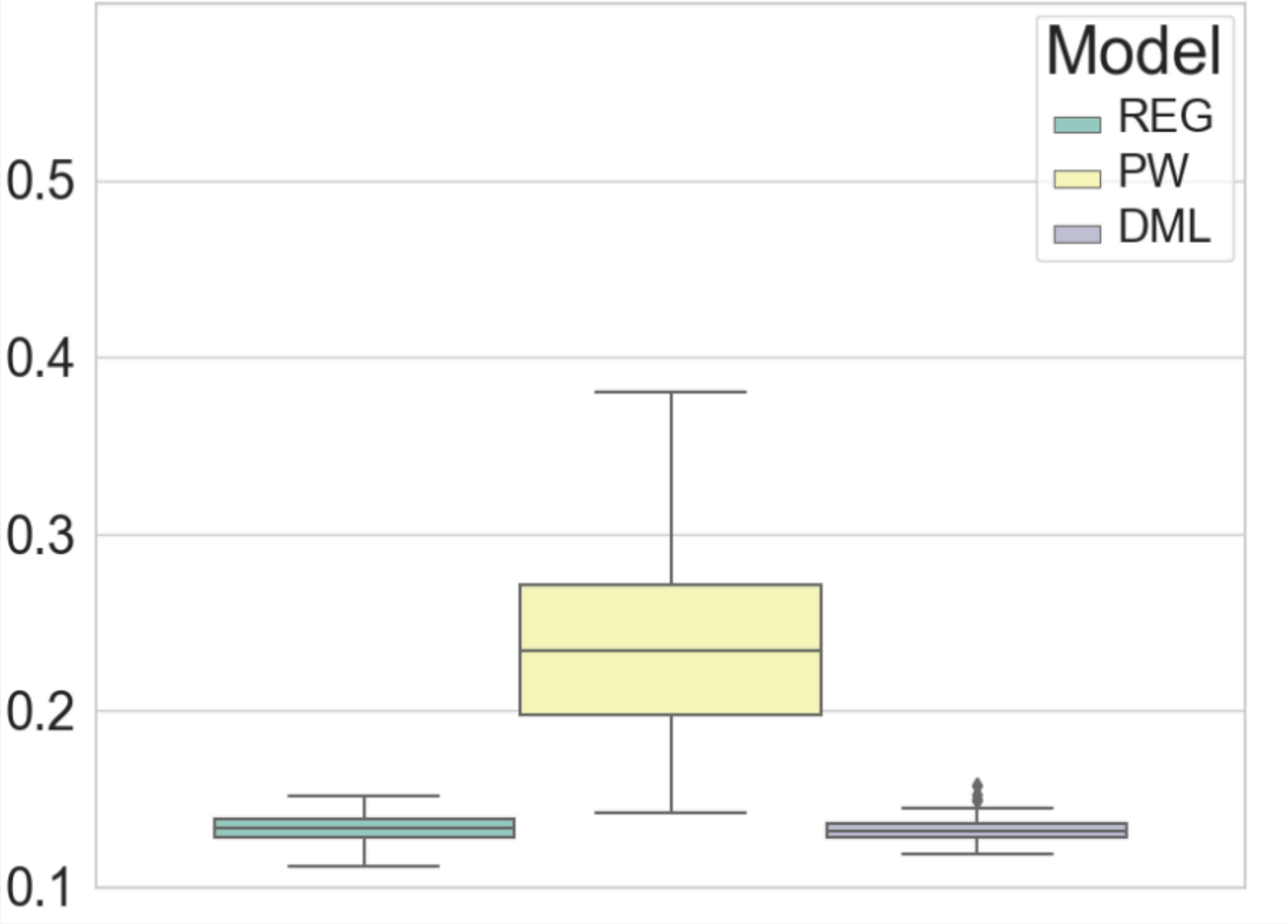


Simulation Results: STAR

Scenario 3



Scenario 4



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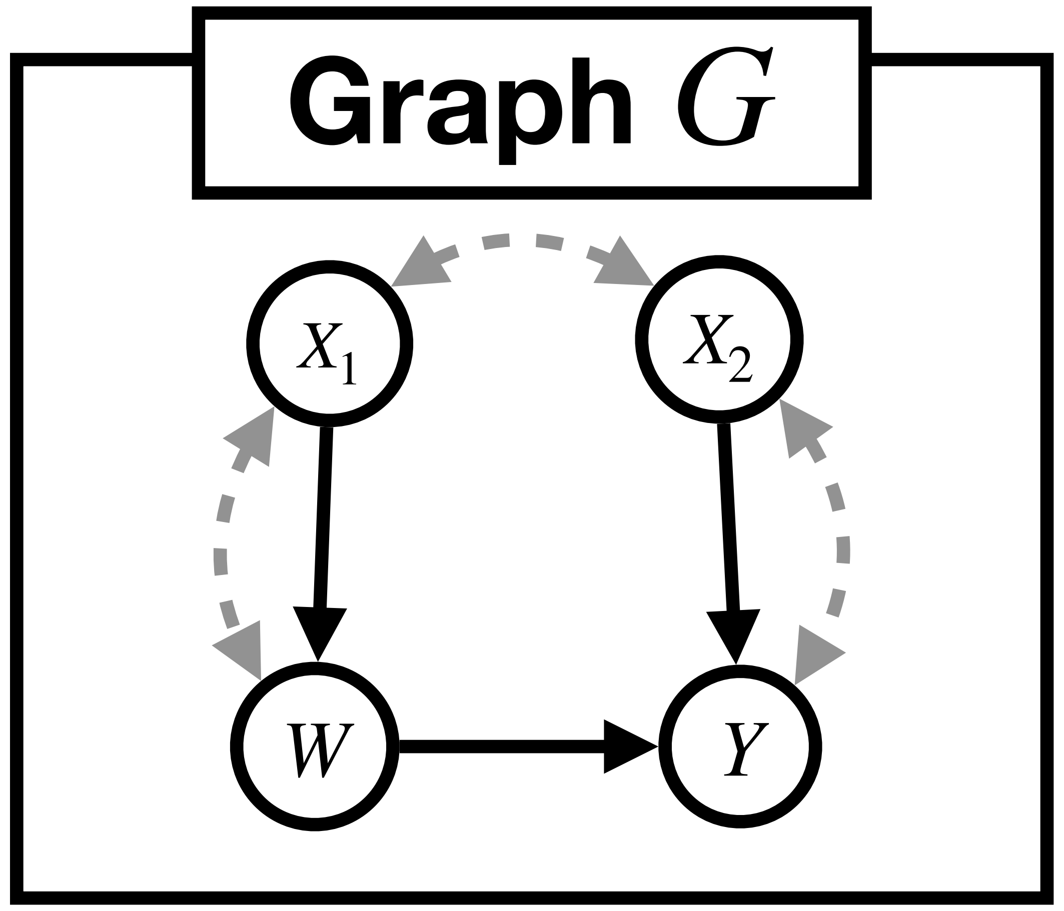
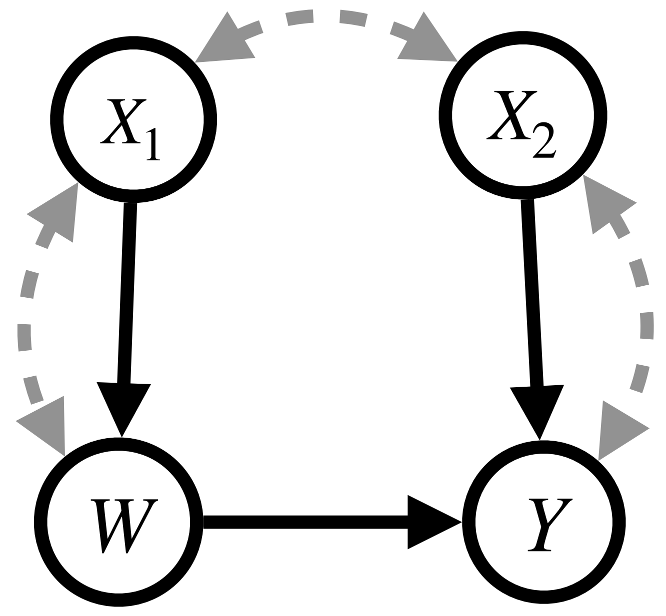
Multiple Treatment Interaction

Multiple Treatment Interaction



Multiple Treatment Interaction

Graph G



Dist. $\{P\}$

$\{P(\mathbf{V} \mid do(x_i)\}$
for $i = 1, \cdots, m$

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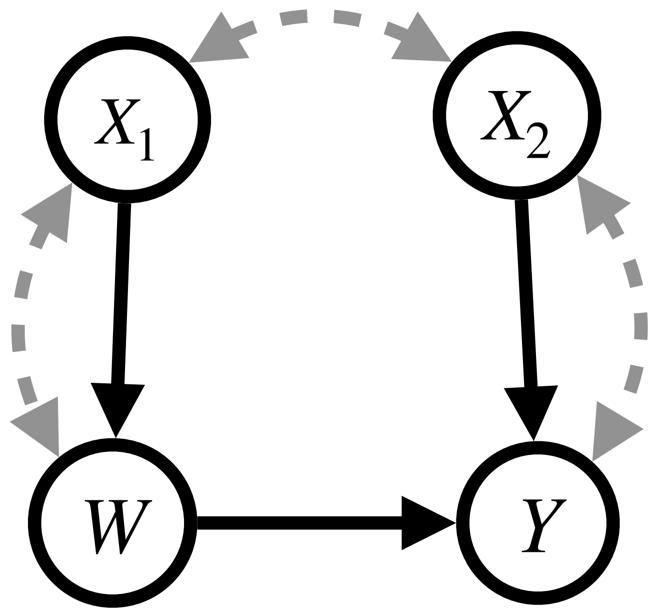

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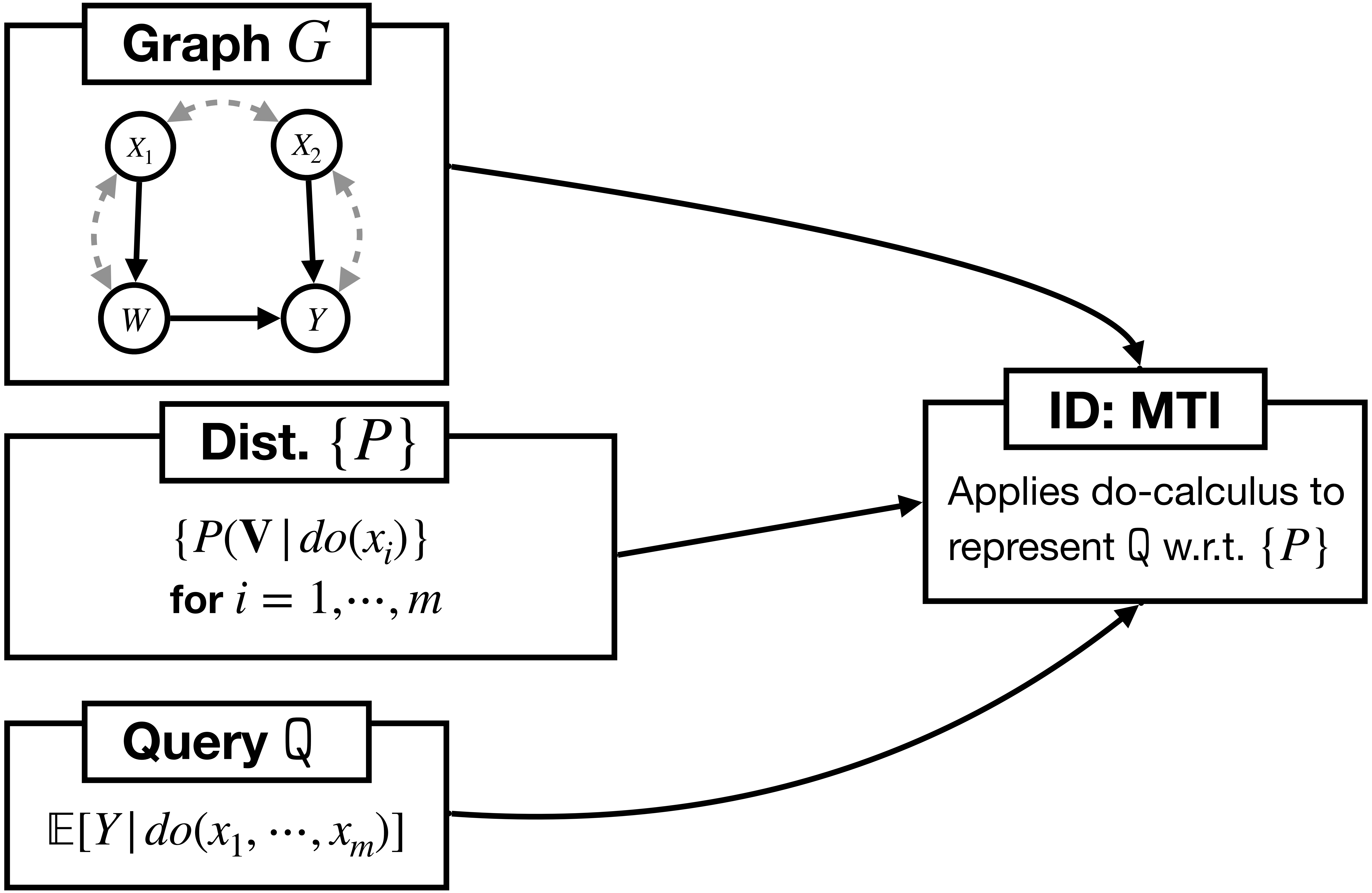
Query $\mathbb{Q}$

$\mathbb{E}[Y \mid do(x_1, \cdots, x_m)]$

Multiple Treatment Interaction



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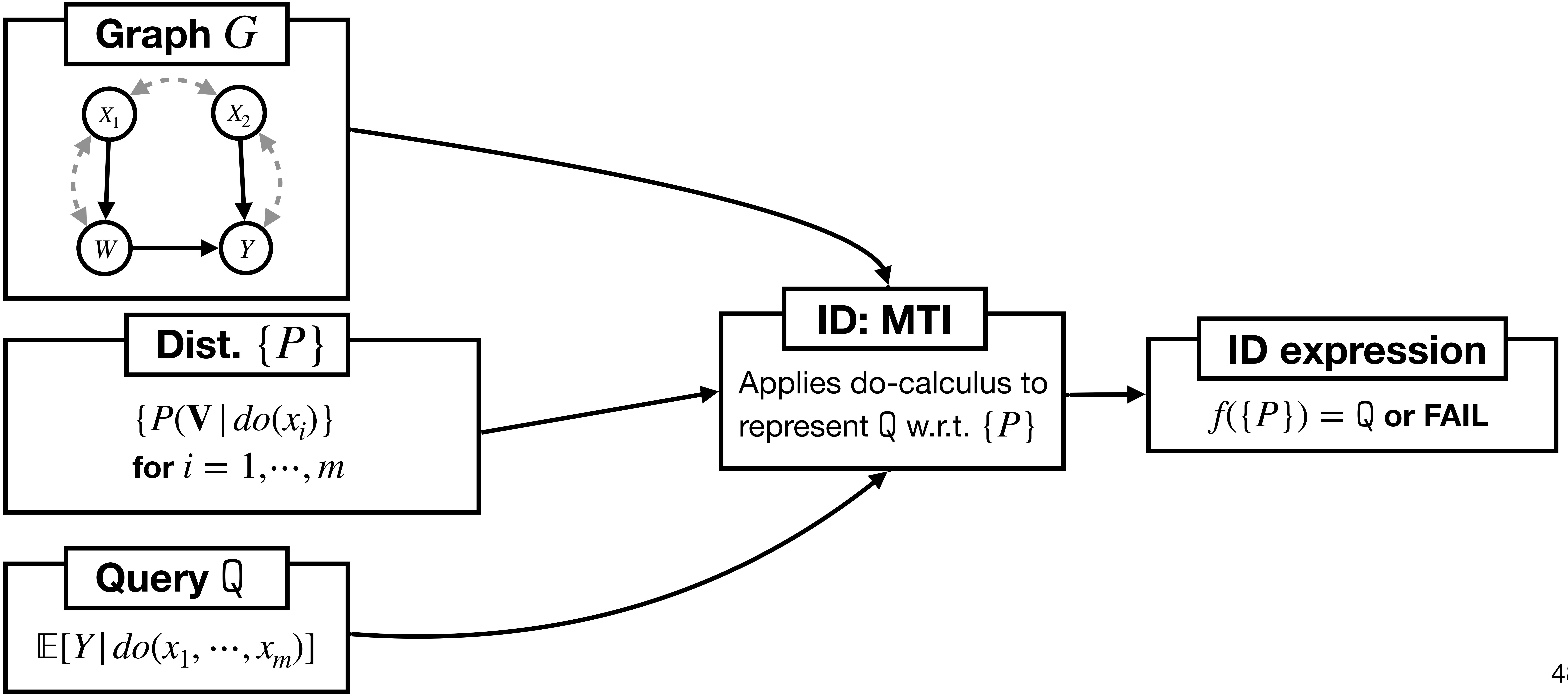
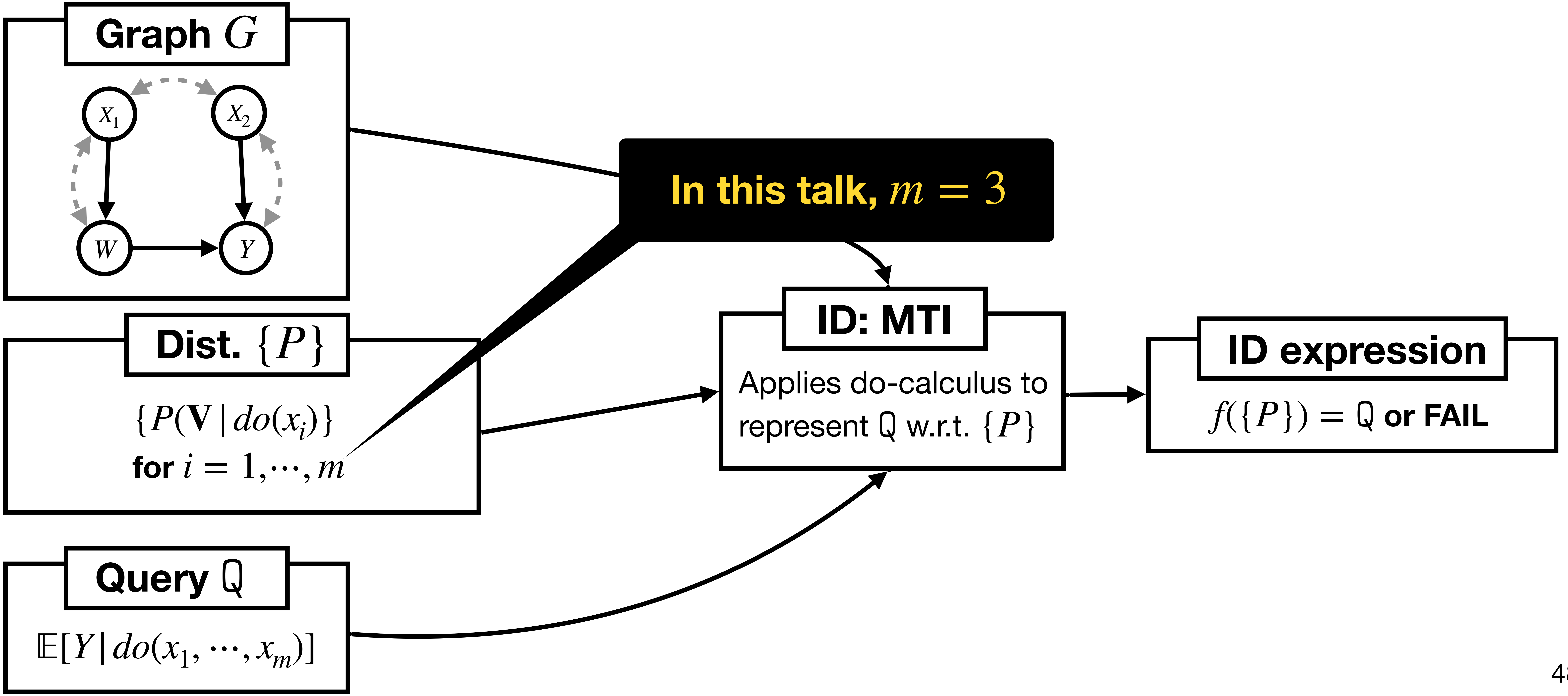


Multiple Treatment Interaction



Sufficient ID Condition: MTI

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Identification Condition for Multiple Treatment Interaction

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Identification Condition for Multiple Treatment Interaction

There is a set of variables W_1, W_2 s.t.

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3. There are no unmeasured confounders between (Y, X_2) given W_2, X_1, W_1 in the post-treatment distribution $X_3 = x_3$.

Sufficient ID Condition: MTI

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Equivalent Condition for Multiple Treatment Interaction

Sufficient ID Condition: MTI

Equivalent Condition for Multiple Treatment Interaction

There is a set of variables W_1, W_2 s.t.

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Multiple Treatment Interaction: Examples

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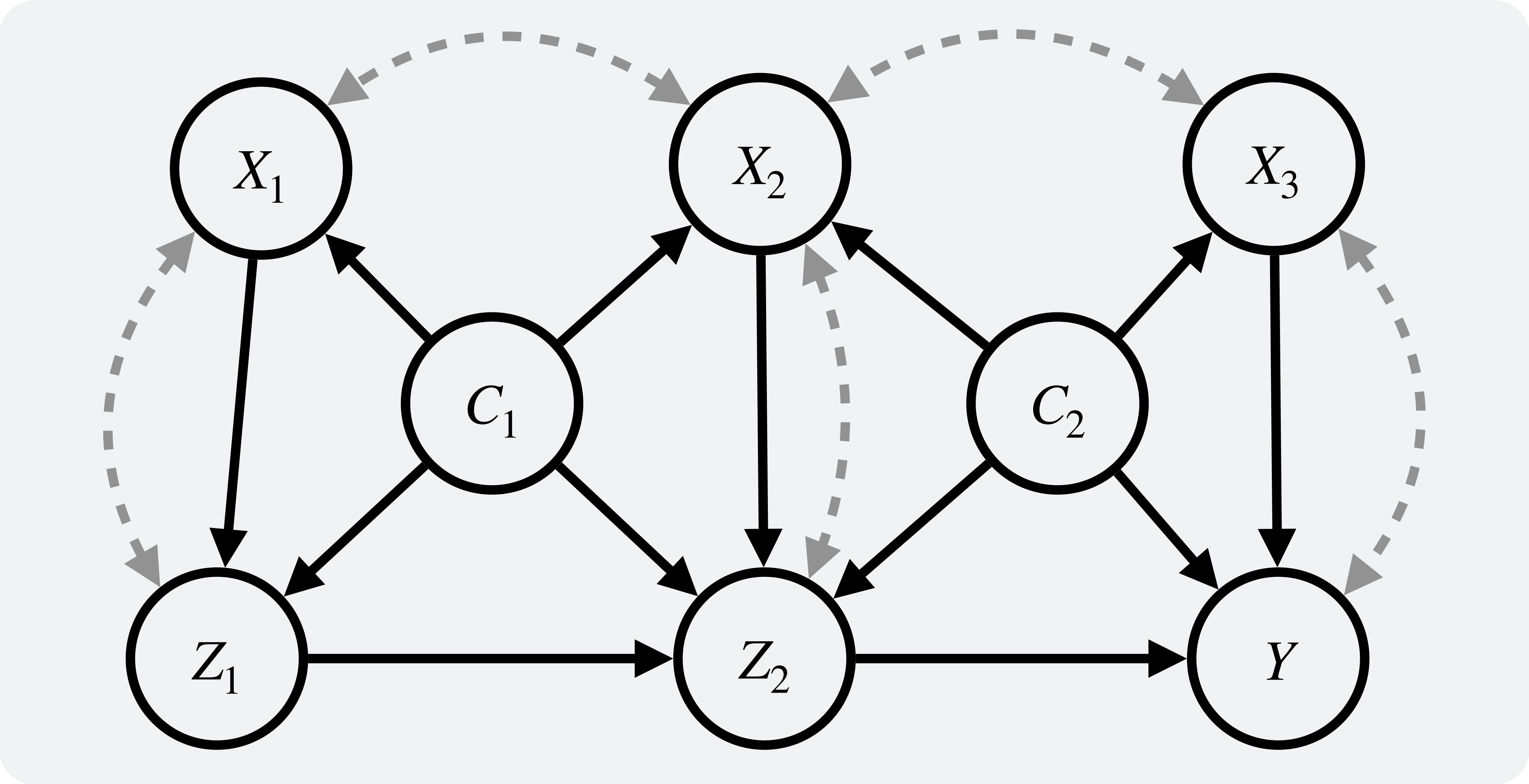

Multiple Treatment Interaction: Examples

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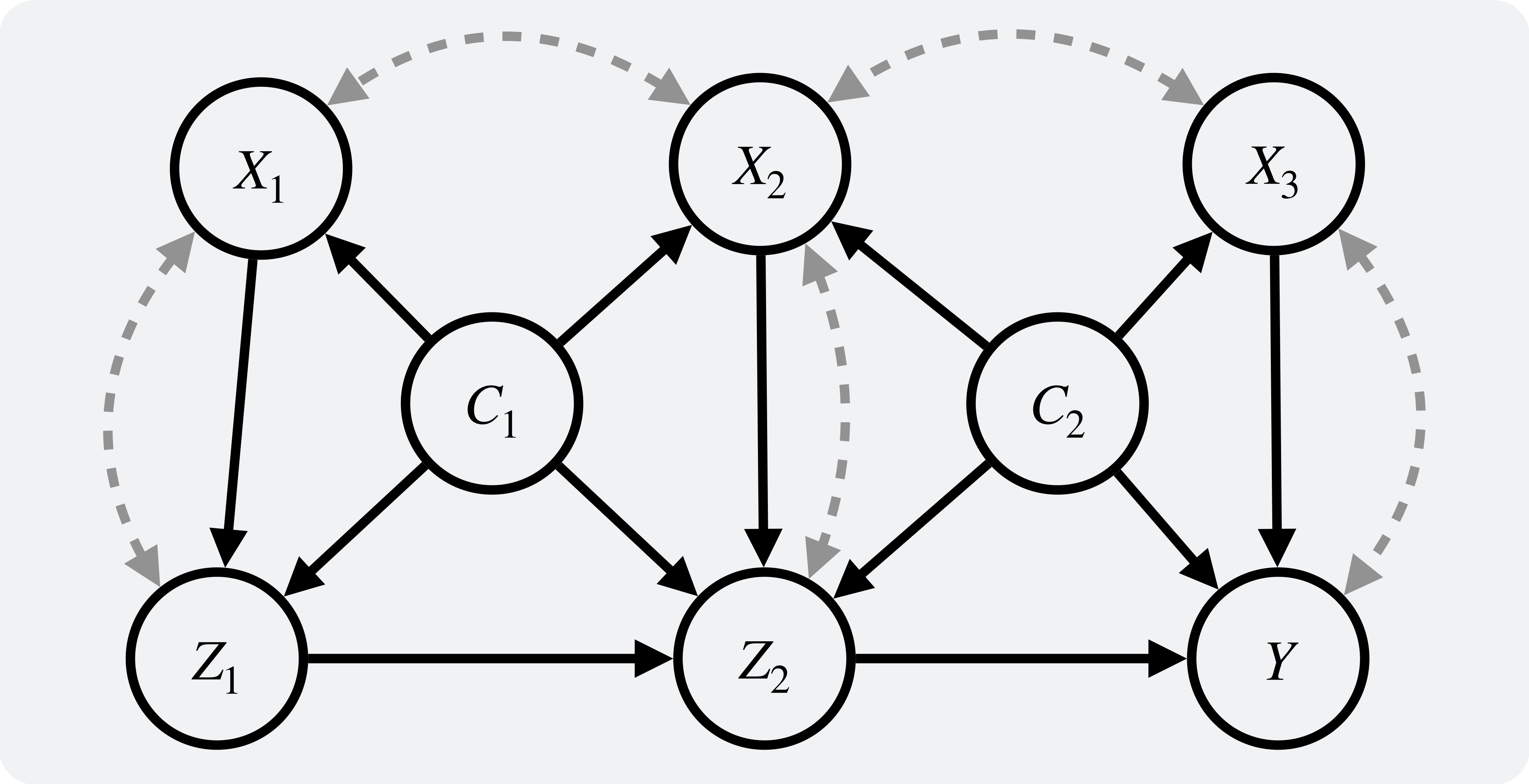

$W_1 = \{Z_1, C_1\}, W_2 = \{Z_2, C_2\}$

Multiple-Treatment Interaction

Identification Condition for Multiple Treatment Interaction

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Then, $\mathbb{E}[Y \mid do(x_1, x_2, x_3)]$ is identifiable from three experimental distributions $\{P(\cdot \mid do(x_1)), P(\cdot \mid do(x_2)), P(\cdot \mid do(x_3))\}$ and $P(\cdot \mid do(x_2))$ as follows:

$$\mathbb{E}[Y \mid do(\mathbf{x})] = \sum_{w_1, w_2 \in \mathscr{W}_1, \mathscr{W}_2} \mathbb{E}[Y \mid do(x_3), w_1, w_2, x_1, x_2] P(w_2 \mid do(x_2), w_1, x_1) P(w_1 \mid do(x_1))$$

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Joint Treatment Effect Estimation : Regression

$$\mathbb{E}[Y \mid do(\mathbf{x})] = \sum_{w_1, w_2 \in \mathscr{W}_1, \mathscr{W}_2} \mathbb{E}[Y \mid do(x_3), w_1, w_2, x_1, x_2] P(w_2 \mid do(x_2), w_1, x_1) P(w_1 \mid do(x_1))$$

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Regression-based Representation

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$$\mathbb{E}[Y|do(\mathbf{x})] = \sum_{w_1, w_2 \in \mathcal{W}_1, \mathcal{W}_2} \mathbb{E}[Y|do(x_3), w_1, w_2, x_1, x_2] P(w_2|do(x_2), w_1, x_1) P(w_1|do(x_1))$$

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Joint Treatment Effect Estimation : DR Estimand

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DR Representation

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DR Representation

$$\mathbb{E}[Y \,|\, do(\mathbf{x})] = \mathbb{E}[\pi_0^2 \pi_0^1 I_{x_1}(X_1) I_{x_2}(X_2)(Y - \mu_0^2(\mathbf{W}, \mathbf{X})) \,|\, do(x_3)]$$

$$+\mathbb{E}[\pi_0^1 I_{x_1}(X_1)(\mu_0^2(\mathbf{W}, X_1, x_2) - \mu_0^1(W_1, X_1)) \,|\, do(x_2)]$$

$$+\mathbb{E}[\mu_0^1(W_1, x_1) \,|\, do(x_1))]$$

Joint Treatment Effect Estimation : DR Estimand

DR Representation

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$$+\mathbb{E}[\pi_0^1 I_{x_1}(X_1)(\mu_0^2(\mathbf{W}, X_1, x_2) - \mu_0^1(W_1, X_1)) \mid do(x_2)]$$

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$$\mathbf{X} := (X_1, X_2)$$
$$\mathbf{W} := (W_1, W_2)$$

Asymptotic Error Analysis

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Assume

1. $\|\pi_k^2 - \pi_0^2\|_{P_3}\|\mu^2 - \mu_0^2\|_{P_3} = o_{P_3}(1)$ $\|\pi_k^1 - \pi_0^1\|_{P_2}\|\mu^1 - \mu_0^1\|_{P_2} = o_{P_2}(1), k \in \{a, b\}$; i.e., $\pi_k^2, \mu_k^2, \pi_k^1, \mu_k^1$ converges to the true parameters.

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1. $\|\pi_k^2 - \pi_0^2\|_{P_3}\|\mu^2 - \mu_0^2\|_{P_3} = o_{P_3}(1)$ $\|\pi_k^1 - \pi_0^1\|_{P_2}\|\mu^1 - \mu_0^1\|_{P_2} = o_{P_2}(1), k \in \{a, b\}$; i.e., $\pi_k^2, \mu_k^2, \pi_k^1, \mu_k^1$ converges to the true parameters.

2. $\|\pi_k^1\pi_k^2(Y - \mu_k^2) - \pi_0^1\pi_0^2(Y - \mu_0^2)\|_{P_3} = O_{P_3}(1)$; i.e., $\pi_k^1\pi_k^2(Y - \mu_k^2) - \pi_0^1\pi_0^2(Y - \mu_0^2)$ is bounded.

3. $\|\pi_k^1(\mu_k^2 - \mu_k^1) - \pi_0^1(\mu_k^2 - \mu_0^1)\|_{P_2} = O_{P_2}(1)$; i.e., $\pi_k^1(\mu_k^2 - \mu_k^1) - \pi_0^1(\mu_k^2 - \mu_0^1)$ is bounded.

Asymptotic Error Analysis

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$$T^{dml} - \mathbb{E}[Y \mid do(\mathbf{x})] = (R_1 + R_2 + R_3)$$

$$+ \sum_{k \in \{a,b\}} O_{P_3}\left(\|\pi_k^2 - \pi_0^2\|_{P_3} \|\mu_k^2 - \mu_0^2\|_{P_3} \right)$$

$$+ \sum_{k \in \{a,b\}} O_{P_2}\left(\|\pi_k^1 - \pi_0^1\|_{P_2} \|\mu_k^1 - \mu_0^1\|_{P_2} \right)$$

Asymptotic Error Analysis

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$$
\begin{aligned}
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\end{aligned}
$$

where R_i (for $i \in \{1,2,3\}$) is a random variable s.t. $\sqrt{n_i} R_i \xrightarrow{d} Z_i \sim \text{normal}(0, \sigma_i^2)$, where $\sigma_1^2 := \mathbb{V}_{P_1}[\mu_0^2]$, $\sigma_2^2 := \mathbb{V}_{P_2}[\pi_0^1(\mu^2 - \mu_0^1)]$, and $\sigma_3^2 := \mathbb{V}_{P_2}[\pi_0^1 \pi_0^2 (Y - \mu_0^2)]$.

Robustness of DML estimator

$$T^{dml} - \mathbb{E}[Y \mid do(\mathbf{x})] = (R_1 + R_2 + R_3) \; + \sum_{k \in \{a,b\}} O_{P_3}\left(\|\pi_k^2 - \pi_0^2\|_{P_3} \|\mu_k^2 - \mu_0^2\|_{P_3} \right) + \sum_{k \in \{a,b\}} O_{P_2}\left(\|\pi_k^1 - \pi_0^1\|_{P_2} \|\mu_k^1 - \mu_0^1\|_{P_2} \right)$$

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Doubly Robustness & Debiasedness

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Doubly Robustness & Debiasedness

1. **Doubly Robustness (DR):** T^{dml} converges to $\mathbb{E}[Y \mid do(\mathbf{x})]$ at $n^{-1/2}$-rate (where $n := \min(n_1, n_2, n_3)$) if $\pi_k^1 = \pi_0^1$ or $\mu_k^1 = \mu_0^1$, and $\pi_k^2 = \pi_0^2$ or $\mu_k^2 = \mu_0^2$.

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2. **Debiasedness (DB):** T^{dml} converges to $\mathbb{E}[Y \mid do(x_1, x_2)]$ at $n^{-1/2}$-rate if $\pi^2, \pi^1, \mu^2, \mu^1$ converges at least at $n^{-1/4}$ rate.

Efficiency of DML estimator

$$T^{dml} - \mathbb{E}[Y \mid do(\mathbf{x})] = (R_1 + R_2 + R_3) \ + \sum_{k \in \{a,b\}} O_{P_3}\left(\|\pi_k^2 - \pi_0^2\|_{P_3} \|\mu_k^2 - \mu_0^2\|_{P_3} \right) + \sum_{k \in \{a,b\}} O_{P_2}\left(\|\pi_k^1 - \pi_0^1\|_{P_2} \|\mu_k^1 - \mu_0^1\|_{P_2} \right)$$

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CAN and Efficiency

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1. **Consistency and Asymptotic Normality (CAN):** Under the {DR,DB} conditions, T^{dml} achieves consistency and asymptotic normality (CAN).

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CAN and Efficiency

1. **Consistency and Asymptotic Normality (CAN):** Under the {DR,DB} conditions, T^{dml} achieves consistency and asymptotic normality (CAN).

2. **Statistical Efficiency:** Under the {DR,DB} conditions, T^{dml} achieves the nonparametric efficiency bound (i.e., achieves the minimum variance).

Omitted Results

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* Z_i can be any set of variables including the empty set.
* Generalization of the case of $p = m$ and $Z_1 = X_1, \cdots, Z_p = X_m$.

Outline of this talk

1. Preliminary and Problem Setup
   (1) Structural Causal Model
2. Treatment-Treatment Interaction
   (1) Identification
   (2) Estimand
   (3) Estimation and Error Analysis
   (4) Simulation Results
3. Multiple-Treatment Interaction (+ Omitted Results)
4. **Future directions & Summary**

Future Research Direction 1

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Scenario: When identification expression is not a covariate adjustment…

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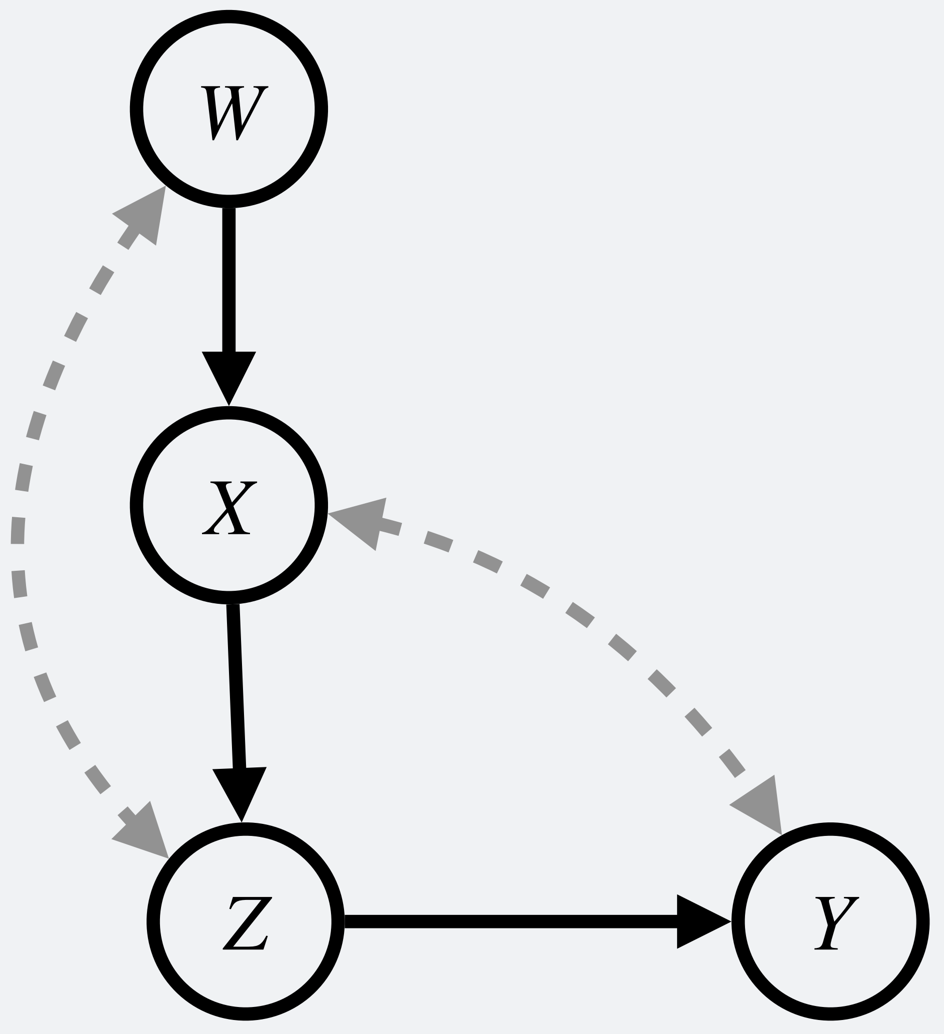

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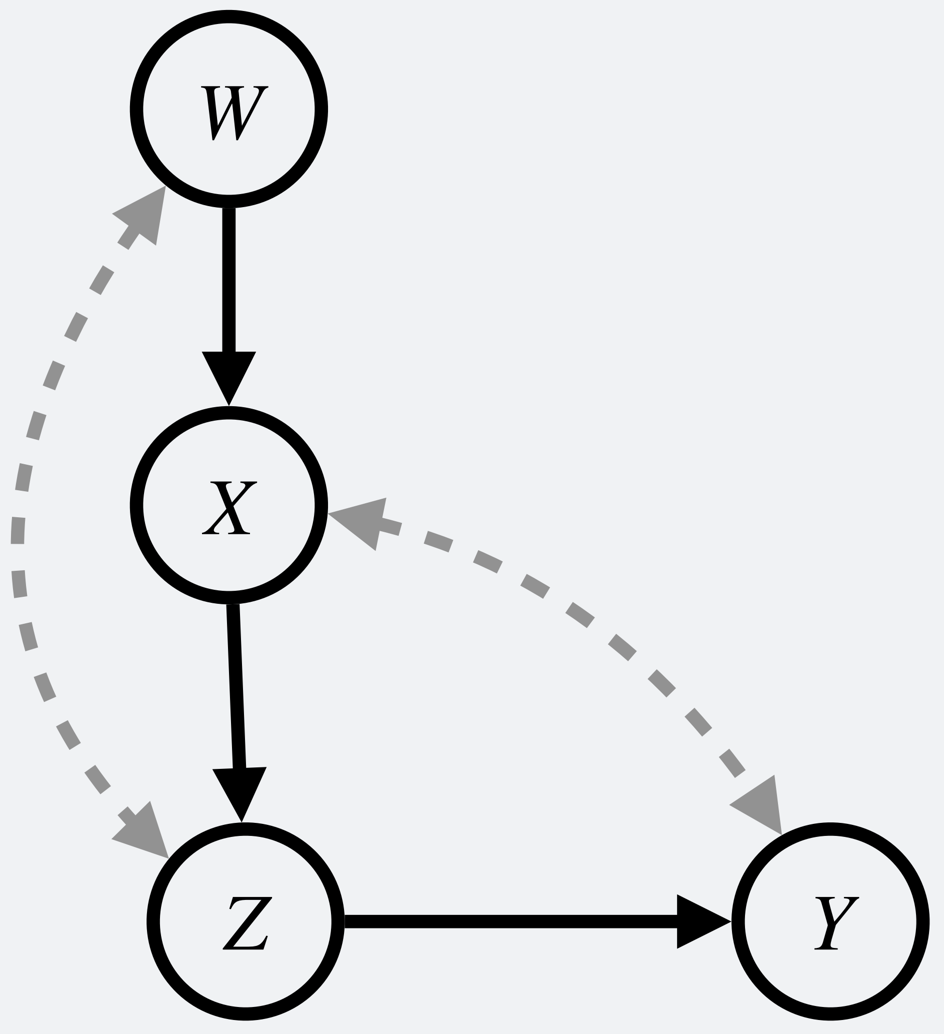

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Estimating Causal Effects Identifiable from a Combination of Observations and Experiments, NeurIPS-23

Future Research Direction 2

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$$\mathbb{P} := \{P(V \mid do(z_1), S = 1), \cdots, P(V \mid do(z_p), S = p)\}$$

$$\mathbb{Q} := \mathbb{E}[Y \mid do(x_1, \cdots, x_m), S = 0]$$

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1. Sufficient graphical criterion for identifying joint treatment effect $\mathbb{E}[Y \mid do(x_1, \cdots, x_m)]$ from marginal distributions $\mathbb{P} := \{P(V \mid do(x_1)), \cdots, P(V \mid do(x_m))\}$.
2. Double/debiased ML-based estimator, which exhibits fast convergence, doubly robustness, and efficiency.